ISSN 2229-5518

Theoretical Studies of Non-Newtonian Behaviors of Blood through the Stenosed Artery

Md.Mohiuddin[1*], A. Hoque[2], Abdul Malek[2], M. A. Halim[1]
Department of Natural Sciences, Daffodil International University, Bangladesh[1]
Department of Mathematics, Rajshahi University, Bangladesh[2]

Abstract

It is known that the stenosis in the artery is responsible for changing the nature of blood flow from its usual state. Therefore, the flow of blood through a cosine-shaped stenosed artery has been investigated, treating blood as Casson fluid. The effects of stenosis height, viscosity, slip velocity and yield stress on blood flow has been obtained. The results have been highlighted that the axial velocity, volumetric flow rate, pressure gradient and wall shear stress decrease with the increasing of viscosity and yield stress but these increases with the increasing of slip velocity. The results have been presented graphically for a better understanding by choosing the suitable parameters.

Index Terms— Steady blood flow, Casson fluid, viscosity, yield stress, stenosed artery, slip velocity.

1 Introduction

It has been suggested that the leading causes of deaths all over the world are cardiovascular diseases. Stenosis or atherosclerosis is one that is produced in the inner wall of the artery due to deposition of fatty substances, cholesterol, celluler waste products, calcium and fibrin. The presence of stenosis in the artery affects the hemodynamical behavior of blood flow. It has been observed that the blood behaves like a Newtonian fluid when it flows through larger diameter arteries at high shear rates and it behaves like a non-Newtonian fluid when it flows through smaller diameter arteries at low shear rates due to which it exhibits a certain yield stress for smooth flow.

Many researchers have proposed various mathematical models to explain the different features of the blood flow. **M. Gaur and M.K. Gupta** [1] studied a Casson fluid model for the steady flow through a stenosed blood vessel. They observed that the velocity, flow rate and pressure gradient increase with the increase in slip velocity and decrease with growth in yield stress. **P. Chaturani and D. Biswas** [2] made a theoretical study of blood flow through stenosed artery with slip velocity at wall. **S. Chakravarty** [3] studied the effects of stenosis on the flow behaviour of blood in an artery. **J. Venkatesan** ***et al*** [4] analyzed a Casson fluid model for blood rheology in stenosed narrow arteries. **A.K. Singh and D.P. Singh** [5] performed in the blood flow obeying Casson fluid equation through an artery with radially non-symmetric mild stenosis. They found that the resistance to flow increases as stenosis height, viscosity, yield stress and flux increase but decrease as stenosis shape increase. **S. Sapna** [6] made an analysis of non-Newtonian fluid in a stenosed artery. She observed that the resistance to flow, viscosity and wall shear stress increases with the stenosis size increases. **D.F. Young** [7] studied the effect of a time-dependent stenosis on flow through a tube. **B.K. Mishra** ***et al*** [8] presented the effect of shear stress, resistance and flow rate across mild stenosis on blood flow through blood vessels. **J.B. Shukla** [9] analysed effects of stenosis on non-Newtonian flow of the blood in an artery. **S. Kumar and C. Diwakar** [10] explained a mathematical model of power law fluid with an application of blood flow through an artery with stenosis. **B. Singh** ***et al*** [11] explored blood flow through an artery having radially non-symmetric mild stenosis by taking blood as a power law fluid.

In this work, we have investigated the effects of stenosis height, blood viscosity, yield stress and slip velocity on blood flow by selecting blood as Casson fluid.

2 Mathematical Formulation

Consider a laminar, steady and fully developed flow of a non-Newtonian incompressible viscous fluid (blood) through an artery with cosine shaped stenosis symmetrical about the axis but non-symmetrical with respect to radial coordinates. Here blood is taken as Casson fluid to describe non-Newtonian behavior of flow. A cylindrical polar coordinate system (r, θ, z) is used to analyze the blood flow, where r and z are the variables taken in the radial and axial directions, respectively, and θ is the azimuthal angle. The mathematical expression for the geometry of stenosis in artery can be written as:

ISSN 2229-5518

$$\frac{R}{R_0}=\begin{cases}1-\dfrac{\delta}{2R_0}\left[1+\cos\dfrac{2\pi}{l_0}\left(z-d-\dfrac{l_0}{2}\right)\right] & d\le z\le d+l_0\\ 1 & Otherwise\end{cases} \quad \ldots (1)$$

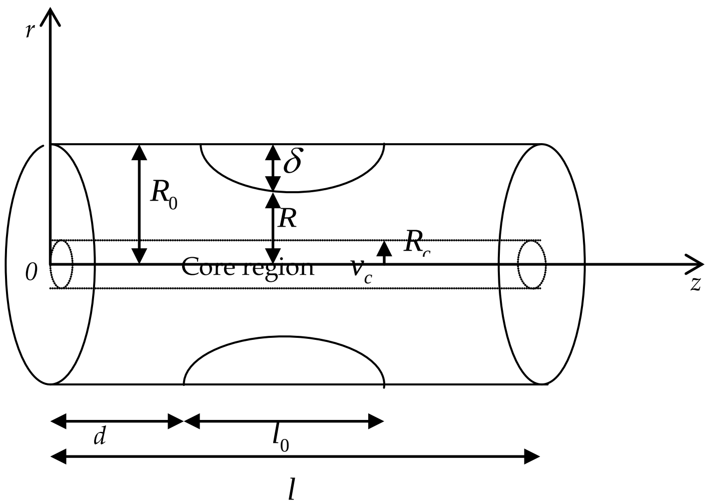


Figure 1: Geometry of stenosed artery.

