Exact Traveling Wave Solutions of Regularized Long Wave (RLW) Equation Using (G'/G)-expansion Method

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Abstract— In this paper, the G'/G - expansion method is used to seek exact traveling wave solutions of Regularized Long Wave (RLW) equation with the aid of symbolic computation. Some new and more general solutions, trigonometric function, hyperbolic function and rational function are constructed. The obtained results are justified the general G'/G **-**expansion method [20-26]. It is found that the solutions obtained by G'/G -expansion method are similar to the solutions given by general G'/G -expansion method. All constructed solutions are verified to satisfy the RLW equation by Maple software and some are found to satisfy the given equation and some are found as the modified forms of other solutions.

Index Terms— Regularized Long Wave (RLW) equation, Traveling Wave, G'/G -expansion, General G'/G -expansion, Nonlinear Evolution Equation (NLEE), Homogeneous balance.

1 Introduction

Nonlinear Evolution Equations (NLEEs) play an important role in different areas of Mathematical Physics such as Fluid Dynamics, Water Wave Mechanics, Plasma Physics, Solid State Physics, Optical Fibers and Quantum Mechanics as well as their applications. Therefore the solutions of NLEEs are highly desirable to the scientists and engineers for analysis and practical purposes. With the invention of symbolic computation software such as Maple, Mathematica and Matlab in the past few decades a number of significant numerical and analytic methods such as Cole-Hopf transformation method [7,8], Variational iteration method [9], Tanh function method [10,11], Jaccob-elliptic function method [12,13], Exp-function method [14], Homogeneous balance method [15], Hirota bilinear method [16], Auxiliary equation method [17], F-expansion method [18,19] and so on have been developed to search explicit traveling wave solutions of NLEEs arising in Mathematical Physics.

Recently, Wang et all. [1] proposed a new method known as G'/G -expansion method to find the exact traveling wave solution of NLEEs. This method is applied in some literatures as for example Z. L. Li [2], Zhang [3], Zayed and Gepreel [4], Malik et al [5] and Bekir [6] to investigate the traveling wave solutions of some useful NLEEs. Moreover, they have shown that the G'/G-expansion method is direct, concise and very effective to solve some NLEEs involving higher order nonlinear terms. Further research has been carried out to modify this method. Recently, Zayed and Zhou [20-25] have been proposed modified G'/G -expansion method. From the literature point of view, it is known that the RLW equation, introduced by Peregrine [27] for modeling the propagation of unidirectional weakly nonlinear and weakly dispersive water waves, is one of the model partial differential equation of the nonlinear dispersive waves which has numerous applications in different areas such as ion-acoustic waves and magneto-hydrodynamic waves in plasma, longitudinal dispersive waves in elastic rods, pressure waves in liquid-gas bubble mixtures and rotating flow down a tube. In spite of its applications in many fields, the equation is

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insufficiently studied to explore the exact traveling wave solutions which are necessary to recapitulate the dynamics of the traveling waves in different applications.

In this paper, the RLW equation is solved using the G'/G -expansion method and the solutions are verified with the solutions obtained by the general G'/G -expansion method. Moreover, the obtained solutions are compared with the solutions constructed by many researchers, such as [28-30].

The rest of the paper is organized as follows: Section 2 describes the G'/G -expansion method to find out exact traveling wave solutions of NLEEs. In section 3, application of this method to the RLW equation is illustrated. Section 4 deals with the use of general G'/G -expansion method to witness the solutions gained in this paper. In section 5, results and discussion with graphical representations are explained. Finally conclusions are given in section 6.

2 The G'/G -expansion method

We assume that the nonlinear evolution equation in two variables, namely x and t is given by

$$F(u,u_x,u_{xt},u_{tt},u_{xx},.....)=0 \quad (1)$$

where $u(x,t)$ is an unknown function, F is a polynomial in $u(x,t)$ and subscripts are indicating partial derivatives involving the highest order derivatives and nonlinear terms. The main steps of the G'/G -expansion method are given as follows:

Step 1:

First we use the following traveling wave transformation to express the independent variables x and t by the variable ξ.

$$u(x,t)=u(\xi),\ \xi=x-ct \quad (2)$$

where ξ is a traveling wave variable and c is the wave velocity, which permits us to transform the equation (1) into the Ordinary Differential Equation (ODE) of the form:

$$F(u,u',u'',.....)=0 \quad (3)$$

Step 2:

