# SIMULATION OF RESERVOIR ENGINEERING ASPECT OF A HORIZONTAL OIL WELL USING FINITE ELEMENT METHOD

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**Abstract**
In Petroleum Engineering, numerical reservoir simulations due to its cheapness when compared with the real physical reservoir simulation are often employed to obtain meaningful and reliable solutions for most actual cases due to extreme complexity of reservoir systems. In numerical simulations, finite element simulation is preferred when accuracy of the result as compared with the analytical is of paramount essence. In this study, reservoir engineering aspect of a horizontal oil well was developed using finite element method. Three-dimensional finite element simulation model was formulated using Darcy's equation, continuity equation and partial differential equations that describe the accumulation and production of the phases in the reservoir media. Finite element algorithm was developed for the model. Initial pressure of 3814 Psi and the reservoir geometry dimensions of 2500 x 2500 x 150ft were used for the model. The COMSOL multiphysics software with time dependent for the purpose of inclusion of the time step which helped to show the various times of simulation was used. Data obtained was validated using chi-square analysis. The pressure slice plots of reservoir pressure depletion with time were obtained from day 0 to 700 with time step interval of 100days. From the simulation at day 0, the average reservoir pressure was 3814 Psi, At day 100, the pressure depleted to 3757.29 Psi, At day 200, the pressure depleted to 3700.29 Psi. Also at days 300, 400, 500 and 600 there was further depletion in the reservoir pressures to 3646.80 Psi, 3591.37 Psi, 3536.50 Psi and 3481.23 Psi respectively. At the final stage of simulation (day 700), the reservoir pressure dropped to an average value of 3371.90 Psi. The decrease in pressures is due to an increase in time of simulation. These results when compared with literature established no significant difference.A finite element simulation model for a reservoir engineering aspect of a horizontal oil well in three-dimensional oil reservoir media has been developed, simulated and results obtained validated; hence it can be used in a real life reservoir simulation study.

**Keywords:** Finite Element method, Reservoir, Simulation, Horizontal oil well, Petroleum Engineering.

**Nomenclature**

$\rho_1$- Density of fluid 1
$\varepsilon_p$- Porosity
$s_1$ – Saturation of fluid 1
$T$ - Reservoir temperature
$t$ – Simulation time

$k$ - Permeability
$u$– Velocity of fluid flow
$n$ – Vector normal to reservoir boundary
$p$ – Reservoir pressure
$g$ – Acceleration of gravity
$D$– Reservoir thickness
$\Phi$- Reservoir fluid potential
$D_c$ – Capillary diffusion
$\mu_1$ – Dynamic viscosity of fluid 1
$k_{r1}$– Relative permeability of fluid 1
$c_1$– Content of fluid 1

## 1. Introduction

A petroleum reservoir or oil and gas reservoir is a subsurface quantity of hydrocarbons contained in porous or fractured rock formations. The naturally occurring hydrocarbons, such as crude oil or natural gas, are trapped in the subsurface by overlying rock formations with lower permeability. The total estimate of petroleum reservoirs include the total quantity of oil that can be recovered and oil residuals that cannot be recovered due to geographical constraints or oil characteristics and financial or technological limitations. The fraction of crude oil reservoirs that can be extracted from the oil field using current technology is classified as reserves. In order to maximize the oil recovery from a field, various techniques are used during the lifetime of a reservoir [1].Petroleum reservoir is a portion of a trap (a permeable bed, containing oil and gas in its pore channels and an overlying bed which prevents further movement of oil and gas) where oil or gas is stored in nature. It varies in sizes, depending upon the size of the trap and how much petroleum is present to accumulate. The physical shapes of petroleum reservoir are classified according to the geological features causing their occurrence. These features are structural folding, structure with faulting, structure with unconformity, structure caused by some deep seated movement of earth materials such as salt domes or serpentine plugs, change in permeability within a formation and combination of two or more of the foregoing. The Petroleum system consists of all the components needed for the accumulation of oil and gas; source rocks, reservoir rocks, overburden rock, trap formation, generation, migration, progression of hydrocarbon to accumulation and preservation. It includes all geological components mandatory for the existence of an accumulation [1]. Oil and gas accumulations occur in underground traps formed by structural and stratigraphic features, the hydrocarbon accumulations usually occur in the more porous and permeable portion of beds, which are mainly sands, sandstones, limestone's and dolomites in the inter-granular openings or in pore spaces caused by joints, fractures and solution activity. In some cases the entire trap is filled with oil and gas, and in these instances the trap and the reservoir are the same. Often the hydrocarbon reservoir is hydraulically connected to a volume of water-bearing rock called an aquifer. Many reservoirs are located in large sedimentary basins and share a common aquifer. When this occurs, the production of fluid from one reservoir will cause the pressure to decline in other reservoirs by fluid communication through aquifer [2]. Also, hydrocarbon fluids are mixtures of molecules containing carbon and hydrogen. Under initial reservoir conditions, the hydrocarbon fluids are in either a single-phase or a two-phase state. A single-phase reservoir fluid may be in a liquid phase (oil) or a gas phase (natural gas). In either case, when produced to the surface, most hydrocarbon fluids will separate into gas and liquid phases. Gas produced at the surface from a fluid that is liquid in the reservoir is called dissolved gas. Therefore, a volume of reservoir oil will produce both oil and the associated dissolved gas at the surface, and both dissolved natural gas and crude oil volumes must be estimated. On the other hand, liquid produced at the surface reservoir is called gas condensate because the liquid condenses from the gas phase. In this case, a volume of reservoir gas will produce both natural gas and condensate at the surface, and both gas and condensate volumes must be estimated. Where the hydrocarbon accumulation is in a two-phase state, the overlying vapour phase is called the gas cap and the underlying liquid phase is called the oil zone. There will be four types of hydrocarbon volumes to be estimated when this occurs: the free gas or associated gas, the dissolved gas, the oil in the oil zone, and the recoverable natural gas liquid (condensate) from the gas cap. Although the hydrocarbons in place are fixed quantities, which are referred to as the reserves, the reserves depend on the mechanisms by which the reservoir is produced. Two main properties of a reservoir rock is the capacity to store and transmit fluid, technically referred to as porosity and permeability [2]. Finite element method is one of the numerical techniques used in solving complex Engineering problems where solution cannot be obtained or too difficult to obtain analytically. Finite element method has become a commonly used tool to solve various engineering problems. Finite element method is applicable to a wide range of boundary value problems in engineering. In boundary value problems, solutions are sought in the region of the body, while on the boundaries the values of the unknown variables (or their derivatives) are prescribed. For the last three to four decades, finite element method has received much attention, due to the increase in the use of high-speed computers and the growing emphasis on numerical methods for engineering study. This is completely understandable, since it is not possible to obtain analytical solutions for many practical engineering problems [3]. An analytical solution is a mathematical expression that can give the values of the desired unknown variables at any location in a continuum, and as a consequence it is applicable for an infinite number of locations in the body. Analytical solutions can be obtained only for certain simple problems. This difficulty, however, can be overcome with the application of the finite element method. The finite element method has been applied to solve Engineering problems with complex geometries, loadings, and material properties where analytical solution cannot be obtained. For problems involving non-isotropic material properties and complex boundary conditions, one has to resort to numerical methods that provide approximate solutions with reasonable accuracies. More so, in most of the numerical methods, the solutions yield approximate values of the unknown variables only at a discrete number of points in the continuum [4]. Furthermore, simulation is the imitation of the operation of a real-world process or system over time. It involves the design of a model of an actual or theoretical physical system, executing the model on a digital computer and analyzing the execution output. Simulation requires that a model be developed, this model represents the key characteristics or behaviours or functions of selected physical or abstract system [5]. Reservoir simulation refers to the construction and operation of a model whose behaviour assumes the appearance of actual reservoir behaviour. It is an area of reservoir engineering in which computer models are used to predict the flow of fluids (typically, oil, water and gas) through a porous media. Reservoir simulation technique makes it possible to gain insight into the recovery processes of a reservoir which helps to understand fluid flow and to evaluate the performance of oil and gas recovery methods [6]. The reservoir simulation is useful in understanding the reservoir process and learning more about the physical nature of the reservoir and the mode of primary recovery. Compaction occurs when the rock compressibility strength is surpassed and it results in the reduction of porosity and permeability when there is a deformation of the rock [7]. In addition, reservoir engineering is used extensively to identify opportunities to increase oil production in heavy oil deposits. This study was therefore design to develop a finite element model to simulate reservoir engineering aspect of a horizontal oil well using COMSOL Multiphysics software package.

### 1.1 Finite Element Method

The Finite Element Method is a numerical method for solving problems of engineering and mathematical physics. FEM is useful for problems with complicated geometries, loadings, and material properties where analytical solutions cannot be obtained. The finite element method was developed in the American and European aircraft industries in the 1950s [8] and was transformed in the 1960s from a physically based procedure with a limited mathematical foundation into the present day method resting upon variational principles [9]. The technique has become increasingly popular with the help of engineers and numerical analysts, and its application has been extended far beyond the original aero-elasticity problems [10], [11], [12]. It has been recognized increasingly often as a general approximation method for boundary value and eigen value problems, often superior to finite difference methods. Newton Raphson iterative scheme was used to solve for the sum and difference of the fluid pressures simultaneously and the occurrences of fingering phenomenon, noise oscillations in numerical results were reported. The application of finite element method for analysing multiphase flow through porous medium such as petroleum reservoir was proposed by Lewis [13]. Over the years, a Galerkin-base finite element method has been used to solve two phase immiscible flow. In this formulation, the variations of unknown in the time domain were approximated in a small time increment by a linear and quadratic polynomial that resulted in a very simple algorithm for solving equations in the time domain, furthermore, a numerical modelling of flow of immiscible fluids in subsurface reservoirs with the use of a self-adapting mesh was used to simulate multiphase flow in two dimensional reservoirs [14]. Settari *et al.* [15] applied numerical techniques to calculate subsidence induced by gas production in the North Adriatic due to the complexity of the reservoir and compaction mechanisms; it is a combined approach of reservoir and geo-mechanical simulators in modelling subsidence, including the level of coupling between the fluid flow and geo-mechanical solution. The research presents that a fully coupled solution had an impact only on the aquifer area, and an explicitly coupled technique was good enough to give accurate results. On grid issues, the preferred approach is to use compatible grids in the reservoir domain and to extend the mesh to geo-mechanical modelling. However, it was also noted that the grids generated for reservoir simulation were often not suitable for coupled models and require modification. Settari [16] also proposes the method of numerical modelling techniques used to explore the effects of small levels of over-consolidation onset of subsidence and the areal extent of the resulting subsidence bowl. The same method could be used for the coupling of the multiphase, compositional simulator and the geo-mechanical simulator in future. Apart from experimental observations, there is also the scope to perform numerical simulations to determine the impact of thermal stress in various categories, such as water injection, gas injection or production etc. Vaziri et al. [17] used a finite element analysis to develop a modified form of the Mohr-Coulomb failure envelope to simulate both tensile and shear-induced failure around deep wellbores in oil and gas reservoirs. There have been studies in various categories of well completion including modelling of coupled fluid flow and mechanical deformation of medium.