The constitutive equation for Casson's fluid (blood) may be written as

$$-\frac{dv}{dr}=f(\tau)=\begin{cases}\dfrac{1}{\mu}\left(\sqrt{\tau}-\sqrt{\tau_0}\right)^2 & \tau\ge\tau_0\\ 0 & \tau\le\tau_0\end{cases} \quad \ldots (2)$$

The equation governing the flow of blood is taken in the form

$$-\frac{dp}{dz}=\frac{1}{r}\frac{d}{dr}(r\tau) \quad \ldots (3)$$

The boundary conditions are

$v=v_s$ at $r=R(z)$ (slip condition)

τ is finite at $r=0$ (regularity condition)

In core region

$v=v_c$ at $r=R_c$

where, v is the velocity component in the axial direction, p the pressure, ρ the density, R_0 the radius of the normal artery, R the radius of the stenosed artery, l_0 the length of the stenosis, l the length of the artery, τ the shear stress, τ_0 the yield stress, μ the viscosity, δ the maximum height of the stenosis and d the distance between equispaced points.

Integrating equation (3), we obtain

$$\tau=\frac{rP}{2} \quad \ldots (4)$$

where, $-\dfrac{dp}{dz}=P$ is pressure gradient.

The wall shear stress is

$$\tau_R=\frac{RP}{2} \quad \ldots (5)$$

From Eqs. (4) and (5) we have

$$\frac{\tau}{\tau_R}=\frac{r}{R} \quad \ldots (6)$$

The yield shear stress is

$$\tau_0=\frac{R_cP}{2}$$

From equation (2), we have

$$-\frac{dv}{dr}=\frac{1}{\mu}\left(\sqrt{\tau}-\sqrt{\tau_0}\right)^2$$

$$=\frac{1}{\mu}\left(\tau-2\sqrt{\tau\tau_0}+\tau_0\right)$$

$$=\frac{1}{\mu}\left(\frac{rP}{2}-2\sqrt{\frac{rP\tau_0}{2}}+\tau_0\right) \quad \ldots (8)$$

Integrating Eq. (8) with respect to r which gives

$$-v=\frac{1}{\mu}\left(\frac{r^2P}{4}-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}r^{3/2}+r\tau_0\right)+A \quad \ldots (9)$$

where, A is an integrating constant.

Using $v=v_s$ at $r=R(z)$ in Eq. (9) we obtain

$$A=-v_s-\frac{1}{\mu}\left(\frac{PR^2}{4}-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}R^{3/2}+R\tau_0\right)$$

From Eq. (9) we can write

$$-v=\frac{1}{\mu}\left(\frac{r^2P}{4}-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}r^{3/2}+r\tau_0\right)-v_s-\frac{1}{\mu}\left(\frac{PR^2}{4}-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}R^{3/2}+R\tau_0\right)$$

$$Or,\ v=v_s+\frac{1}{\mu}\left(\frac{PR^2}{4}-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}R^{3/2}+R\tau_0\right)-\frac{1}{\mu}\left(\frac{r^2P}{4}-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}r^{3/2}+r\tau_0\right)$$

$$=v_s+\frac{1}{\mu}\left\{\frac{P}{4}\left(R^2-r^2\right)-\frac{4}{3}\sqrt{\frac{P\tau_0}{2}}\left(R^{3/2}-r^{3/2}\right)+\tau_0(R-r)\right\} \quad \ldots (10)$$

$$=v_s+\frac{R}{2\mu}\left\{\frac{P}{2}\left(R-\frac{r^2}{R}\right)-\frac{8}{3}\sqrt{\frac{P\tau_0}{2}}\left(R^{1/2}-\frac{r^{3/2}}{R}\right)+2\tau_0\left(1-\frac{r}{R}\right)\right\}$$

$$=v_s+\frac{R}{2\mu}\left\{\left(\tau_R-\frac{\tau r}{R}\right)-\frac{8}{3}\sqrt{\tau_0}\left(\tau_R^{1/2}-\frac{\tau^{1/2}r}{R}\right)+2\tau_0\left(1-\frac{r}{R}\right)\right\}$$

$$=v_s+\frac{R}{2\mu}\left\{\left(\tau_R-\frac{\tau^2}{\tau_R}\right)-\frac{8}{3}\sqrt{\tau_0}\left(\tau_R^{1/2}-\frac{\tau^{3/2}}{\tau_R}\right)+2\tau_0\left(1-\frac{\tau}{\tau_R}\right)\right\}$$

$$or, v=v_s+\frac{R}{2\mu\tau_R}\left\{\left(\tau_R{}^2-\tau^2\right)-\frac{8}{3}\sqrt{\tau_0}\left(\tau_R{}^{3/2}-\tau^{3/2}\right)+2\tau_0\left(\tau_R-\tau\right)\right\} \quad \ldots (11)$$

This is axial velocity along $R_c \le r \le R$.

Using $v = v_c$ at $r = R_c$ in Eq. (11) we have

$$v_c = v_s + \frac{R}{2\mu\tau_R}\left\{\left(\tau_R^{\,2} - \tau_0^{\,2}\right) - \frac{8}{3}\sqrt{\tau_0}\left(\tau_R^{3/2} - \tau_0^{3/2}\right) + 2\tau_0\left(\tau_R - \tau_0\right)\right\}$$

$$= v_s + \frac{R}{2\mu\tau_R}\left(\tau_R^{\,2} - \tau_0^{\,2} - \frac{8}{3}\tau_0^{1/2}\tau_R^{3/2} + \frac{8}{3}\tau_0^{\,2} + 2\tau_0\tau_R - 2\tau_0^{\,2}\right)$$

$$v_c = v_s + \frac{R}{2\mu\tau_R}\left(\tau_R^{\,2} - \frac{8}{3}\tau_0^{1/2}\tau_R^{3/2} - \frac{1}{3}\tau_0^{\,2} + 2\tau_0\tau_R\right). \qquad \dots (12)$$

This is core velocity along $0 \le r \le R_c$.