We suppose that the solution of equation (3) can be expressed by a polynomial in $\frac{G'}{G}$ as follows:

$$u(\xi)=\sum_{i=0}^{n} a_i\left(\frac{G'}{G}\right)^i \quad (4)$$

where a_i's are constants to be determined such that a_n cannot be zero at the same time and $G=G(\xi)$ satisfies the following second order nonlinear ODE

$$G''+\lambda G'+\mu G=0 \quad (5)$$

where λ and μ are real constants and

$$G'=\frac{dG}{d\xi}, G''=\frac{d^2G}{d\xi^2}.$$

Step 3:

To determine the positive integer n appearing in equation (4), we consider the homogeneous balance between the highest order derivative and highest order nonlinear term appearing in the equation obtained by substituting equation (4) into equation (3).

Step 4:

Substituting n into equation (4) and then equation (4) to the equation (3) yields a polynomial in $\frac{G'}{G}$.

Equating the coefficients of this polynomial to zero, we obtain a set of algebraic equations in a_i, c, λ and μ. Solving the system by algebraic computation, values of a_i, c, λ and μ can be found.

Step 5:

Finally, substituting the general solution of equation (5), values of a_i, c, λ, μ and equation (2) into the equation (4) we avail more traveling wave solutions of equation (1).

3 Application of the method

We consider the following Regularized Long Wave Equation to apply the $\frac{G'}{G}$ - expansion method.

$$u_t + u_x + \alpha u u_x - \beta u_{xxt} = 0 \qquad (6)$$

where α and β are non-zero arbitrary constants. Using the traveling wave transformation (2), the equation (6) transformed into the nonlinear ODE

$$-cu'(\xi) + u'(\xi) + \alpha u(\xi)u'(\xi) + c\beta u'''(\xi) = 0 \qquad (7)$$

Integrating equation (7) with respect to ξ reduces to

$$-cu(\xi) + u(\xi) + \frac{1}{2}\alpha u^2(\xi) + c\beta u''(\xi) + K = 0 \qquad (8)$$

where K is an arbitrary constant and prime denotes the derivative with respect to ξ.

Now, substituting equation (4) into equation (8) and making the homogeneous balance between u'' and u^2 we have $n = 2$. Therefore the solution of the equation (6) is of the following form:

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2 \qquad (9)$$

where a_0, a_1 and a_2 are non-zero constants.

Using equation (9) in equation (8) with the help of equation (5) yields a polynomial in $\frac{G'}{G}$ and setting the coefficients of $\left(\frac{G'}{G}\right)^i, i = 0,1,2$ equal to zero we obtain a system of algebraic equations in $a_0, a_1, a_2, \lambda, \mu$ and K as follows:

$$6\beta c a_2 + \frac{1}{2}\alpha a_2^2 = 0$$

$$10\beta c\lambda a_2 + 2c\beta a_1 + \alpha a_1 a_2 = 0$$

$$-ca_2 + 3\beta c\lambda a_1 + 4\beta c\lambda^2 a_2 + 8\beta c\mu a_2 + \alpha a_0 a_2 + a_2 + \frac{1}{2}\alpha a_1^2 = 0$$

$$6\beta c\lambda\mu a_2 + c\beta\lambda^2 a_1 + 2c\mu\beta a_1 + \alpha a_0 a_1 - ca_1 + a_1 = 0$$

$$-ca_0 + \beta c\lambda\mu a_1 + \frac{1}{2}\alpha a_0^2 + a_0 + 2c\mu^2\beta a_2 + K = 0$$

Solving the above system of equations for $a_0, a_1, a_2, \lambda, \mu$ and K by Maple -17 software we have the following results:

$$a_0 = -\frac{c\beta\lambda^2 + 8c\mu\beta - c + 1}{\alpha}, \quad a_1 = -\frac{12\beta c\lambda}{\alpha},$$
$$a_2 = -\frac{12\beta c}{\alpha}, \quad c = c \text{ and}$$
$$K = -\frac{c^2\beta^2\lambda^4 - 8c^2\mu\beta^2\lambda^2 + 16c^2\mu^2\beta^2 - c^2 + 2c - 1}{2\alpha}. \qquad (10)$$

where λ and μ are free parameters and $\alpha \neq 0$.