### 1.2 Reservoir Simulation

The practice of reservoir simulation has been in existence since the beginning of petroleum engineering in the 1930's. But the term "numerical simulation" only became common in the early 1960's as predictive methods that evolved into relatively sophisticated computer programs. These computer programs represented a major advancement because they allowed solution of large sets of finite-difference equations describing two and three-dimensional, transient, multiphase flow in heterogeneous porous media. This advancement was made possible by rapid evolution of large-scale, high-speed digital computers and development of numerical mathematical methods for solving large systems of finite-difference equations [18]. Many numerical methods which can be utilized to proffer solution to modelling problems are in existence, and some of the methods are Finite difference method; Finite element method; Vortex element method; Monte Carlo method and many others. Fluid flow in petroleum reservoirs (porous media) is very complex phenomena, and as such analytical solutions to mathematical models are only obtainable after making simplifying assumptions regarding reservoir geometry, properties and boundary conditions. However, simplifications of this nature are often invalid for most fluid flow problems and in many cases, it is impossible to develop analytical solutions for practical issues due to the complex behaviours of multiphase flow, non-linearity of the governing equations, and the heterogeneity and irregular shape of a reservoir system. Due to these limitations in the use of analytical method, these models must be solved with numerical methods such as finite difference. The first numerical method for calculating pressure distributions for unsteady-state gas flow in homogeneous reservoirs was developed by [19] and since then, there has been improvement and application of reservoir simulation of computer technology. Darman in his work [20] described reservoir simulation as one of the most effective tools for reservoir engineers; it involves developing mathematical equations or computable procedure that are employed to understand the behaviour of the real reservoir. Reservoir simulation uses a computer model to describe fluid flow thereby allowing more in-depth analysis of the reservoir. It is the science of combining physics, mathematics, reservoir engineering, and computer programming to develop a tool for predicting hydrocarbon reservoir performance under various operating strategies [21]. The pressure-saturation relationships in form of non-linear differential equations are created which can be solved using finite difference or finite element; reservoir modelling is the use of numerical solutions to solve these equations. The mathematical equations derived are usually in form of non-linear partial differential equations in which the solutions can rarely be sought using analytical methods because of the relationship of pressure and saturation changes with time throughout the medium and the presence of specialized boundary conditions [20]. According to Fanchi [22] reservoir modelling is an incorporation of seismic interpretation, petro-physics, geological model, fluid properties, wells, facilities, tubing curves and model grid effects. However, this research would be based on the approach of FEM for reservoir's simulation. In order to predict the future performance of a reservoir as precise as possible, reservoir simulations are carried out using the finite difference or finite element formulation. Prediction of reservoir performance and ultimate recovery is the major reason for simulation studies [23]. Before a reservoir simulation can be carried out, it is mandatory to know the objectives of the study. The procedures involved in carrying out a reservoir simulation include; geological review, reservoir performance review, data gathering, selection of model, initialize, history match, predictions and recording output [24]. Numerical reservoir simulation is frequently used as an important tool to help investment decisions on major exploitation and development study. It involves different decisions which include determining commerciality, optimizing field development plans and initiating secondary and enhanced oil recovery methods on oil and gas projects basically on reservoir simulation results. However, the usefulness of the computer generated results depends on the software and the input data used [25]. According to Archer [26] two particular conditions for reservoir modelling process are the ability of equations to represent the physics of the fluid flow, the equilibrium in the reservoir and well system and the description to represent three dimensional reservoirs for grid properties. A two-dimensional model to describe the areal performance or to simulate the vertical conformance in a reservoir. The 2-D areal model as shown in Figure 1(a) is used if there is very little vertical movement of fluids as in thin sand. It could also be used for thick sands when there is no much difference in permeability i.e. when there is no permeability layering. The two dimensional cross sectional model (Figure 1(b)) is often used to simulate a slice of a field. It is useful in determining completion intervals and stratification effects. A radial 2-D model to evaluate the behaviour of individual wells. They are sometimes referred to as “coning models” based on their early applications for studying the effects of coning phenomena.  A 3-D model (Figure 1(c)) that can account for areal and vertical conformance simultaneously. It is effective for different simulation studies [21].

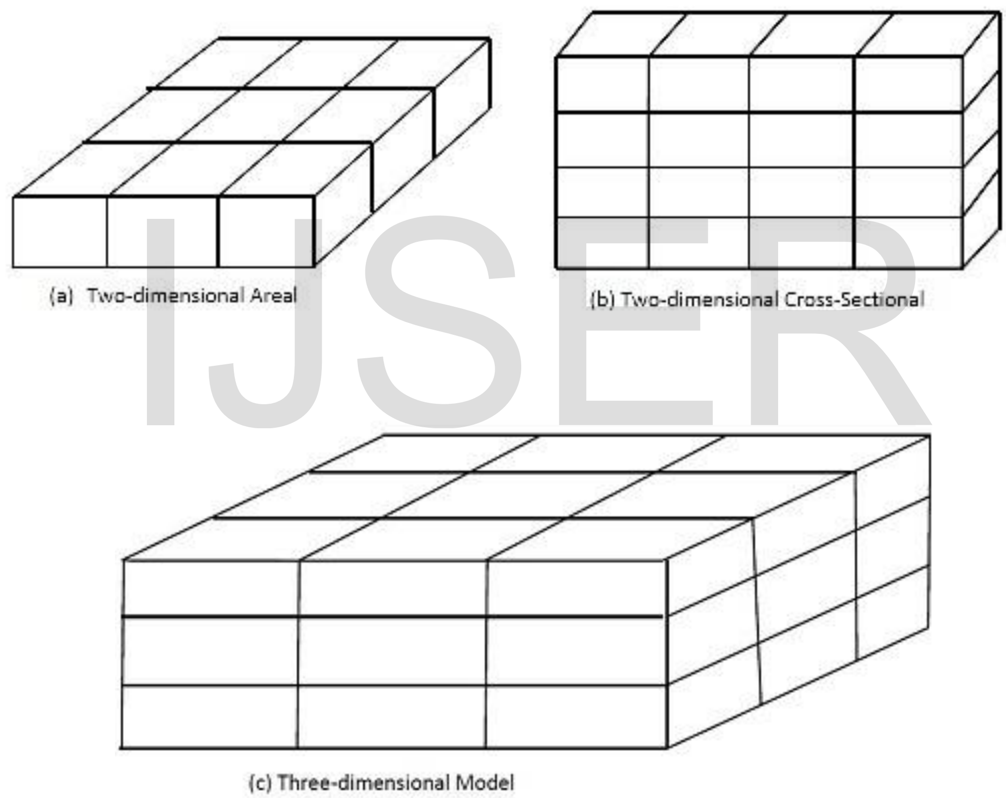



**Figure 1**: Schematics of reservoir models

## 2.0 Methodology

The methodology of this project followed the use of finite element analysis software called COMSOL Multiphysics for the numerical analysis. By simply utilizing the predefined modules on the software, one can easily carry out a finite element simulation by defining the pertinent physical quantities like properties of material, boundary conditions etc. without having to write out the governing equations. This software by itself computes all the necessary equations that might be needed, as specified through some selected physics by the user.

## 2.1 The Governing Equations

### 2.1.1 Darcy's Equation for Momentum Transfer in Porous Media

In a porous medium, global transport of momentum by shear stresses within the fluid is usually negligible; this is because the pore walls impede momentum transport to the fluid outside the individual pores. Darcy's law together with the continuity equation and equation of state for the pore fluid (or gas) provide a complete mathematical model suitable for a wide variety of applications involving porous media flows, for which the pressure gradient is the major driving force. Darcy's equation describes fluid movement through interstices in a porous medium because the fluid loses considerable energy to frictional resistance within pores and flow velocities in porous media are very low. The Darcy's Law interface in the subsurface flow module applies to water moving in an aquifer or stream bank, oil migrating to a well. Also set up multiple Darcy's Law interfaces to model multiphase flows involving more than one mobile phase. Darcy's law portrays flow in porous media driven by gradients in the hydraulic potential field has units of pressure. For many applications, it is convenient to represent the total hydraulic potential or the pressure and the gravitational components with equivalent heights of fluid. The physics interface also supports specifying boundary conditions and result evaluation using hydraulic head and pressure head. In the physics interface, pressure is always the dependent variable. Darcy's law applies when the gradient in hydraulic potential drives fluid movement in the porous medium.

According to Darcy's law, the net flux across a face of porous surface gives

$$u = -\frac{k}{\mu}(\nabla p + \rho g \nabla D) \quad (1)$$

In this equation, u is the Darcy velocity or specific discharge vector (m/s);
$k$ is the permeability of the porous medium ($m^2$)
$\mu$ is the fluid's dynamic viscosity (Pa.s)
p is the fluid's pressure (Pa)
$\rho$ is its density ($kg/m^3$)
g is the magnitude of gravitational acceleration ($m/s^2$) and
$\nabla D$ is a unit vector in the direction over which the gravity acts.
Here the permeability $k$, represents the resistance to flow over a representative volume consisting of many solid grains and pores (COMSOL Multiphysics 5.0).

$$\frac{\partial}{\partial t}(\rho \varepsilon_p) + \nabla \cdot (\rho u) = 0 \quad (2)$$

$$\varphi = p - \rho g \nabla D \quad (3)$$

$$\rho = s_1 \rho_1 + s_2 \rho_2 \quad (4)$$

$$\frac{1}{\mu} = s_1 \frac{k_{r1}}{\mu_1} + s_2 \frac{k_{r2}}{\mu_2} \quad (5)$$

$$\frac{\partial}{\partial t}(\varepsilon_p c_1) + \nabla \cdot (c_1 u) = \nabla \cdot D_c \nabla c_1 \quad (6)$$

$$c_1 = s_1 \rho_1 \quad (7)$$

## 2.2 Boundary Conditions

No slip boundary condition exists at the reservoir domain wall and the riser wall.