The volumetric flow rate Q is,

$$Q = \int_0^R 2\pi r v\,dr$$

$$= \int_0^{R_c} 2\pi r v_c\,dr + \int_{R_c}^{R} 2\pi r v\,dr$$

$$= \pi R_c^{\,2} v_c + 2\pi\int_{R_c}^{R} r\left[v_s + \frac{1}{\mu}\left\{\frac{P}{4}\left(R^2 - r^2\right) - \frac{4}{3}\sqrt{\frac{P\tau_0}{2}}\left(R^{3/2} - r^{3/2}\right) + \tau_0\left(R - r\right)\right\}\right]dr$$

$$= \pi R_c^{\,2}\left[v_s + \frac{R}{2\mu\tau_R}\left(\tau_R^{\,2} - \frac{8}{3}\tau_0^{1/2}\tau_R^{3/2} - \frac{1}{3}\tau_0^{\,2} + 2\tau_0\tau_R\right)\right] + 2\pi\left[\frac{\left(R^2 - R_c^{\,2}\right)}{2}v_s\right.$$

$$+\frac{1}{\mu}\left\{\frac{P}{4}\left(\frac{1}{4}R^4 - \frac{R^2R_c^{\,2}}{2} + \frac{1}{4}R_c^{\,4}\right) - \frac{4}{3}\sqrt{\frac{P\tau_0}{2}}\left(\frac{3}{14}R^{7/2} - \frac{1}{2}R^{3/2}R_c^{\,2} + \frac{2}{7}R_c^{7/2}\right)\right.$$

$$\left.\left.+\tau_0\left(\frac{1}{6}R^3 - \frac{1}{2}RR_c^{\,2} + \frac{1}{3}R_c^{\,3}\right)\right\}\right]$$

$$= \pi R^2 v_s + \frac{\pi R R_c^{\,2}}{2\mu\tau_R}\left(\tau_R^{\,2} - \frac{8}{3}\tau_0^{1/2}\tau_R^{3/2} - \frac{1}{3}\tau_0^{\,2} + 2\tau_0\tau_R\right) + \frac{\pi R^3}{\mu}\left\{\left(\frac{1}{4}\tau_R - \frac{1}{2}\frac{\tau_0^{\,2}}{\tau_R} + \frac{1}{4}\frac{\tau_0^{\,4}}{\tau_R^{\,3}}\right)\right.$$

$$\left.-\frac{4}{7}\tau_0^{1/2}\tau_R^{1/2} + \frac{4}{3}\tau_0\frac{\tau_0^{5/2}}{\tau_R^{3/2}} - \frac{16}{21}\frac{\tau_0^{\,4}}{\tau_R^{\,3}} + \tau_0\left(\frac{1}{3} - \frac{\tau_0^{\,2}}{\tau_R^{\,2}} + \frac{2}{3}\frac{\tau_0^{\,3}}{\tau_R^{\,3}}\right)\right\}$$

$$= \pi R^2 v_s + \frac{\pi R^3}{\mu}\left(\frac{1}{2}\frac{\tau_0^{\,2}}{\tau_R} - \frac{4}{3}\frac{\tau_0^{5/2}}{\tau_R^{3/2}} - \frac{1}{6}\frac{\tau_0^{\,4}}{\tau_R^{\,3}} + \frac{\tau_0^{\,3}}{\tau_R^{\,2}} + \frac{1}{4}\tau_R - \frac{1}{2}\frac{\tau_0^{\,2}}{\tau_R} + \frac{1}{4}\frac{\tau_0^{\,4}}{\tau_R^{\,3}} - \frac{4}{7}\tau_0^{1/2}\tau_R^{1/2}\right.$$

$$\left.+\frac{4}{3}\frac{\tau_0^{5/2}}{\tau_R^{3/2}} - \frac{16}{21}\frac{\tau_0^{\,4}}{\tau_R^{\,3}} + \frac{1}{3}\tau_0 - \frac{\tau_0^{\,3}}{\tau_R^{\,2}} + \frac{2}{3}\frac{\tau_0^{\,4}}{\tau_R^{\,3}}\right)$$

$$= \pi R^2 v_s + \frac{\pi R^3}{\mu}\left(\frac{1}{4}\tau_R - \frac{1}{84}\frac{\tau_0^{\,4}}{\tau_R^{\,3}} - \frac{4}{7}\tau_0^{1/2}\tau_R^{1/2} + \frac{1}{3}\tau_0\right)$$

$$= \pi R^2 v_s + \frac{\pi R^3\tau_R}{\mu}\left(\frac{1}{4} - \frac{1}{84}\frac{\tau_0^{\,4}}{\tau_R^{\,4}} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}} + \frac{1}{3}\frac{\tau_0}{\tau_R}\right)$$

since $\tau_0/\tau_R \ll 1$ so the term $\frac{1}{84}\frac{\tau_0^{\,4}}{\tau_R^{\,4}}$ is negligible and replacing $1/3$ by $16/49$ we obtain

$$Q = \pi R^2 v_s + \frac{\pi R^3\tau_R}{\mu}\left(\frac{1}{4} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}} + \frac{16}{49}\frac{\tau_0}{\tau_R}\right) \qquad \dots (13)$$

$$Q = \pi R^2 v_s + \frac{\pi R^3\tau_R}{\mu}\left(\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}}\right)^2$$

or, $$\left(\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}}\right)^2 = \frac{\mu Q - \pi\mu R^2 v_s}{\pi R^3\tau_R}$$

or, $$\left(\frac{1}{2} - \frac{4}{7}\sqrt{\frac{\tau_0}{\tau_R}}\right) = \sqrt{\frac{\mu Q - \pi\mu R^2 v_s}{\pi R^3\tau_R}}.$$