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The general solution of equation (5) is given by

$$G(\xi) = C_1 e^{\left(-\frac{1}{2}\lambda+\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)\xi} + C_2 e^{\left(-\frac{1}{2}\lambda-\frac{1}{2}\sqrt{\lambda^2-4\mu}\right)\xi} \tag{11}$$

Substituting the equation (11) in equation (4) yields the following solutions:

I Hyperbolic function solution

When $\lambda^2 - 4\mu > 0$ we obtain

$$u_1(\xi) = a_0 + a_1\left(-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\frac{U\sinh\left(\frac{\xi\sqrt{\Delta}}{2}\right)+V\cosh\left(\frac{\xi\sqrt{\Delta}}{2}\right)}{U\cosh\left(\frac{\xi\sqrt{\Delta}}{2}\right)+V\sinh\left(\frac{\xi\sqrt{\Delta}}{2}\right)}\right) + a_2\left(-\frac{\lambda}{2}+\frac{\sqrt{\Delta}}{2}\frac{U\sinh\left(\frac{\xi\sqrt{\Delta}}{2}\right)+V\cosh\left(\frac{\xi\sqrt{\Delta}}{2}\right)}{U\cosh\left(\frac{\xi\sqrt{\Delta}}{2}\right)+V\sinh\left(\frac{\xi\sqrt{\Delta}}{2}\right)}\right)^2 \tag{12}$$

where $\Delta = \lambda^2 - 4\mu,\ U = C_1 + C_2,\ V = C_1 - C_2$ and $\xi = x - ct$

II Trigonometric function solution

When $\lambda^2 - 4\mu > 0$ we obtain

$$u_2(\xi) = a_0 + a_1\left(-\frac{\lambda}{2}-\frac{\sqrt{\Delta}}{2}\frac{U\sin\left(\frac{\xi\sqrt{\Delta}}{2}\right)-V\cos\left(\frac{\xi\sqrt{\Delta}}{2}\right)}{U\cosh\left(\frac{\xi\sqrt{\Delta}}{2}\right)+V\sinh\left(\frac{\xi\sqrt{\Delta}}{2}\right)}\right) + a_2\left(-\frac{\lambda}{2}-\frac{\sqrt{\Delta}}{2}\frac{U\sin\left(\frac{\xi\sqrt{\Delta}}{2}\right)-V\cos\left(\frac{\xi\sqrt{\Delta}}{2}\right)}{U\cosh\left(\frac{\xi\sqrt{\Delta}}{2}\right)+V\sinh\left(\frac{\xi\sqrt{\Delta}}{2}\right)}\right)^2 \tag{13}$$

where $\Delta = \lambda^2 - 4\mu,\ U = C_1 + C_2,\ V = C_1 - C_2$ and $\xi = x - ct$

II Rational function solution

When $\lambda^2 - 4\mu > 0$ we obtain

$$u_3(\xi) = a_0 + a_1\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\xi}\right) + a_2\left(-\frac{\lambda}{2}+\frac{C_2}{C_1+C_2\xi}\right)^2 \tag{14}$$

where $\Delta = \lambda^2 - 4\mu,\ U = C_1 + C_2,\ V = C_1 - C_2$ and $\xi = x - ct$

Substituting (10) into the general solutions given by equation (12) to (14) we get the following family of exact traveling wave solutions:

Family1. (Hyperbolic function solutions)

Case 1. $(U, V \neq 0)$

$$u_{1,1}(\xi) = -\frac{c\beta\lambda^2 + 8c\mu\beta - c + 1}{\alpha} + -\frac{12\beta c\lambda}{\alpha}\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\frac{U\sinh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right) + V\cosh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}{U\cosh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right) + V\sinh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}\right)$$

$$-\frac{12\beta c}{\alpha}\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\mu}}{2}\frac{U\sinh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right) + V\cosh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}{U\cosh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right) + V\sinh\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}\right)^2 \quad (15)$$

where $\xi = x - ct$

Case 2. $(U = 0, V \neq 0)$

$$u_{1,2}(\xi) = \frac{1}{\alpha}\left(-3c\beta\lambda^2\coth\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2 + 12c\mu\beta\coth\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2 + 2c\beta\lambda^2 - 8c\mu\beta + c - 1\right) \quad (16)$$

where $\xi = x - ct$

Case 3. $(U \neq 0, V = 0)$

$$u_{1,3}(\xi) = \frac{1}{\alpha}\left(-3c\beta\lambda^2\tanh\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2 + 12c\mu\beta\tanh\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2 + 2c\beta\lambda^2 - 8c\mu\beta + c - 1\right) \quad (17)$$

where $\xi = x - ct$

Family2. (Trigonometric function solutions)