Final outlet pressure equals zero since the process of exploration considered for this study does not require the use of any pump

$$-n \cdot D_c \nabla c_1 = 0 \quad (8)$$

Steady state fluid flow
The fluid matrix and the tube properties are constant
Gravitational pull is evident
Other body force effects are negligible

## 2.2 Assumptions

The assumptions used in this study are stated below:
The three-dimensional fluid flow is laminar
The reservoir temperature is constant
The fluid and the reservoir properties are the volumetric averages of the units' properties.
The reservoir boundaries are impermeable and the flow across the reservoir is catered for by production. The pressure and saturation values are not changing drastically. The porosity of the reservoir is constant i.e. the reservoir rock matrix is incompressible

## 2.3 Modelling of Geometry

To optimally simulate for the real life environment, the three-dimensional drawing of the horizontal oil well reservoir was modelled on SolidWorks. SolidWorks was used because it gives well-defined 3D geometries inputted by the user and produces results that are compatible on COMSOL [27] without any complication.

**Table 1**: Parameters of the horizontal oil well [1]

| Parameter | Value and Unit |
|---|---|
| Reservoir length | 2500ft |
| Reservoir breath | 2500ft |
| Reservoir thickness | 150ft |

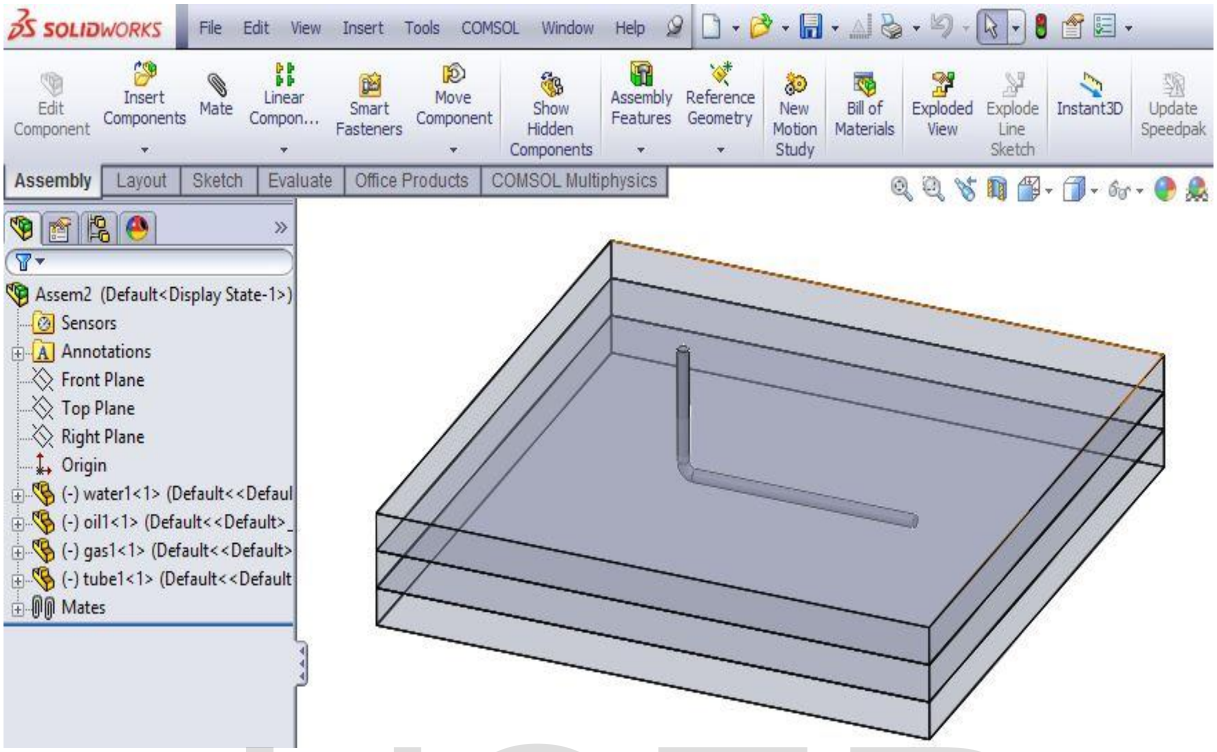
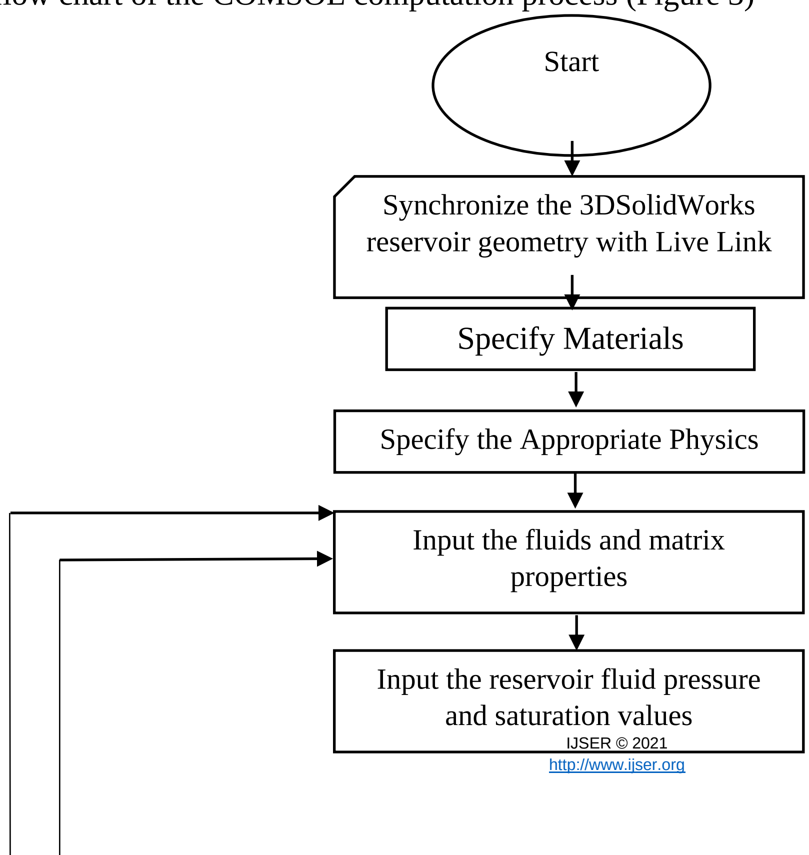



**Figure 2**: Reservoir geometry modelled on Solid Works



## 2.4 Flow Chart for the simulation on COMSOL

The flow chart illustrates the processes that are followed in the computation process. Below is the flow chart of the COMSOL computation process (Figure 3)



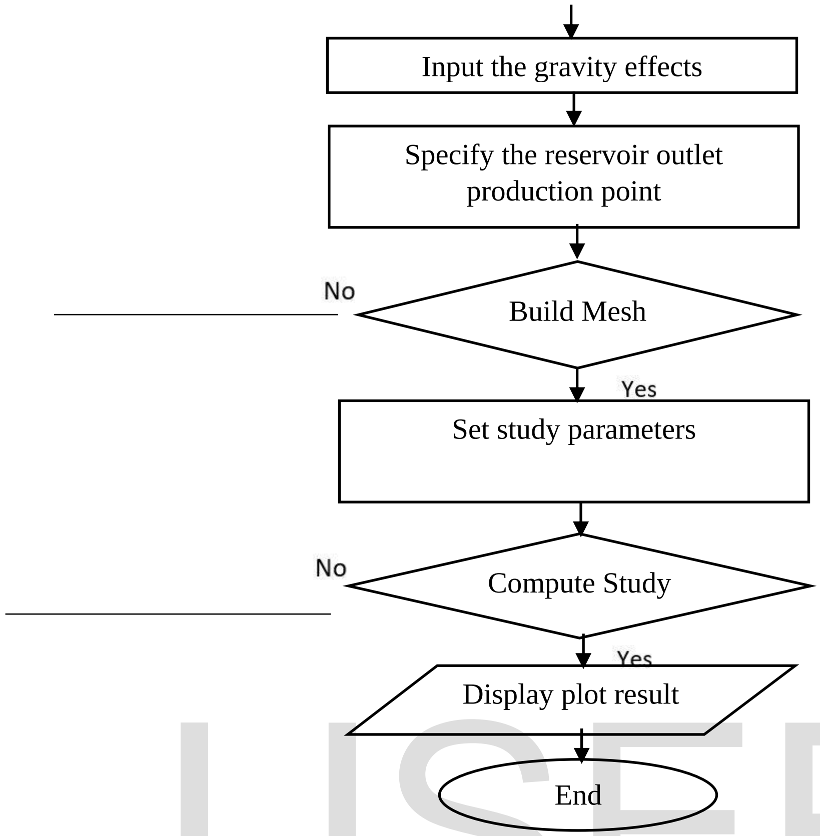



**Figure 3: Flow Chart for the simulation on COMSOL**

## 2.5 Importing of the Geometry

The modelled geometry was imported to the environment of COMSOL Multiphysics, after which the material properties were added. Below is the model when sychronized from SolidWorks to COMSOL