Multiplying by $2\sqrt{\tau_R}$ we obtain

$$\sqrt{\tau_R} - \frac{8}{7}\sqrt{\tau_0} = 2\sqrt{\frac{\mu Q - \pi\mu R^2 v_s}{\pi R^3}}$$

or, $$\sqrt{\tau_R} = 2\sqrt{\frac{\mu Q - \pi\mu R^2 v_s}{\pi R^3}} + \frac{8}{7}\sqrt{\tau_0}$$

or, $$\tau_R = \left(2\sqrt{\frac{\mu Q - \pi\mu R^2 v_s}{\pi R^3}} + \frac{8}{7}\sqrt{\tau_0}\right)^2$$

... (14)

From Eqs. (5) and (14) we have

$$\frac{dp}{dz} = -\frac{2}{R}\left[\frac{8}{7}\sqrt{\tau_0} + 2\sqrt{\frac{\mu Q - \pi\mu v_s R^2}{\pi R^3}}\right]^2. \qquad \dots (15)$$

Integrating Eq. (15) with respect to z using the conditions $p = p_1$ at $z = 0$ and $p = p_2$ at $z = l$ we obtain

$$p_1 - p_2 = \frac{128\tau_0}{49R_0}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz + \frac{64}{7}\int_0^l\sqrt{\frac{\mu Q\tau_0 - \pi\mu v_s\tau_0 R^2}{\pi R^5}}dz + 8\int_0^l\left(\frac{\mu Q - \pi\mu v_s R^2}{\pi R^4}\right)dz$$

or, $$\frac{\Delta p}{Q} = \frac{128\tau_0}{49R_0 Q}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz + \frac{64}{7}\int_0^l\sqrt{\frac{\mu\tau_0}{\pi Q R^5} - \frac{\mu v_s\tau_0}{Q^2R^3}}dz + \frac{8\mu}{\pi R_0^{\,4}}\int_0^l\left(\frac{R}{R_0}\right)^{-4}dz$$

$$-\frac{8\mu v_s}{QR_0^{\,2}}\int_0^l\left(\frac{R}{R_0}\right)^{-2}dz \qquad \dots (16)$$

The resistance of flow is defined by

$$\lambda = \frac{\Delta p}{Q} \qquad \dots (17)$$

From Eqs. (16) and (17) we obtain

$$\lambda = \frac{128\tau_0}{49R_0Q}\int_0^l\left(\frac{R}{R_0}\right)^{-1}dz + \frac{64}{7}\int_0^l\sqrt{\frac{\mu\tau_0}{\pi QR_0^{\,5}}\left(\frac{R}{R_0}\right)^{-5} - \frac{\mu v_s\tau_0}{Q^2R_0^{\,3}}\left(\frac{R}{R_0}\right)^{-3}}dz + \frac{8\mu}{\pi R_0^{\,4}}\int_0^l\left(\frac{R}{R_0}\right)^{-4}dz$$

$$-\frac{8\mu v_s}{QR_0^{\,2}}\int_0^l\left(\frac{R}{R_0}\right)^{-2}dz$$

$$= \frac{128\tau_0}{49R_0Q}\left[\int_0^d\left(\frac{R}{R_0}\right)^{-1}dz + \int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-1}dz + \int_{d+l_0}^{l}\left(\frac{R}{R_0}\right)^{-1}dz\right]$$

$$+\frac{64}{7}\left[\int_0^d\sqrt{\frac{\mu\tau_0}{\pi QR_0^{\,5}}\left(\frac{R}{R_0}\right)^{-5} - \frac{\mu v_s\tau_0}{Q^2R_0^{\,3}}\left(\frac{R}{R_0}\right)^{-3}}dz\right.$$

$$+\int_d^{d+l_0}\sqrt{\frac{\mu\tau_0}{\pi QR_0^{\,5}}\left(\frac{R}{R_0}\right)^{-5} - \frac{\mu v_s\tau_0}{Q^2R_0^{\,3}}\left(\frac{R}{R_0}\right)^{-3}}dz$$

$$\left.+\int_{d+l_0}^{l}\sqrt{\frac{\mu\tau_0}{\pi QR_0^{\,5}}\left(\frac{R}{R_0}\right)^{-5} - \frac{\mu v_s\tau_0}{Q^2R_0^{\,3}}\left(\frac{R}{R_0}\right)^{-3}}dz\right]$$

$$+\frac{8\mu}{\pi R_0^{\,4}}\left[\int_0^d\left(\frac{R}{R_0}\right)^{-4}dz + \int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-4}dz\right.$$

$$\left.+\int_{d+l_0}^{l}\left(\frac{R}{R_0}\right)^{-4}dz\right] - \frac{8\mu v_s}{QR_0^{\,2}}\left[\int_0^d\left(\frac{R}{R_0}\right)^{-2}dz + \int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-2}dz\right.$$

ISSN 2229-5518

$$+\int_{d+l_0}^{l}\left(\frac{R}{R_0}\right)^{-2}dz\Bigg]. \quad \text{... (18)}$$

Suppose, $a_1=\frac{128\tau_0}{49R_0Q}$, $a_2=\frac{64}{7}\sqrt{\frac{\mu Q\tau_0-\mu\pi v_s\tau_0R_0^2}{\pi Q^2R_0^5}}$, $a_3=\frac{8\mu}{\pi R_0^4}$, $a_4=\frac{8\mu v_s}{QR_0^2}$

$$I_1=\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-1}dz,$$

$$I_2=\frac{64}{7}\int_d^{d+l_0}\sqrt{\frac{\mu\tau_0}{\pi QR_0^5}\left(\frac{R}{R_0}\right)^{-5}-\frac{\mu v_s\tau_0}{Q^2R_0^3}\left(\frac{R}{R_0}\right)^{-3}}dz$$

$$I_3=\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-4}dz,\quad I_4=\int_d^{d+l_0}\left(\frac{R}{R_0}\right)^{-2}dz.$$

