

# Implementation of MUSIC Algorithm for a Smart Antenna System for Mobile Communications

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**Abstract**— This paper presents practical design of a smart antenna system based on direction-of-arrival estimation and adaptive beam forming. Direction-of-arrival (DOA) estimation is based on the MUSIC algorithm for identifying the directions of the source signals incident on the sensor array comprising the smart antenna system. Adaptive beam forming is achieved using the LMS algorithm for directing the main beam towards the desired source signals and generating deep nulls in the directions of interfering signals. The smart antenna system designed involves a hardware part which provides real data measurements of the incident signals received by the sensor array. Results obtained verify the improved performance of the smart antenna system when the practical measurements of the signal environment surrounding the sensor array are used. This takes the form of sharper peaks in the MUSIC angular spectrum and deep nulls in the LMS array beam pattern.

**Index Terms**— Smart antennas, DOA estimation, adaptive beam forming, least mean

## 1 INTRODUCTION

Wireless networks face ever-changing demands on their spectrum and infrastructure resources. Increased minutes of use, capacity-intensive data applications, and the steady growth of worldwide wireless subscribers mean carriers will have to find effective ways to accommodate increased wireless traffic in their networks. However, deploying new cell sites is not the most economical or efficient means of increasing capacity. Smart antennas provide greater capacity and performance benefits than standard antennas because they can be used to customize and fine-tune antenna coverage patterns to the changing traffic or radio frequency (RF) conditions in a wireless network [1][6]. A smart antenna is a digital wireless communications antenna system that takes advantage of diversity effect at the source (transmitter), the destination (receiver), or both. Diversity effect involves the transmission and/or reception of multiple RF-waves to increase data speed and reduce the error rate. In conventional wireless communications, a single antenna is used at the source, and another single antenna is used at the destination. Such systems are vulnerable to problems caused by multipath effects.

Basic diagram of smart antenna is shown in Fig.1

From this which is explained about the beam forming one represents the desired user and interfering user towards this direction antenna will focus the radiation beam forming. Maximum radiation pattern provide towards the desired user and less beam provide the interfering user (unwanted user).

Functional Block Diagram of a smart antenna system as shown in fig 2



Fig.1. Basic diagram of smart antenna

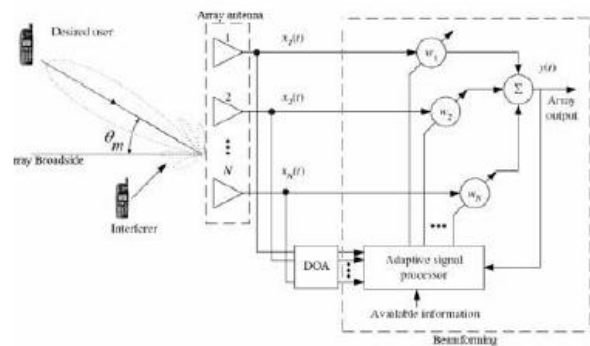


Fig.2 A functional block diagram of a smart antenna system [3].

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The use of smart antennas can reduce or eliminate these problems resulting in wider coverage and greater capacity. A smart antenna system at the base station of a cellular mobile system is depicted in Fig. 3. It consists of a uniform linear antenna array for which the current amplitudes are adjusted by a set of complex weights using an adaptive beam forming algorithm.

The adaptive beam forming algorithm optimizes the array output beam pattern such that maximum radiated power

is produced in the directions of desired mobile users and deep nulls are generated in the directions of undesired signals representing co-channel interference from mobile users in adjacent cells. Prior to adaptive beam forming, the directions of users and interferes must be obtained using a direction-of arrival (DOA) estimation algorithm [2]. DOA estimation as well as adaptive beam forming these models and simulations was based on pre-defined input data signals that simulate the signal environment surrounding the sensor array. This paper investigates the performance of smart antenna algorithms using an experimental setup that involves a hardware part used to collect real data measurements of the received signals impinging on the smart antenna sensor array. In this way, a more realistic and accurate description of the signal environment surrounding the sensor array is used to provide the input for the MUSIC algorithm being investigated.