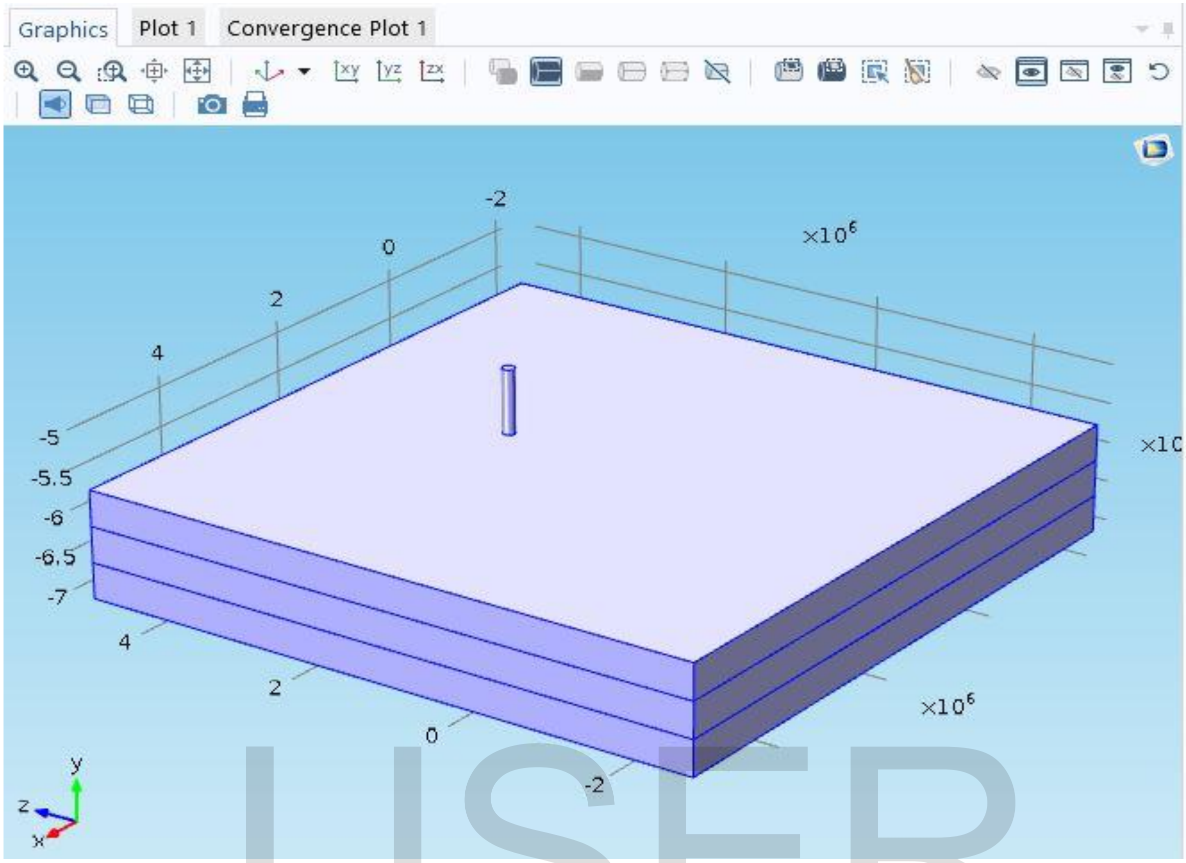



**Figure 4**: Image of the geometry as imported



## 2.6 Specification of Materials

Five different materials were utilized for this study.
The materials were:
Crude oil
Natural gas
Water
High density polyethylene(HDPE) and
Sandstone (quartzitic)
Crude oil: Crude oil was specially prepared from COMSOL's blank material.
Its density is 800kg/ $m^3$
Dyanmic viscosity is 0.0032Pa.s.
Relative permeability of crude oil: 0.4
Saturation of crude oil: 0.5
Natural gas: The natural gas too was prepared from the software's blank material.
It has a density of 0.7kg/$m^3$
Dynamic viscosity of 0.000014962845 Pa.s.
Relative permeability of natural gas: 0.2
Saturation of natural gas: 0.2
Water: This material was selected from the software's default materials.

It has a density of 1000kg/$m^3$
Dynamic viscosity of 1000 Pa.s.
Saturation of water: 0.3
High density polyethylene: This is the material of the riser tube that conveys the crude oil from the horizontal oil well reservoir to the top of the well.
It was also added from the COMSOL's predefined materials.
Sandstone: The matrix was principally of sandstone material, and its parameters were supplied by COMSOL with a porosity of 0.23

## 2.7 Inputting of Parameters

Because the simulation entails a horizontal oil well which is usually a porous media, 'Two Phase darcy's Law' Multiphysics was used. Darcy law comprises a set of momentum and conservation equations which could be a derivative of Navier Stokes equations.
Defining this physics requires the setting of 5 nodes. They are Fluid and Matrix Properties 1, No Flux 1, Initial Values 1, Outlet 1, Fluid and Matrix Properties 2.
The nodes are further expantiated below:
Fluid and Matrix Properties 1: The fluid properties like relative permeability, porosity,viscosity and density are specified on this node.
Temperature: The reservoir temperature was 361.483K
Capillary diffusion: The value for the capillary diffusion for any possible capillary model was $1 \times 10^6 m^2/s$.
No Flux 1: This node caters for the effects of impervious boundaries that denies any fluid flow i.e. there is no fluid flow across some impermeable boundaries. The governing equations are:
$n \cdot \rho u = 0, \quad n \cdot \nabla c_1 = 0$
Initial Values 1: The initial values are principally the initial pessures of the reservoir and the saturation values of the fluid in the pore spaces. The initial reservoir pressure was pegged at 3814 psi [28] and the saturation of fluid value for crude oil was 0.5.
Outlet 1: This node supplies the appropriate boundary condition for the outlet boundary that is dominated. The pressure at this point was left at 0.
Fluid and Matrix Properties 2: This node of the Fluid and Matrix Properties caters for the water. This helps to turn the fluid flow to three phases.

## 2.8 Meshing

The meshing of all the domains were caried out with the mesh element been set to normal while allowing the sequence to be monitored by the physics i.e. the meshes were set automatically.

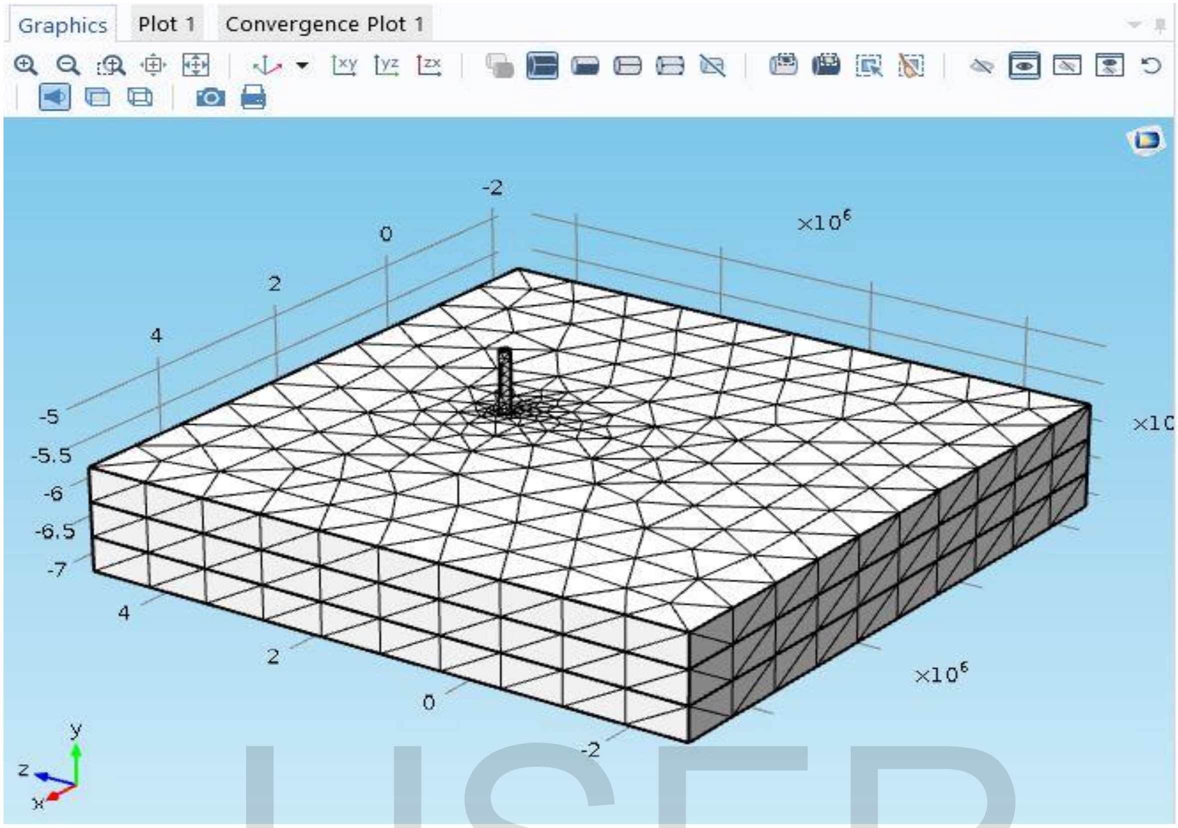



Figure 5: The meshed form of the geometry



Geometry statistics

Table **2:** Mesh geometry statistics table

| Property | Value |
|---|---|
| Minimum element quality | 0.009473 |
| Average element quality | 0.6011 |
| Tetrahedral elements | 5351 |
| Pyramid elements | 0 |
| Prism elements | 0 |
| Hexahedral elements | 0 |
| Triangular elements | 2004 |
| Quadrilateral elements | 0 |
| Edge elements | 274 |
| Vertex elements | 28 |

The products used for this project work involve COMSOL Multiphysics, CAD Import Module, CFD Module, LiveLink™ for SOLIDWORKS®.

## 3. Results and discussion

The results of the numerical simulation of the finite element simulation of the three-dimensional horizontal oil well reservoir on COMSOL Multiphysics is in here presented. After completing the simulation process on COMSOL, the reservoir pressure results were extracted and ploted on SPSS tool for proper interpretation..The process of computation suceeds that of the selection of physics, addition of materials and specification of paramters. For the purpose of inclusion of the time step, which is necessary to show the various times of the simulations, the study was made to be time dependent. The time dependent study is utlized when media variables change with time. The total time permitted for the simulation was 700 days while starting from day 0. Results of the pressure plots of the petroleum reservoir media Figures 1 to 8 below show the pressure plots results of the petroleum reservoir media. At day 0 (figure 1), when the reservoir is at initial stage, the reservoir pressure is maintained at its initial value of 3814. This shows that there was very little of no drop in the average reservoir pressure. This is of course expected because no simulation has been carried out at this stage.