Using these in Eq. (18) we obtain

$$\lambda=a_1(l-l_0+I_1)+(a_2l-a_2l_0+I_2)+a_3(l-l_0+I_3)-a_4(l-l_0+I_4)$$
$$=(l-l_0)(a_1+a_2+a_3-a_4)+(a_1I_1+I_2+a_3I_3-a_4I_4).$$

If there is no stenosis i.e. $R=R_0$, then the resistance of flow is given by

$$\lambda_N=l(a_1+a_2+a_3-a_4).$$

The resistance to flow is

$$\bar{\lambda}=\frac{\lambda}{\lambda_N}=1-\frac{l_0}{l}+\frac{(a_1I_1+I_2+a_3I_3-a_4I_4)}{l(a_1+a_2+a_3-a_4)}. \quad \text{... (19)}$$

From Eq. (14) we have

$$\tau_R=\frac{64}{49}\tau_0+\frac{32}{7}\sqrt{\frac{\mu Q\tau_0-\mu\pi v_s\tau_0R^2}{\pi R^3}}+4\left(\frac{\mu Q-\mu\pi v_sR^2}{\pi R^3}\right)$$

If there is no stenosis then

$$\tau_N=4\left(\frac{\mu Q-\mu\pi v_sR_0^2}{\pi R_0^3}\right)$$

The wall shear stress is

$$\overline{\tau_R}=\frac{\tau_R}{\tau_N}=\frac{16\pi R_0^3\tau_0}{49(\mu Q-\mu\pi v_sR_0^2)}+\frac{8}{7}\frac{\sqrt{\pi\mu Q\tau_0R_0^3-\mu\pi^2v_s\tau_0R_0^5\left(\frac{R}{R_0}\right)^2}}{(\mu Q-\mu\pi v_sR_0^2)}\left(\frac{R}{R_0}\right)^{-3/2}$$
$$+\frac{\left(\mu Q-\mu\pi v_sR_0^2\left(\frac{R}{R_0}\right)^2\right)}{(\mu Q-\mu\pi v_sR_0^2)}\left(\frac{R}{R_0}\right)^{-3}. \quad \text{... (20)}$$

3 . Figures:

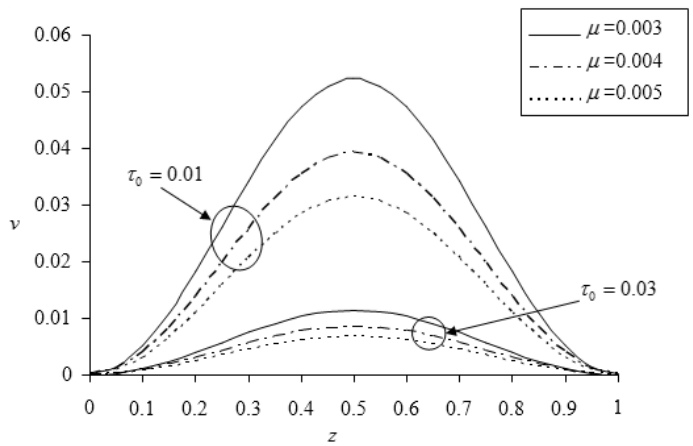


Figure 1(a): Variations of axial velocity v with axial distance z for different values of the viscosity μ and yield stress τ_0 with some fixed values,

$\tau_R=0.07,\ \tau=0.4,v_s=0.0,\ \frac{\delta}{R_0}=0.1,\ d=0,\ l_0=1$

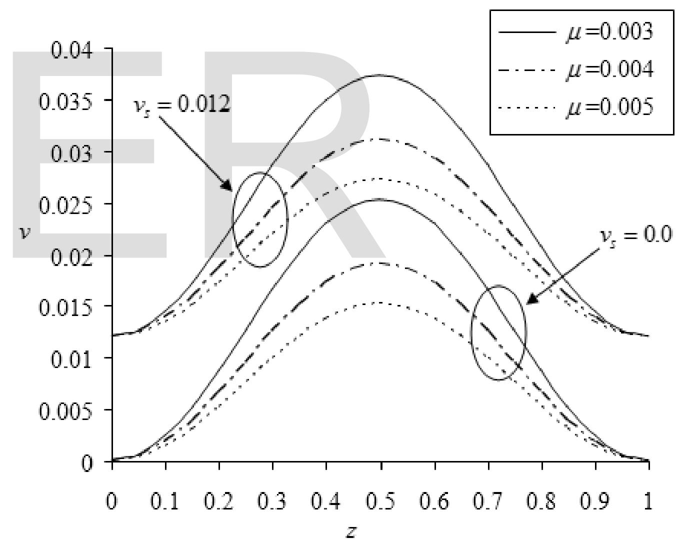


Figure 1(b): Variations of axial velocity v along axial distance z for different values of the viscosity μ and slip velocity v_s with some fixed values

$\tau_R=0.07,\tau=0.04,\ \tau_0=0.02,\ \delta/R_0=0.1,\ d=0,\ l_0=1$

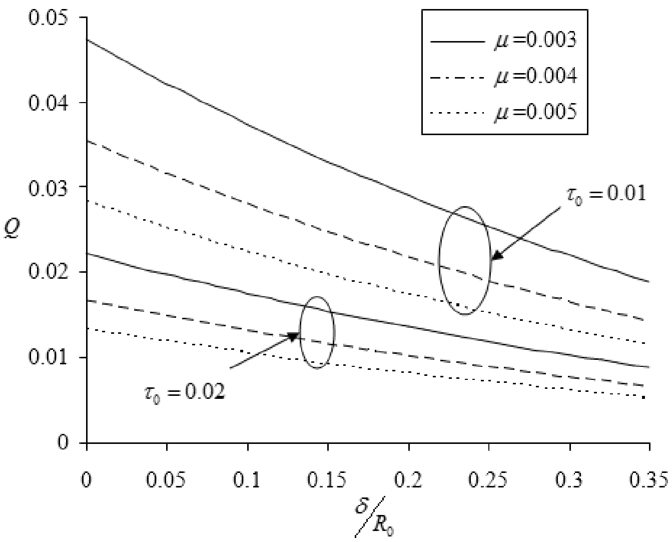


Figure 2(a): Variations of volumetric flow rate Q along the stenosis height δ/R_0 for different values of the viscosity μ and yield stress τ_0 with some fixed values

$\tau_R = 0.07,\ v_s = 0.0,\ z = 2,\ d = 0.4,\ l_0 = 0.6$.