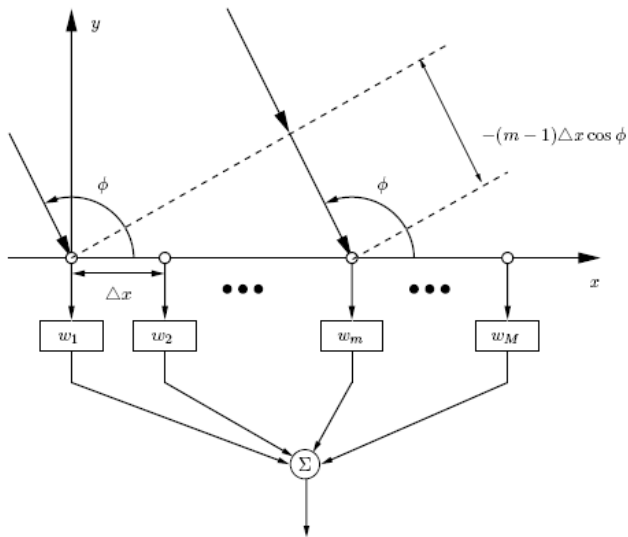


Fig.3 Uniform linear array with M elements

The paper is discussed as follows: Section 1 Introduction Section 2 develops the theory of smart antenna systems. Section 3 Signal Model and its MATLAB Implementation, Section 4 presents performance results for the designed smart antenna system. Finally, conclusions are given in Section 5.

## 2. THEORETICAL MODEL FOR AN ANTENNA ARRAY

### Intelligent antenna definition

An "intelligent" antenna is an array of spatially separated antennas whose outputs are fed into a weighting network. The part that makes the antenna array "intelligent" is the signal processing unit which calculates the weights that produce the desired radiation pattern of the array.

### 2.1 Conventional Charging

An antenna array can be arranged in any arbitrary fashion, but in this paper we will limit ourselves to linear arrays with uniformly spaced sensors (see figure 3). Let M be the number of antenna elements [4][5]. Usually an array has a reference element. Let's suppose that this reference element is located at the

origin. The coordinates of the  $m^{th}$  element are noted  $(x_m, y_m, z_m)$ .

There are L incoming signals due to L sources. The signals as they travel across the array undergo a phase shift. The phase shift between a signal received at the reference element and the same signal received at element m is given by

$$\Delta\gamma_m = \gamma_m(t) - \gamma_1(t) = -kx_m \cos \phi \sin \theta - ky_m \sin \phi \sin \theta - kz_m \cos \phi \quad (1)$$

Where  $k = \frac{2\pi}{\lambda}$  is the propagation constant in free space for linear array of equispaced elements with element spacing  $\Delta_x = d$  aligned along the x-axis such that the first element is situated at the origin, we have  $x_m = (m-1)d$  and  $y_m = z_m = 0$ . as we suppose that the signals are coming in horizontally we also have  $\theta = \pi/2$ .

$$\Delta\gamma_m = -kd(m-1) \cos \phi \quad (2)$$

Note that  $\Delta\gamma_1 = 0$ . Let's define the incoming signal at the array element 1 due to the  $l^{th}$  source by

$$s_l(t) = m_l(t) e^{j2\pi f_0 t} \quad (3)$$

Where  $m_l(t)$  is the modulating function of the  $l^{th}$  source and  $f_0$  the frequency of the carrier signal. The incoming signal at element m will be in that case

$$x_m(t) = m_l(t) e^{j(2\pi f_0 t + \Delta\gamma_m)} + n_m(t) = s_l(t) a_m(\phi) + n_m(t) \quad (4)$$

Where

$$a_m(\phi) = e^{j\Delta\gamma_m} = e^{-jkd(m-1) \cos \phi} \quad (5)$$

and where  $n_m(t)$  is a random noise component on the  $m^{th}$  element, which includes background noise and electronic noise generated in the  $m^{th}$  channel. It is assumed to be temporally white with zero mean and variance equal to  $\sigma_n^2$ .