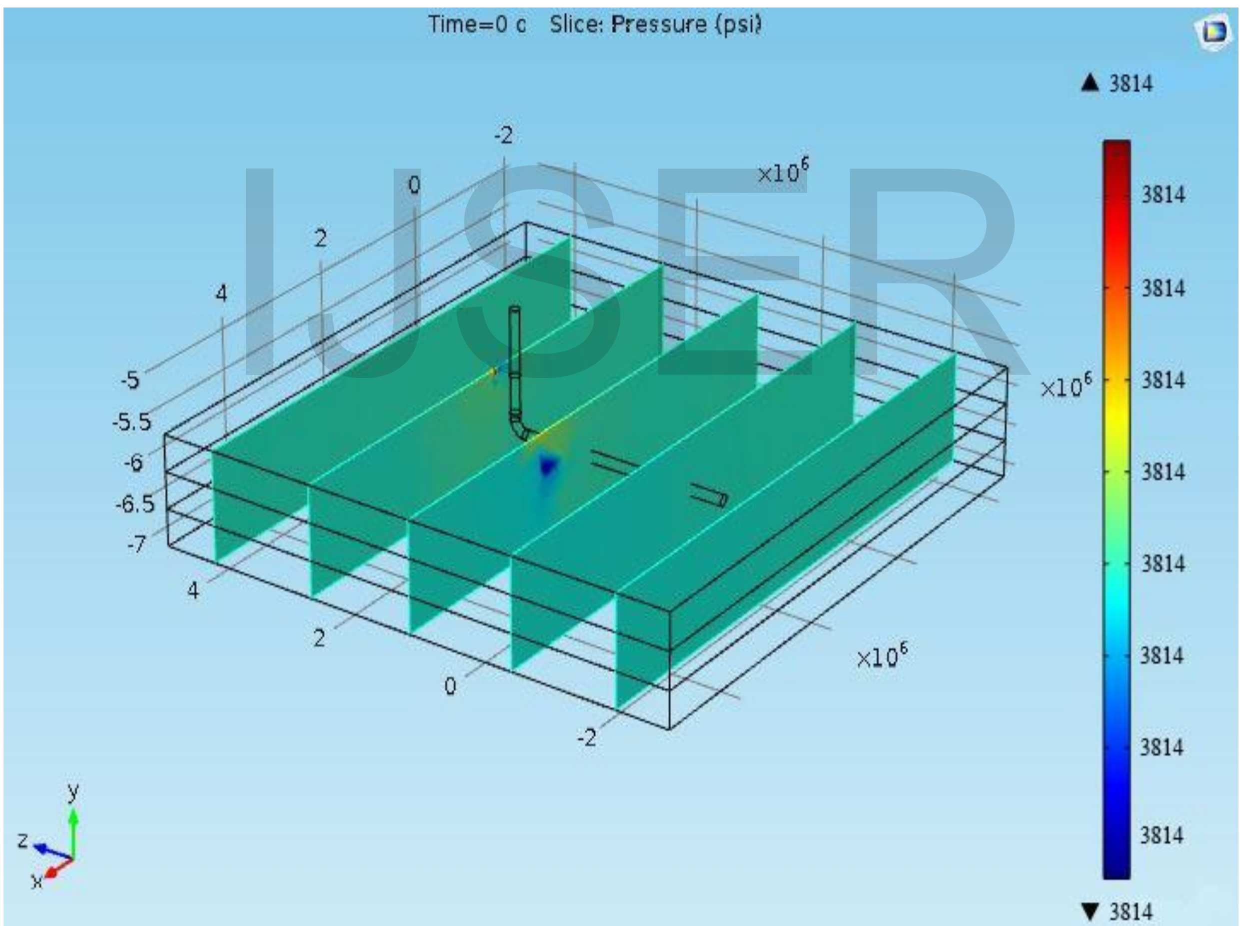



**Figure 6:** Pressure plot of the reservoir media at time = 0 day

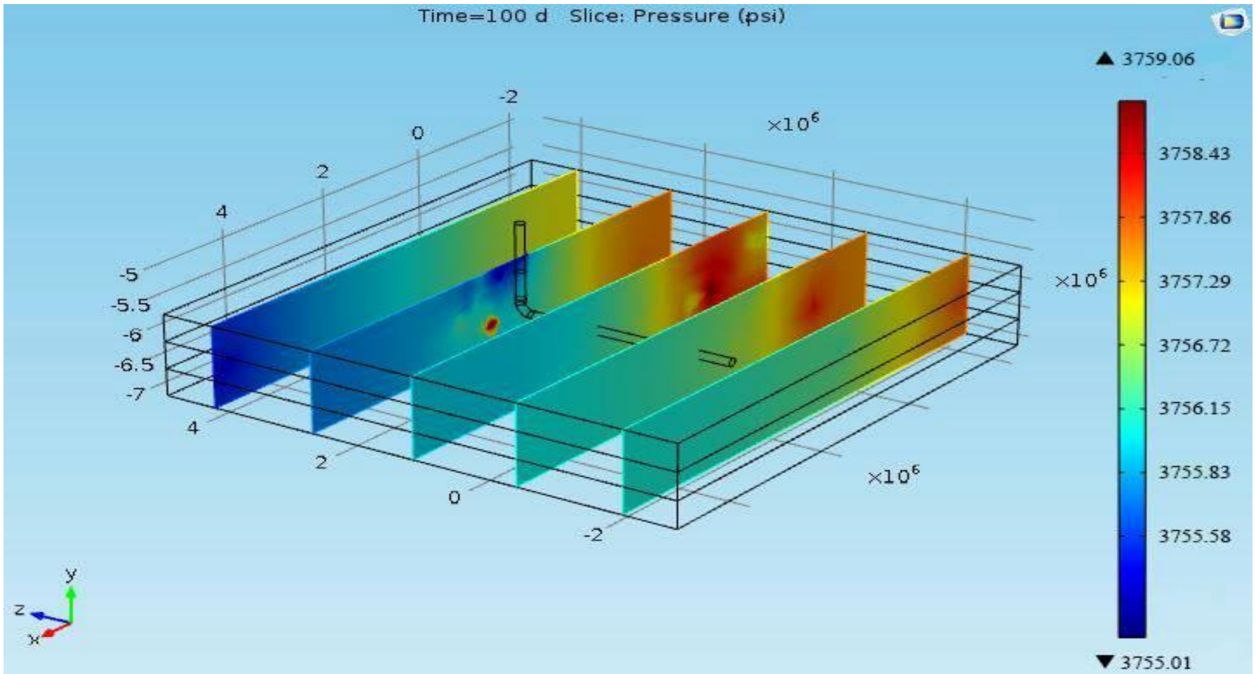



**Figure7**: Pressure plot of the reservoir media at time= 100 days

Figure 7 above at 100th day of simulation show that the pressure value dropped from the initial 0th day value of 3814psi to a maximum value of 3759.06psi. The least pressure value was 3755.01psi, leaving the average value to about 3757.29psi. These interpretations were deduced from the pressure colour scale at the right hand side of the model.



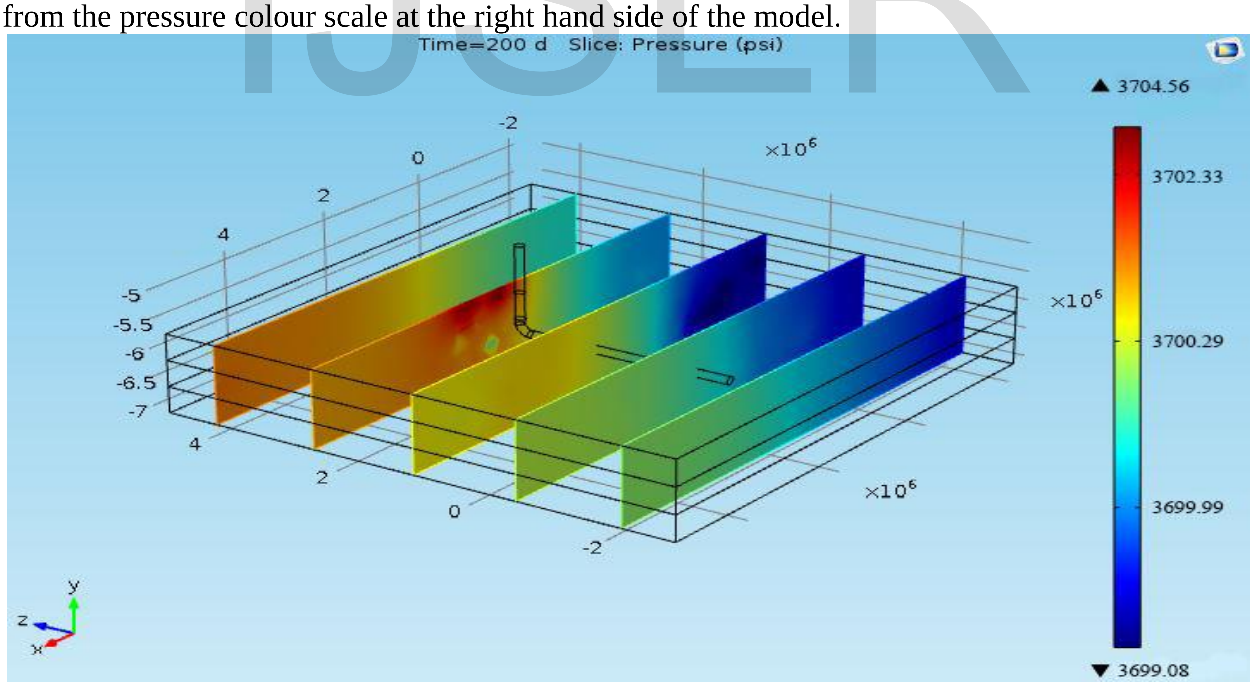



**Figure 8**: Pressure plot of the reservoir media at time = 200 days

Figure 8 above at 200th day of simulation also shows that the pressure further dropped from an average value of 3757.29psi to about 3700.29psi. The least pressure value noticed was 3699.08psi and these interpretations were deduced from the pressure colour scale at the right hand side of the model.