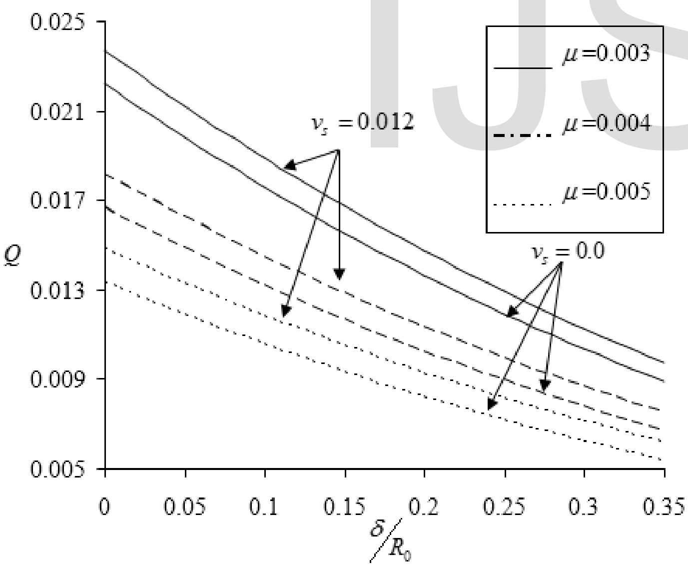
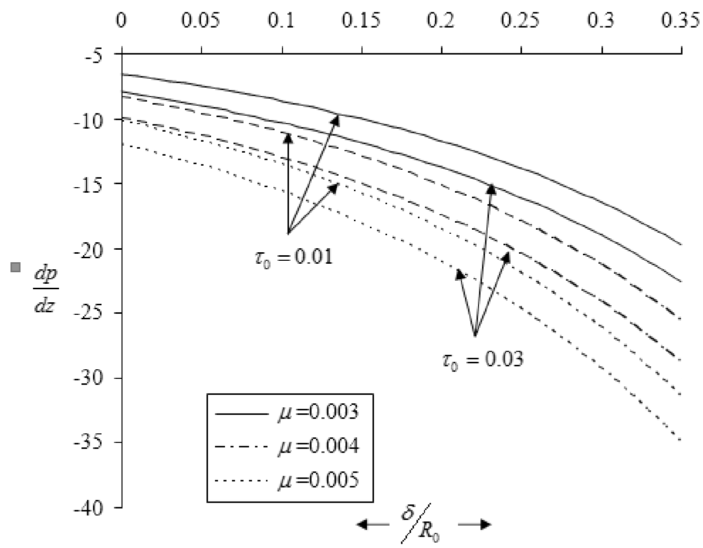


Figure 2(b): Variations of volumetric flow rate Q along the stenosis height δ/R_0 for different values of the viscosity μ and slip velocity v_s with some fixed values

$\tau_R = 0.07,\ \tau_0 = 0.02,\ z = 2,\ d = 0.4,\ l_0 = 0.6$.

Figure 3(a): Variations of pressure gradient P along the



stenosis height for different values of viscosity μ and yield stress τ_0 with some fixed values

$\tau_R = 0.07,\ v_s = 0.0,\ z = 2,\ d = 0.4,\ l_0 = 0.6,\ Q = 1$

.

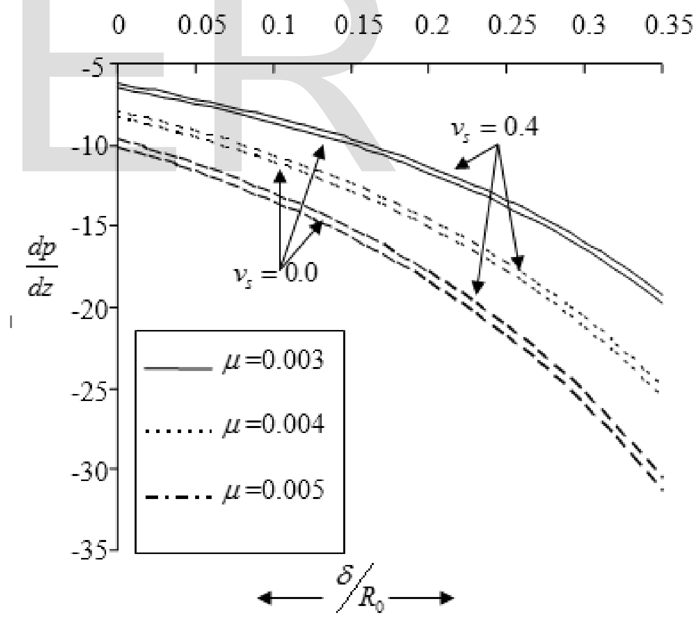


Figure 3(b): Variations of pressure gradient P along the stenosis height δ/R_0 for different values of viscosity μ and slip velocity v_s with some fixed values

$\tau_R = 0.07,\ \tau_0 = 0.01,\ z = 2,\ d = 0.4,\ l_0 = 0.6,\ Q = 1$

.