We can now define the steering vector as

$$a(\phi) = \begin{pmatrix} 1 \\ a_2(\phi) \\ \dots \\ a_m(\phi) \\ \dots \\ a_M(\phi) \end{pmatrix} \quad (6)$$

Now if we consider all sources simultaneously, the signal at the  $m^{th}$  element will be

$$x_m = \sum_{l=1}^L m_l(t) e^{j(2\pi f_0 t + \Delta_{\gamma_m})} + n_m(t) = \sum_{l=1}^L s_l(t) a_m(\phi_l) + n_m(t) \quad (7)$$

Let's define the array signal vector by

$$X(t) = (x_1(t) x_2(t) x_3(t) \dots x_m(t) \dots x_M(t))^T \quad (8)$$

the incoming signal vector by

$$S(t) = (s_1(t) s_2(t) s_3(t) \dots s_l(t) \dots s_L(t))^T \quad (9)$$

the noise vector by

$$n(t) = (n_1(t) n_2(t) n_3(t) \dots n_m(t) \dots n_M(t))^T \quad (10)$$

and the steering matrix (dimensions M by L) by

$$A = (a(\phi_1) a(\phi_2) a(\phi_3) \dots a(\phi_l) \dots a(\phi_L)) \quad (11)$$

We can now write in matrix notation

$$X(t) = As(t) + n(t) \quad (12)$$

Let's denote the weights of the beam former as

$$W = (w_1 w_2 w_3 \dots w_m \dots w_M)^T \quad (13)$$

where w is called the array weight vector. The total array output will be

$$y(t) = \sum_{m=1}^M w_m^* x_m = W^H X(t) \quad (14)$$

Where superscripts T and H, respectively, denote the transpose and complex conjugate transpose of a vector or matrix. If the components of  $x(t)$  can be modeled as zero mean stationary processes, then for a given  $w$ , the mean output power of the process is given by

$$P = E[y(t) y^*(t)] = W^H R_{xx} W \quad (15)$$

where  $E[\cdot]$  denotes the expectation operator and  $R_{xx}$  is the array correlation matrix defined by

$$R_{xx} = E[X(t) X^H(t)] \quad (16)$$

Algebraic manipulation leads to

$$R_{xx} = ASA^H + \sigma_n^2 I \quad (17)$$

where I is an identity matrix of dimensions M and S (L by L matrix) denotes the source correlation

### 3. SIGNAL MODEL AND ITS MATLAB IMPLEMENTATION

We know that for a real antenna, vector x represents the signals received at the different antenna elements

#### The signal vector:

We have seen in the previous section

$$R_{xx} = ASA^H + \sigma_n^2 I \quad (18)$$

and

$$X(t) = As(t) + n(t) \quad (19)$$

Where s represents the incoming signal vector where each element represents the signal sent by one User. One of these signals is modeled by

$$s_l(t) = m_l(t) e^{j2\pi f_0 t} \quad (20)$$

Where  $m_l(t)$  denotes the complex modulating function. The structure of the modulating function reflects the particular modulation used in the system.

#### 3.1 The MUSIC Algorithm

##### DoA Estimation Methods

We have seen that for some beam formers, but also for other applications, it can be useful to gather information about the users' positions. Methods which extract this information from the incoming signals are called Direction of Arrival (DoA) Estimation Methods.

There are mainly two categories of such methods. The methods of the first one are called Spectral Estimation Methods. They include the MVDR (Minimum Variance Distortionless Response) Estimator, Linear prediction method, MEM (Maximum Entropy Method), MLM (Maximum Likelihood Method). The second category is formed by the Eigen structure Methods.

They include the MUSIC (Multiple Signal Classification) algorithm, the Min-Norm Method, the CLOSEST Method, the ESPRIT (Estimation of Signal Parameters via Rotational Invariance Technique) algorithm, and others. There are some more methods that do not fit in any of these two categories.