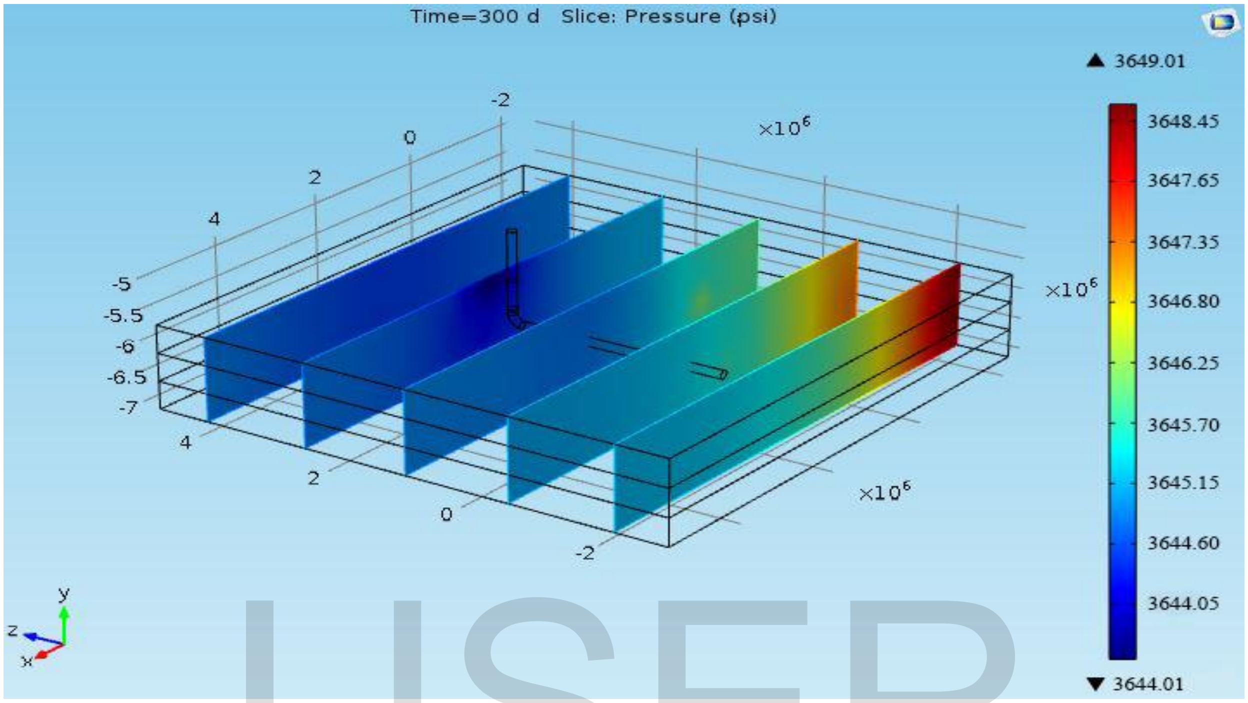



**Figure 9**: Pressure plot of the reservoir media at time = 300 days

At 300 day (figure 4.4), the simulation again shows that the pressure dropped from an average value of 3700.29psi to about 3646.80psi. The least pressure value noticed was 3644.01psi.

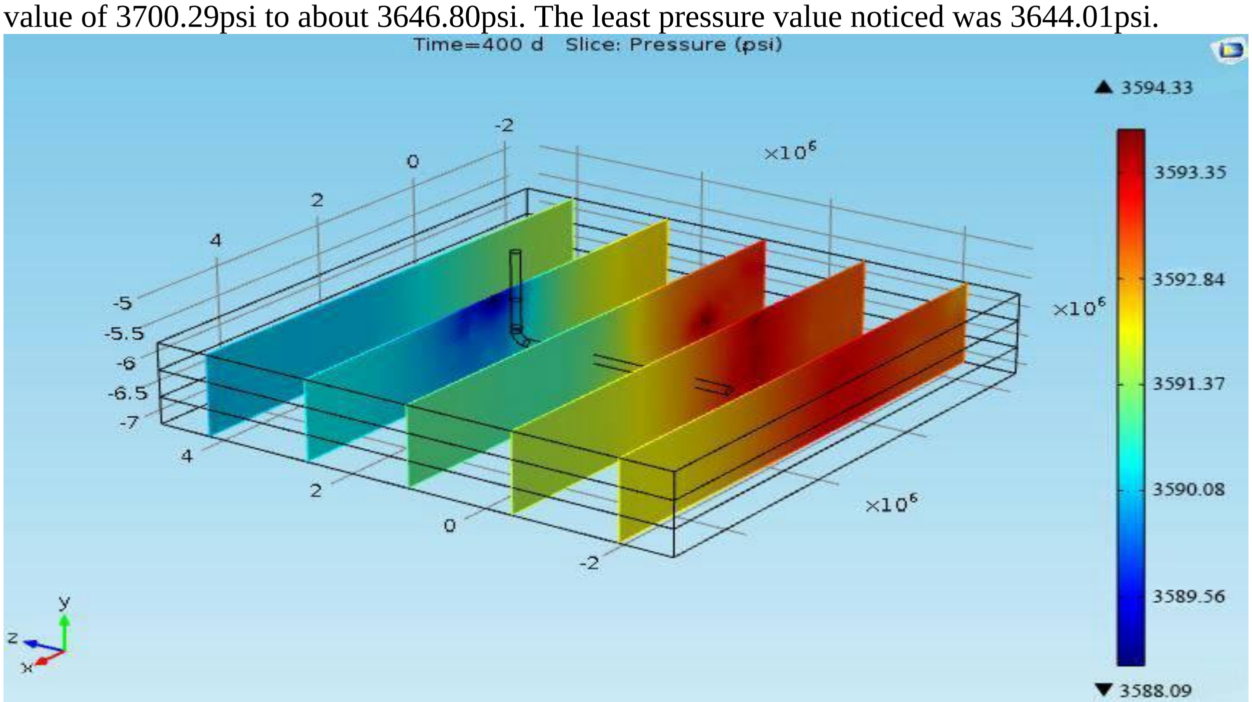



**Figure 10:** Pressure plot of the reservoir media at time = 400 days

In figure 10 at 400th day of simulation, the pressure further dropped from an average value of 3646.80psi to about 3591.37. The least pressure value was 3588.09 psi.

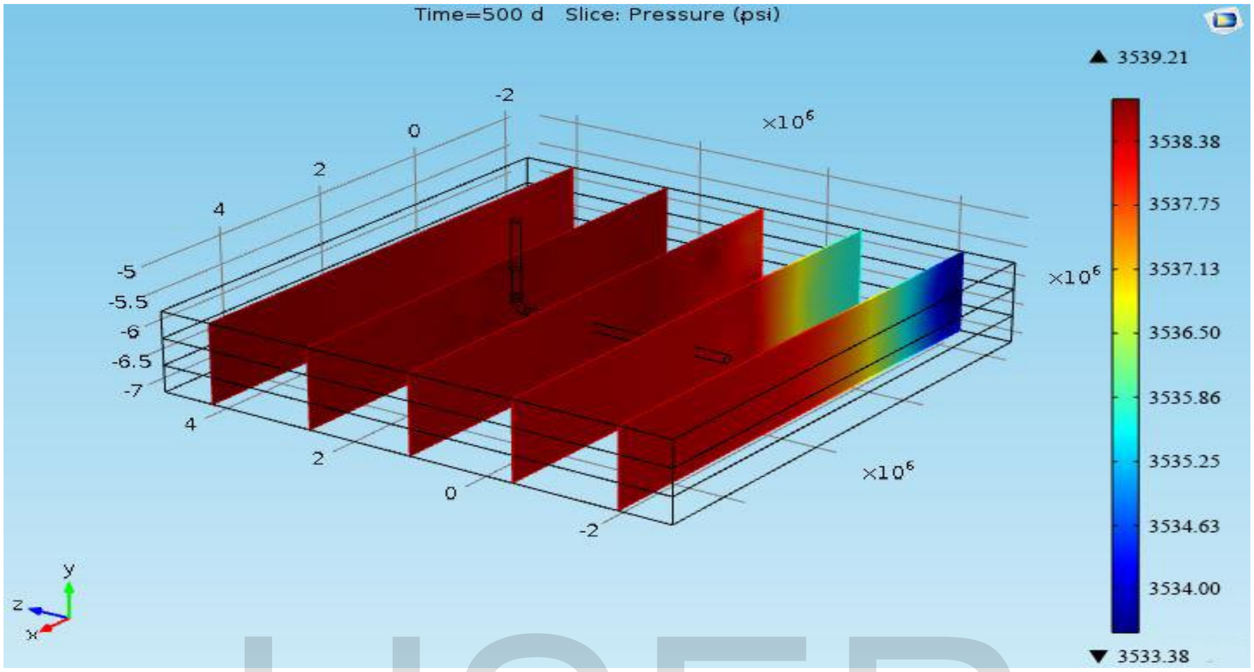



**Figure 11**: Pressure plot of the reservoir media at time = 500 days

The pressure further dropped from an average value of 3591.37psi to roughly 3536.50psi in figure 11 at 500th day of simulation. The least pressure value was 3533.38psi.