ISSN 2229-5518

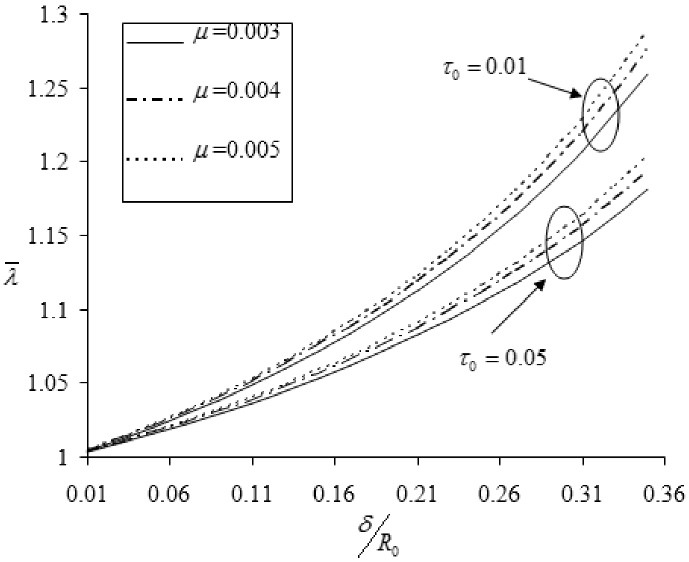


Figure 4: Variations of resistance of flow $\overline{\lambda}$ along the stenosis height δ/R_0 for different values of the viscosity and yield stress with some fixed values

$v_s = 0.0,\ Q = 0.025,\ z = 2,\ l = 2, d = 0.4,\ l_0 = 0.6$

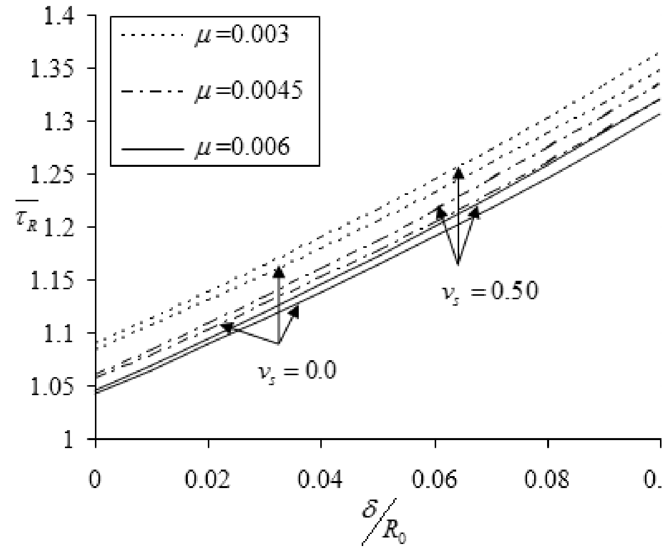


Figure 5(b): Variations of wall shear stress $\overline{\tau_R}$ along the stenosis height δ/R_0 for different values of the viscosity μ and slip velocity v_s with some fixed values

$\tau_0 = 0.03,\ Q = 1,\ z = 2,\ d = 0.4,\ l_0 = 0.6$

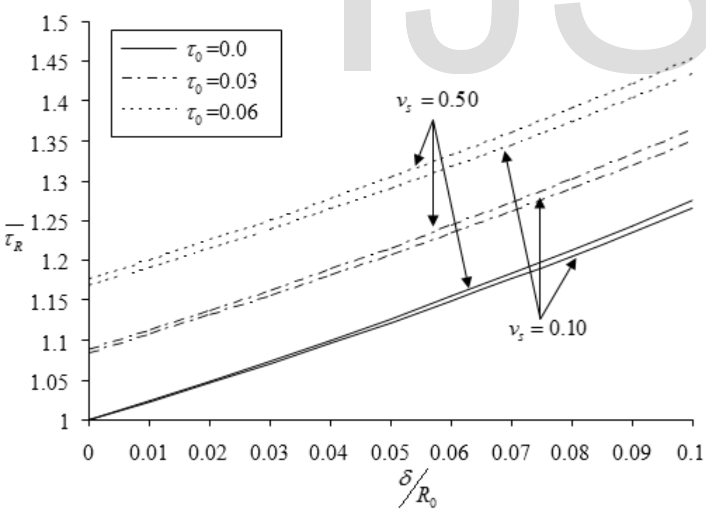


Figure 5(a): Variations of wall shear stress $\overline{\tau_R}$ along the stenosis height δ/R_0 for different values of the yield stress τ_0 and slip velocity v_s with some fixed value

$\mu = 0.003,\ Q = 1,\ z = 2,\ d = 0.4,\ l_0 = 0.6$

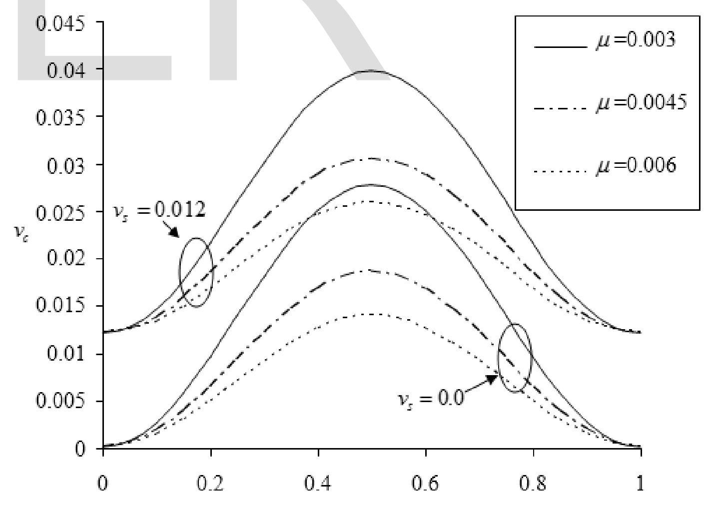


Figure 6: Variations of core velocity v_c along axial dis tance z for different values of the viscosity μ and slip velocity v_s with some fixed values