#### 3.2 Mathematical background of MUSIC

Computation of the correlation matrix The MUSIC algorithm requires the correlation matrix  $R_{xx}$ . In practice however, the exact value of

$$R_{xx} = E[x(t)x^H(t)] \quad (21)$$

is not available. Instead we have to replace  $R_{xx}$  with its estimate. An estimate of  $R_{xx}$  using N samples  $x(n)$ ,  $n = 0, 1,$

2,.....N - 1 of the array signals may be obtained using a simple averaging scheme

$$R_{xx} = \frac{1}{N} \sum_{n=0}^{N-1} X[n]X^H[n] \quad (22)$$

Where  $x(n)$  denotes the array signal sample, also known as the array snapshot, at the  $n$ th instant of time, with  $t$  replaced by  $nT$  and the sampling time  $T$  omitted for the ease of notation. For simplicity of notation, the estimate of  $R_{xx}$  is called  $R_{xx}$  too.

**Eigenvalue decomposition of the correlation matrix**

Next we have to do an eigenvalue decomposition of  $R_{xx}$ .  $R_{xx}$  is of dimension  $M$  by  $M$ , so it has generally  $M$  eigenvalues. Denoting the  $M$  eigenvalues of  $R_{xx}$  in descending order by  $\lambda_m$ ,  $m = 1, \dots, M$  and their corresponding unit-norm eigenvectors by  $U_m$ , the matrix takes the following form:

$$R_{xx} = \sum \Lambda \sum^H \quad (23)$$

with the diagonal matrix

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ & \lambda_m & \\ 0 & 0 & \lambda_M \end{pmatrix} \quad (24)$$

containing the eigenvalues in it's diagonal and

$$\sum = (U_1 \dots U_1 \dots U_M) \quad (25)$$

**3.3 The MUSIC spectrum**

The MUSIC spectrum is defined as

$$P(\phi) = \frac{1}{a^H(\phi) \sum_N \sum_N^H a(\phi)} \quad (26)$$

Where

$$\sum_N = (U_{L+1} \dots U_M) \quad (27)$$

is the collection of the noise eigenvectors and  $a(\phi)$  The matrix  $\sum \Lambda \sum_N^H$  is a projection matrix onto the noise subspace. For steering vectors that are orthogonal to the noise subspace, the denominator of (26) will become very small, and thus peaks will occur in  $P(\phi)$  corresponding to the angle of arrival of the signal. When the ensemble average of the array input covariance matrix is known and the noise can be considered uncorrelated and identically distributed between the elements, the peaks of the MUSIC

spectrum are guaranteed to correspond to the true angle of arrival of the signals incident on the array.

Some peaks in the MUSIC spectrum  $P(\phi)$  are usually very high, so that it is more convenient to plot the log of the MUSIC spectrum. The positions of these peaks are then determined by looking for the zero crossings of the derivate of  $\log(P(\phi))$ . Note that in order to make the MUSIC algorithm work, the incident signals mustn't be correlated.

**MATLAB implementation**

Important preliminary remark: In all the subsequent MATLAB simulations, the direction of a source is parameterized by the variable  $\theta$  (theta) and not by the variable  $\phi$ . In this paper we observed both **Static Cases and Dynamic case**

**4 SIMULATION RESULTS**

**4.1 Static Case**

Let's consider a simple case where  $M$  is 5 and  $L$  is 3. Signal to noise ratio (SNR) is 20 dB. The step size for calculating the MUSIC spectrum is 0.01 and the number of snapshots  $N$  is 10. Figure 4 shows from top to bottom the spectrum  $P$ ,  $\log(P)$  and it's derivate.

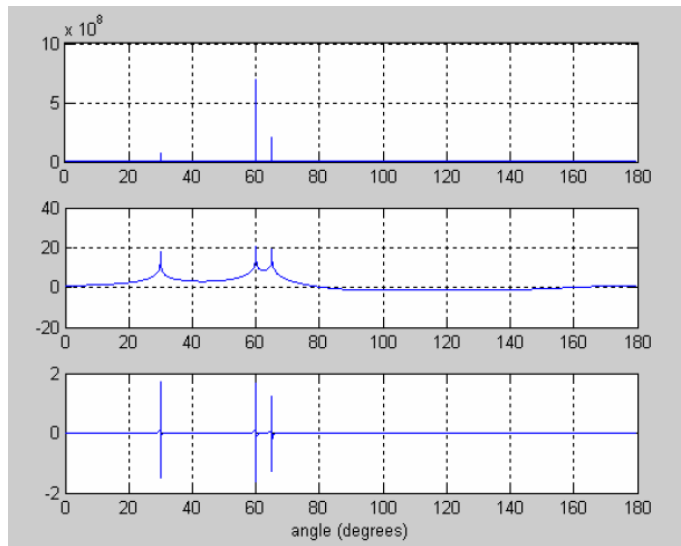


Figure 4 Spectrum  $P$ ,  $\log(P)$  and its derivate which id represented as linear scale