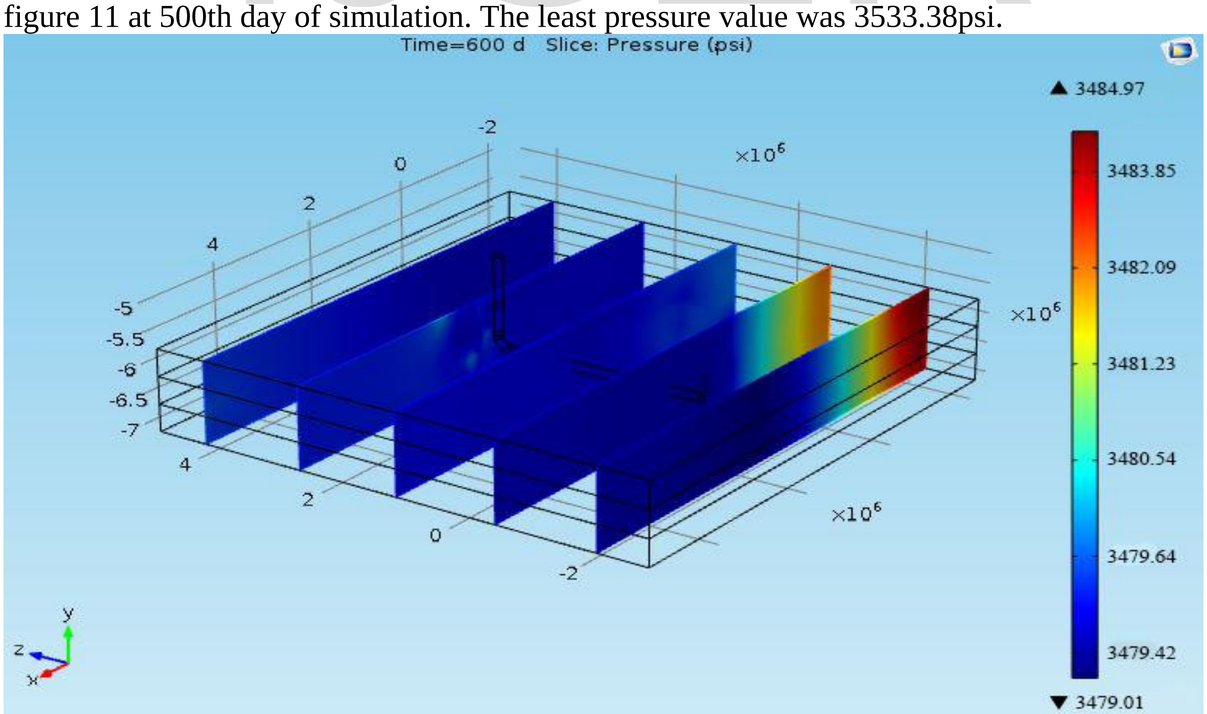



**Figure 12**: Pressure plot of the reservoir media at time= 600 days

At figure 12 of 600th day of simulation, the pressure further dropped from an average value of 3536.50psi to about 3481.23psi. The least pressure value noticed was 3479.01psi.

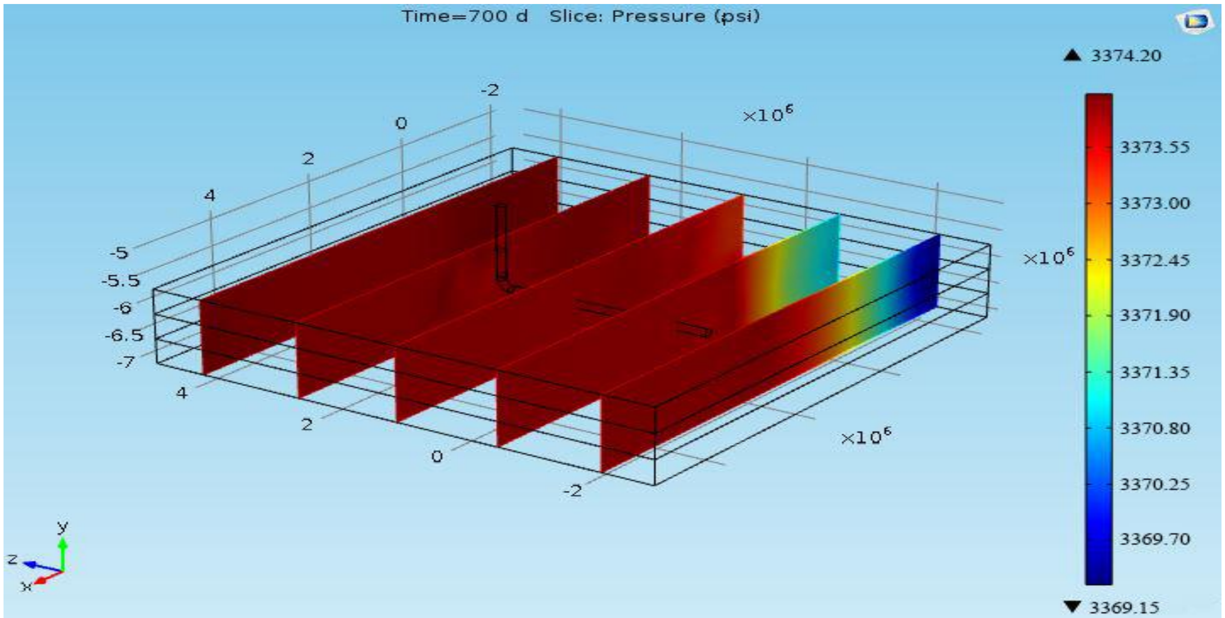



**Figure 13**: Pressure plot of the reservoir media at time = 700 days

At the last 700th day of simulation, the pressure again dropped from an average value of 3481.23psi to about 3371.90psi. The least pressure value noticed was 3369.15psi while the maximum was 3374.20.

## 4.4 Reservoir Fluid Potential

The reservoir fluid potential is the pressure difference between the reservoir pressure and the built up pressure caused by the effect of gravity and head. The mathematical formula is represented below:
$\varphi = p - \rho g \nabla D$. The table below shows the times; average reservoir media pressure values and reservoir fluid potential.

**Table 3: Table of average reservoir media pressure values and fluid potential**

| Time (days) | Average reservoir media pressure(psi) | Maximum reservoir media pressure(psi) | Minimum reservoir media pressure (psi) | Reservoir Fluid potential (psi) |
|---|---|---|---|---|
| 0 | 3814.00 | 3814.00 | 3814.00 | 3768.50 |
| 100 | 3757.29 | 3759.06 | 3755.01 | 3711.79 |
| 200 | 3700.29 | 3704.56 | 3699.08 | 3654.79 |
| 300 | 3646.80 | 3649.01 | 3644.01 | 3601.30 |
| 400 | 3591.37 | 3594.33 | 3588.09 | 3545.87 |
| 500 | 3536.50 | 3539.21 | 3533.38 | 3490.00 |
| 600 | 3481.23 | 3484.97 | 3479.01 | 3435.73 |
| 700 | 3371.90 | 3374.20 | 3369.15 | 3326.40 |

For easier interpretation of the results for the COMSOL, the Chart of mean average reservoir media pressure against Time was plotted below in figure 14

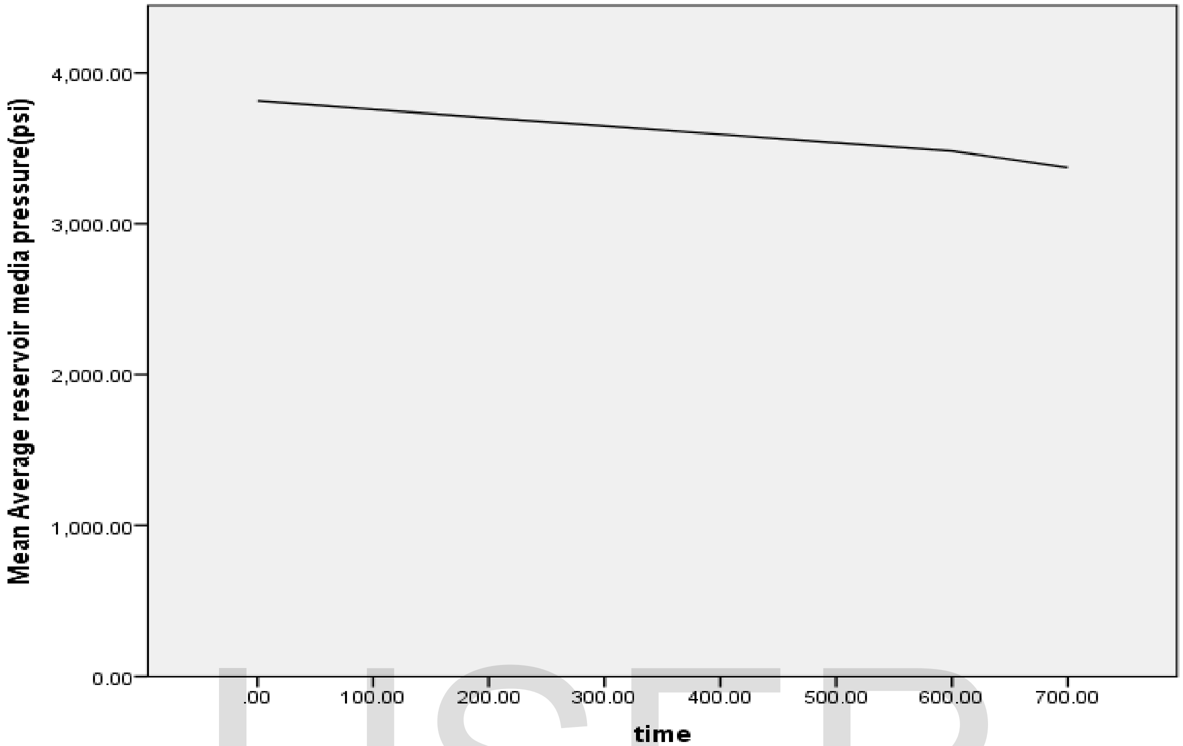



**Figure 14**: Chart of mean average reservoir pressure against Time

Table 4: Table of reservoir media pressure values [29]

| Time (days) | Average reservoir media pressure(psi) | Maximum reservoir media pressure(psi) | Minimum reservoir media pressure(psi) |
|---|---|---|---|
| 0 | 4000.00 | 4000.00 | 4000.00 |
| 100 | 3943.40 | 3943.50 | 3941.00 |
| 200 | 3886.70 | 3887.00 | 3884.50 |
| 300 | 3830.00 | 3830.00 | 3827.50 |
| 400 | 3773.20 | 3773.50 | 3771.00 |
| 500 | 3716.40 | 3716.50 | 3714.00 |
| 600 | 3659.50 | 3660.00 | 3657.50 |
| 700 | 3600.00 | 3603.50 | 3601.00 |