$\tau_R = 0.07,\ \tau_0 = 0.02,\ \delta/R_0 = 0.1,\ d = 0,\ l_0 = 1$

4 *Results and Discussions*:

In this study, the effects of flow parameters on the flow quantities such as axial velocity, volumetric flow rate, pressure gradient, resistance to flow and wall shear stress have been discussed. The following parameters with their ranges mentioned as:

$Q:0-1, \mu = 0.003-0.006,\ \tau_0 = 0.0-0.06\ dyne/cm^2,$
$l = 2\,cm,\ \ d = 0.0-0.4\,cm, l_0 = 0.6\,cm, R_0 = 0.2\,cm$
$v_s = 0.0-0.50\,cms^{-1},\ \delta/R_0 = 0.01-0.35\,cm$

are used to deduce the expressions of these flow quantities and get data for plotting the figures.

In Figs. 1(a) and 1(b) the variations of axial velocity obtained through equation (11) with axial distance for different values of viscosity, yield stress and slip velocity have been shown respectively. Both figures show that the axial velocity fluctuates (i.e., the axial velocity first increases and after a certain point, it starts decreasing and again increases along the axial distance). It is seen that from figure 1(a) the axial velocity decreases as viscosity and yield stress both increase. It is also seen that from figure 1(b), the axial velocity increases as slip velocity increases.

The variations of volumetric flow rate derived by the equation (13) with stenosis height for different values of viscosity, yield stress and slip velocity have been displayed in Figs. 2(a) and 2(b). it is observed that from these figures that, the volumetric flow rate decreases as stenosis height, viscosity and yield stress increase. But figure 2(b) shows the volumetric flow rate increases as slip velocity increases.

Figs. 3(a) and 3(b) show the variations of the pressure gradient obtained in equation (15) along the stenosis height for various values of the viscosity, yield stress and slip velocity. It is observed that from both figures, the pressure gradient decreases as stenosis height, viscosity increase. Also figure 3(a) depicts that the pressure gradient decreases as yield stress increases and figure 3(b) represents the pressure gradient increases as slip velocity increases.

The variations of the resistance to flow obtained through equation (19) along the stenosis height for different values of viscosity and yield stress have been presented in figure (4). This figure depicts that the resistance to flow increases as stenosis height increases. It is also clarified from this figure, the resistance to flow increases as viscosity increases and decreases as yield stress increases.

In Figs. 5(a) and 5(b) the changes in the wall shear stress are drawn against the stenosis height for different values of viscosity, yield stress and slip velocity. These figures show that the wall shear stress increases as stenosis height and slip velocity increase. Figure 5(a) also shows that the wall shear stress increases as yield stress increases. On the other hand, Fig. 5(b) shows that the wall shear stress decreases as viscosity increases.

The changes in the core velocity obtained in equation (12) along the axial distance for various values of viscosity and slip velocity have been shown in figure (6). It depicts that the core velocity decreases when slip velocity increases.

5 *Conclusion* :

In this work, the aim of the researchers was to develop a mathematical study on the various blood flow properties through a stenosed artery by choosing blood as a Casson fluid. The study exhibits that the viscosity, Slip velocity, yield stress and stenosis affect the blood flow. It is concluded that the axial velocity, volumetric flow rate,pressure gradient and wall shear stress decrease as viscosity and yield stress increase but these flow properties increase as slip velocity increases. The resistance to flow increases as stenosis height and viscosity increase but it decreases as yield stress increases. In view of these arguments, the present study may be more useful to control the blood flow in diseased life.

6 *References*:

[1] Gaur M, Gupta M.K: A Casson fluid model for the steady flow through a stenosed blood vessel. 'British Journal of Mathematics and Computer Sci.', 2014; 4(11): 1629-1641.

[2] Chaturani P, Biswas D: A theoretical study of blood flow through stenosed artery with velocity slip at the wall. 'Proc. First Int. Symposium on Physiological Fluid Dynamics, IIT Madras, India', 1983; 23-26.

[3] Chakravarty S: Effects of stenosis on the flow behavior of blood in an artery, Int. J. Engg. Sci.', 1987; 25: 1003-1016.

[4] Venkatesan J, Sankar D.S, Hemalatha K, Yazariah Yatim: Mathematical analysis of Casson fluid model for blood rheology in stenosed narrow arteries. 'Journal of Applied Mathematics', 2013; 10: 11-55.

[5] Singh A.K, Singh D.P: Blood flow obeying Casson fluid equation through an artery with radially non-symmetric mild stenosis. 'American Journal of Applied Mathematics', 2012; 1: 11-14.

[6] Sapna S: Analysis of non-Newtonian fluid flow in a stenosed artery. 'International Journal of Physical Science', 2009; 4(11): 663-671.

[7] Young D.F: Effect of time-dependent stenosis on flow through a tube. 'J. Manuf. Sci. Eng.', 1968; 90(2): 248-254.

[8] Mishra B.K, Prodhan P, Panda T.C: Effect of shear stress, resistance and flow rate across mild stenosis on blood flow through blood vessels. 'The Cardiol.', 2010; 5: 4-11.

[9] Shukla J.B, Parihar R.S, Rao B.R.P: Effects of stenosis on non-Newtonian flow of the blood in an artery. 'Ball. Math. Biol.', 1980; 42: 283-294.

[10] Kumar S, Diwkar C: A mathematical model of power law fluid with an application of blood flow through an artery with stenosis. 'Advances in Applied Math-

ISSN 2229-5518
ematical Bioscience', 2013; 4(1): 25-36.

[11] Singh B, Joshi P, Joshi B.K: Blood flow through an artery having radially non-symmetric mild stenosis. 'Applied Mathematical Sciences', 2010; 4(22): 1065-1072.