figure 4 represents the user positions  $P$  i.e. user one, user two, user three, represents the MUSIC spectrum of the signals (three user signals) ( $\log(P)$ ) and represents the zero crossing (its derivate) As you can see, the peaks in  $P$  are very high

**INFLUENCE OF SIGNAL TO NOISE RATIO**

In this section, we analyze the influence of the SNR. Fig 5 is taken from the previous section (SNR = 20 dB and  $N = 10$ ). We know from the previous section that this situation gives very good results for the positions.

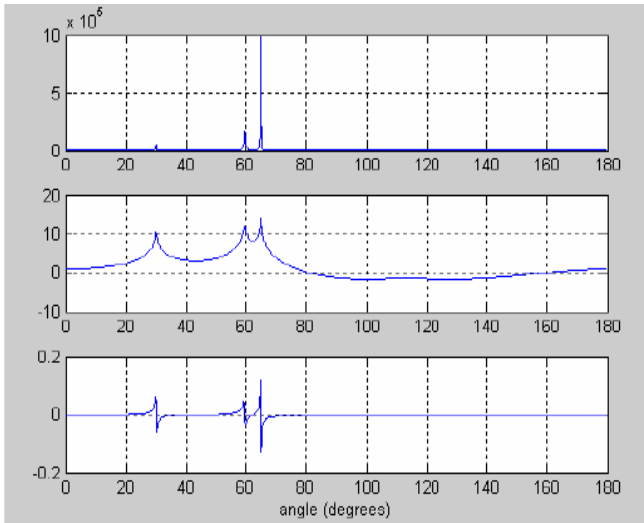


Fig: 5 results for SNR=20dB and N=10

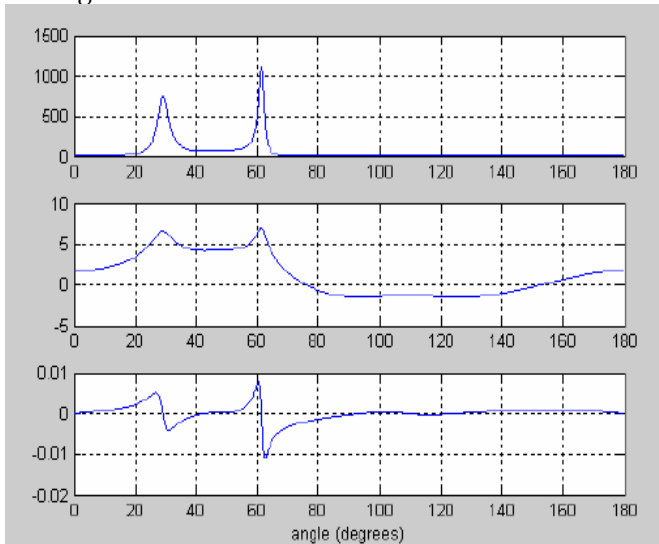


Fig: 6 results for SNR=10dB and N=10

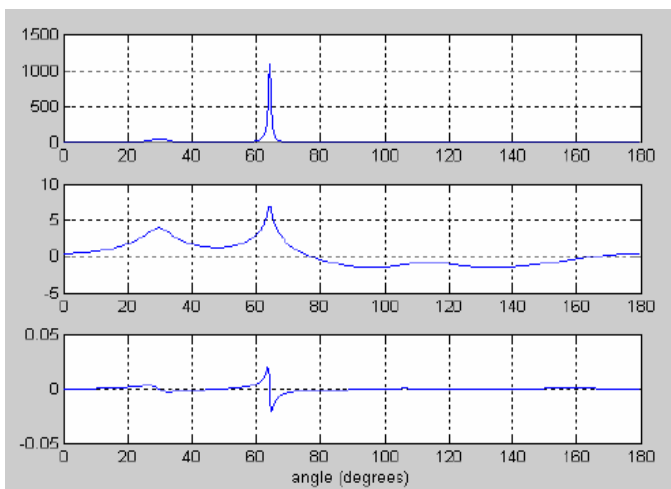


Fig: 7 results for SNR=0dB and N=10

**Dynamic case:**

Let's now have a look at the simulations for the dynamic case. This time we have  $M = 9$  and  $L = 5$  and the main user is moving from position  $50^\circ$  to position  $90^\circ$ . The window size is 10 samples. SNR is 20 dB. The spectrum step size is 0.1 and not 0.01 in order to make the simulation faster. We now consider a first example where the user moves from  $50^\circ$  to  $90^\circ$  in 1000 samples which means that we have 100 windows with 10 samples. This corresponds to a speed of  $0.04 \text{ deg/sample}$ . The result is shown in Fig.8 Of course; at the moment when the moving user is crossing the user at  $80^\circ$  the algorithm will