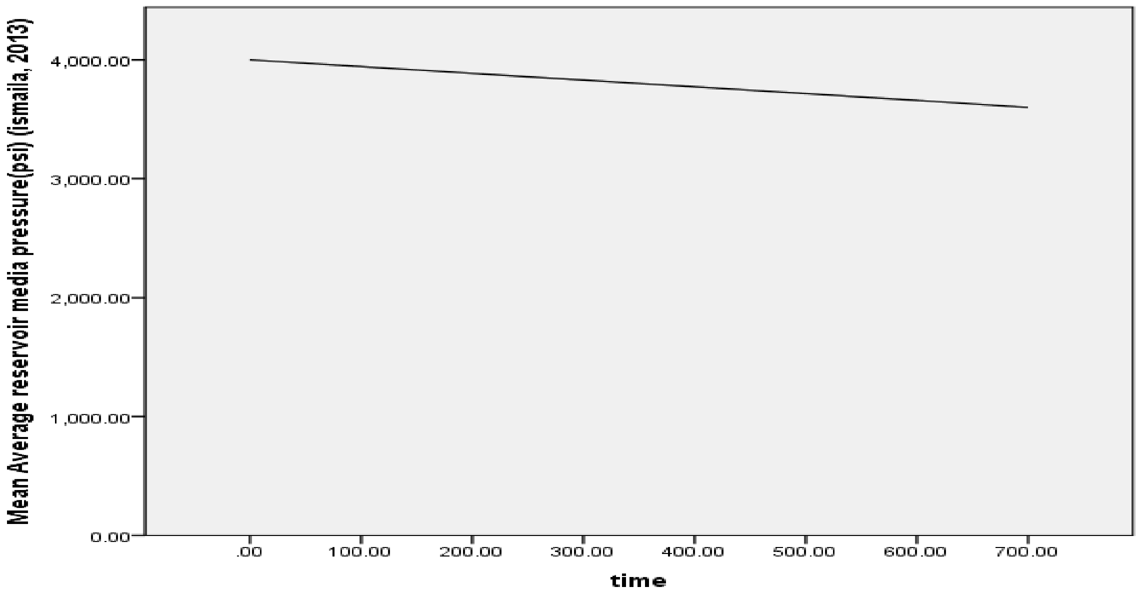



**Figure 15:** Chart of mean average reservoir pressure against Time [29]

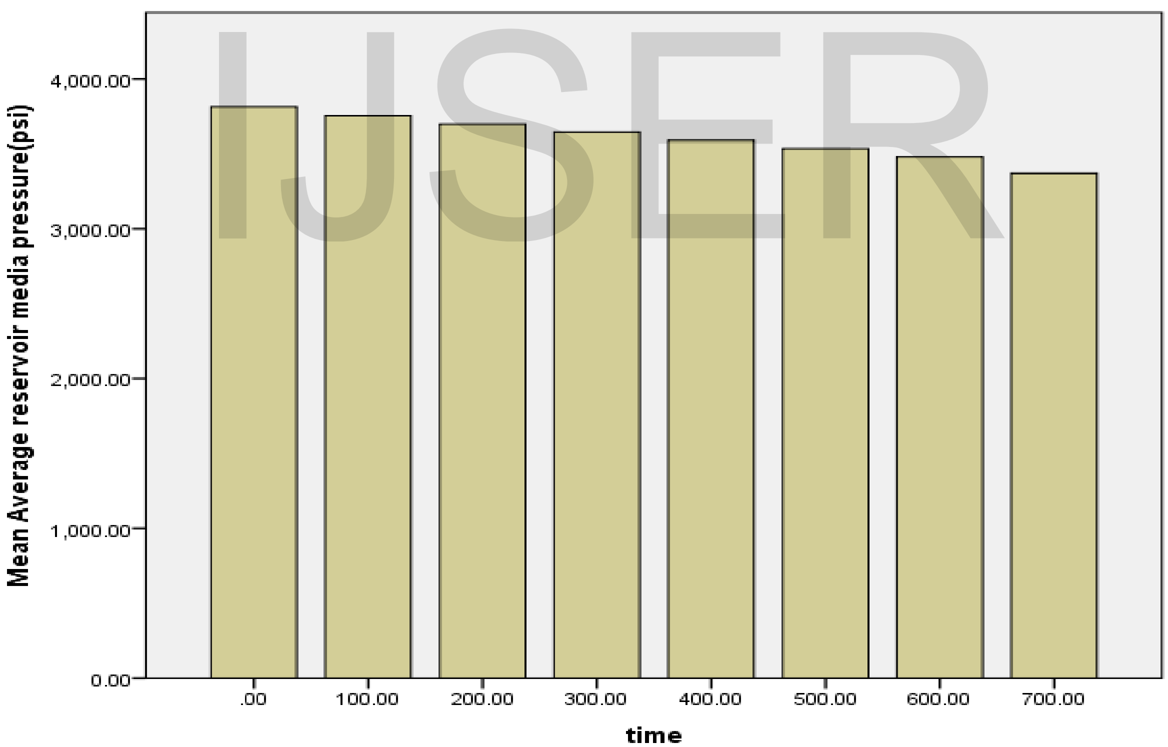



**Figure 16**: Bar plot of mean average reservoir pressure against time

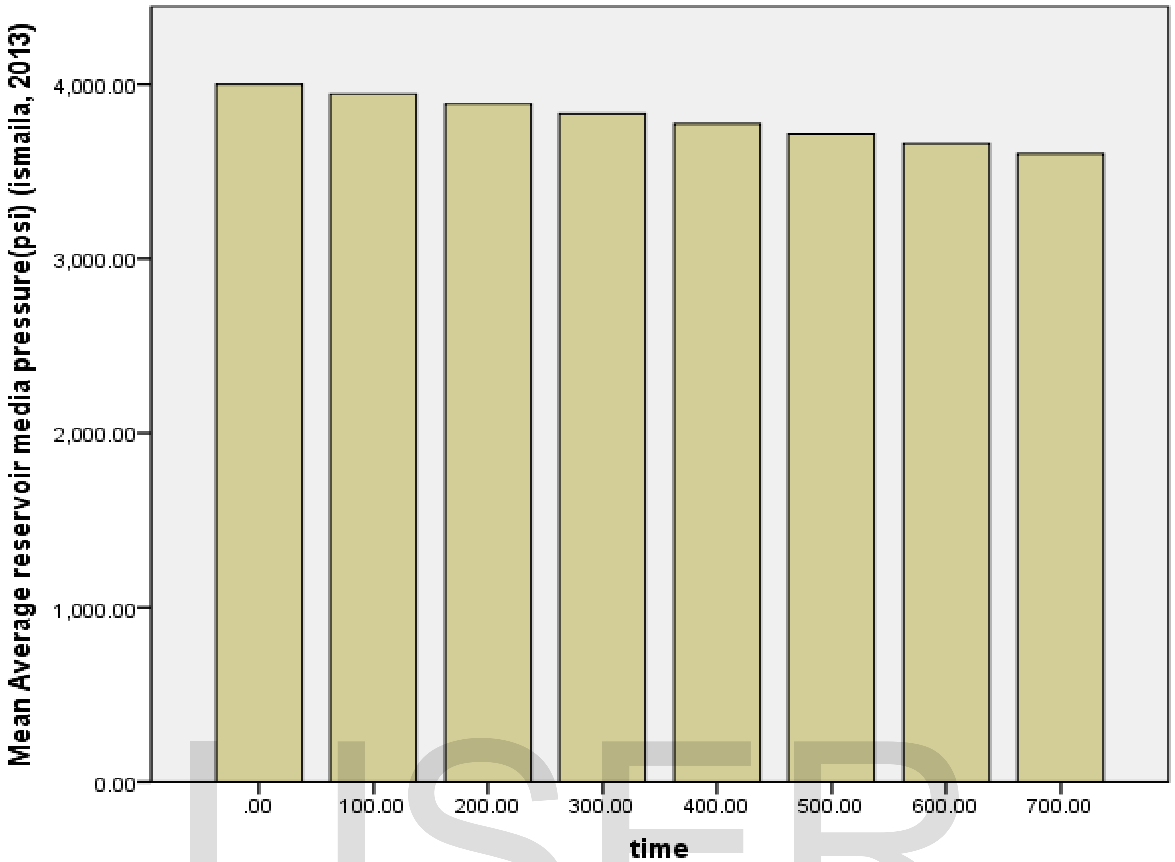



**Figure 17**: Bar plot of mean aveage reservoir pressure against time [29]

Table 5: Chi-Square Tests to Validate Findings

| | Value | Degree of Freedom | Asymp. Sig. (2-sided) |
|---|---|---|---|
| Pearson Chi-Square | 56.000[a] | 49 | .229 |
| Likelihood Ratio | 33.271 | 49 | .958 |
| Linear-by-Linear Association | 6.931 | 1 | .008 |
| No. of Valid Cases | 8 | | |

The above table presents Chi-square analysis which is used to validate the research findings, by comparing it with a previous study of Ismaila [30], this is to know if there is significant difference between his findings and the outcome of this study. Chi-square test is a non-parametric analytical technique that is used to know the differences and variations between variables of the same kind. From the table above, the significant value at 0.05 significant level is 0.229. This value is however greater than significant level of 0.05, this means the result is not significant. Relating this to the study means there is no difference between the study outcome and Ismaila's findings.

**Conclusion**

A review of the work done by many authors on porous media has given an insight that the complexities involved in treating a three phase reservoir problem in three dimensions justify the two phase two dimensional approximations. Hence, it is believed that a model of such sophistication has three phases-three dimensional reservoir models and should be studied only if the reservoir thickness is large. However, the importance of the models in some other conditions makes their study very important and thus despite the odds the model in this study was developed. This section contains the deductions from the finite element simulation of the flow characteristics of a horizontal oil well using COMSOL Multiphysics. The visual results generated by COMSOL were afterwards interpreted using SPSS tool. The deductions are presented below: The development of a reservoir engineering aspect of a horizontal oil well finite element model was achieved. It can be deduced evidently that the rate of increase of the time of simulation is inversely proportional to the mean average reservoir pressure. This study as completed validates the study of (Ismaila, 2013), which was on three dimensional numerical simulators for expansion-drive reservoir.

**Recommendation**

In this section, some recommendations for future studies are mentioned. It was realized that after completing this study, some areas of the simulation of petroleum reservoir media were not considered. Some of them are: Other pertinent factor that affect fluid flow like thermal effects and forces dues to electromagnetic forces were not considered. These factors should be considered in future works. More funding should be provided by various companies that may need to have some of their reservoir problems solve experimentally to carry out further simulations in small scales.



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