only detect 4 users, because the two crossing users mask each other. Let's now consider a second example where the user moves from  $50^\circ$  to  $90^\circ$  in 100 samples. This means that we only have 10 windows with 10 samples.

This corresponds to a speed of  $0.4 \text{ deg/sample}$ , so the user moves by 4 degrees during one window. As you can see in Fig. 9, the results are much more imprecise this time

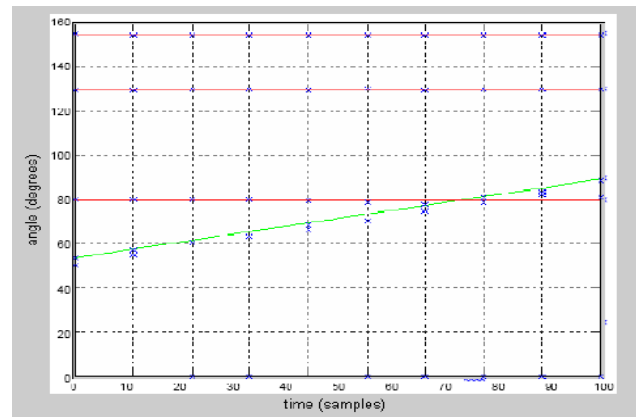


Fig 8: Real user positions and calculated user positions for user speed of  $0.04 \text{ deg/sample}$ .

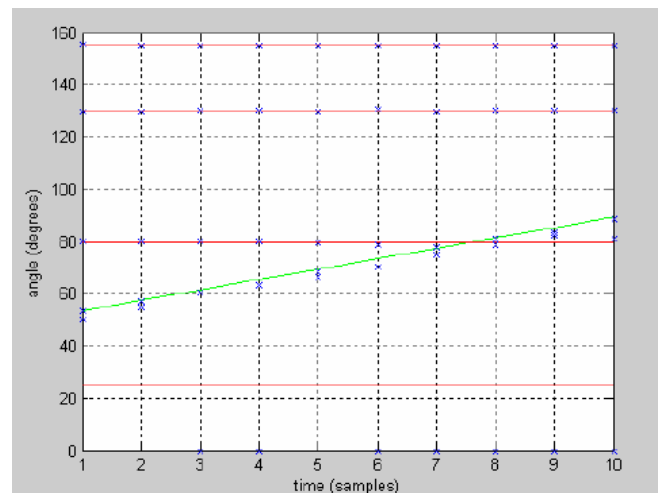


Fig.9 Real user positions and calculated user positions for user speed of  $0.4 \text{ deg/sample}$  in linear scale

## 5 CONCLUSION

Referring to the paper, one can say that the goals have been largely reached. A direction of arrival algorithm (MUSIC) has been implemented in MATLAB, followed by a parametric study of the static case. After that, a possible solution for the dynamic case has been proposed a detailed treatment of various methods of estimating the DOA's has been provided by including the description, limitation, and capability of each method and their performance comparison as well as their sensitivity to parameter perturbations. This paper provides references to studies where array beam-forming and DOA schemes are considered for mobile communications systems.

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