Case 1. $(U,V\neq 0)$

$$u_{2,1}(\xi)=-\frac{c\beta\lambda^2+8c\mu\beta-c+1}{\alpha}+-\frac{12\beta c\lambda}{\alpha}\left(-\frac{\lambda}{2}-\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{U\sin\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)+V\cos\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}{U\cos\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)+V\sin\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}\right)$$
$$-\frac{12\beta c}{\alpha}\left(-\frac{\lambda}{2}-\frac{\sqrt{\lambda^2-4\mu}}{2}\frac{U\sin\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)+V\cos\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}{U\cos\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)+V\sin\left(\frac{\xi\sqrt{\lambda^2-4\mu}}{2}\right)}\right)^2 \tag{18}$$

where $\xi=x-ct$

Case 2. $(U=0,V\neq 0)$

$$u_{2,2}(\xi)=\frac{1}{\alpha}\left(-3c\beta\lambda^2\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2+12c\mu\beta\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2+2c\beta\lambda^2-8c\mu\beta+c-1\right) \tag{19}$$

where $\xi=x-ct$

Case 3. $(U\neq 0,V=0)$

$$u_{2,3}(\xi)=\frac{-2c\beta\lambda^2\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2+8c\mu\beta\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2+3c\beta\lambda^2-c\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2-12c\mu\beta+\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2}{\alpha\cot\left(\frac{1}{2}\xi\sqrt{\lambda^2-4\mu}\right)^2} \tag{20}$$

where $\xi=x-ct$

Family3. (Rational function solutions)

Case 1. $(C_1,C_2\neq 0)$

$$u_{3,1}(\xi)=-\frac{c\beta\lambda^2+8c\mu\beta-c+1}{\alpha}+\frac{6c\beta\lambda\left(C_2\xi\lambda+C_1\lambda-2C_2\right)}{\alpha\left(C_2\xi+C_1\right)}-\frac{3c\beta\left(C_2\xi\lambda+C_1\lambda-2C_2\right)^2}{\alpha\left(C_2\xi+C_1\right)^2} \tag{21}$$

where $\xi=x-ct$ and C_1,C_2 are arbitrary constants.

Case 2. $(C_1 = 0, C_2 \neq 0)$

$$u_{3,2}(\xi) = -\frac{c\beta\lambda^2 + 8c\mu\beta - c + 1}{\alpha} + \frac{6c\beta\lambda(\xi\lambda - 2)}{\alpha\xi} - \frac{3c\beta(\xi\lambda - 2)^2}{\alpha\xi^2} \tag{22}$$

where $\xi = x - ct$

As far as I know that the solutions presented by equation (15) to equation (22) have not been constructed in former research. Although the integrating constant K, occurred in equation (8), have significant effect on solving the system of equations but it has not taken into account in the previous literatures [28-30].

4 Verification of the solutions

In this section, the general $\frac{G'}{G}$- expansion method is applied to check the constructed solutions in section 3. Assume that the solution of equation (6) can be express in the following form

$$u(\xi) = a_{-2}\left(\frac{G'}{G}\right)^{-2} + a_{-1}\left(\frac{G'}{G}\right)^{-1} + a_0 + a_1\left(\frac{G'}{G}\right) + a_2\left(\frac{G'}{G}\right)^2 \tag{23}$$

where a_{-2}, a_{-1}, a_0, a_1 and a_2 are non-zero constants.

Using equation (23) and equation (5) in equation (8) gives a polynomial in $\frac{G'}{G}$ and setting the coefficients of $\left(\frac{G'}{G}\right)^i, i = -2,-1,0,1,2$ equal to zero we obtain a system of algebraic equations in $a_{-2}, a_{-1}, a_0, a_1, a_2, \lambda, \mu$ and K as follows:

$$6\beta c a_2 + \frac{1}{2}\alpha a_2^2 = 0$$

$$10\beta c\lambda a_2 + 2c\beta a_1 + \alpha a_1 a_2 = 0$$

$$-ca_2 + 3\beta c\lambda a_1 + 4\beta c\lambda^2 a_2 + 8\beta c\mu a_2 + \alpha a_0 a_2 + a_2 + \frac{1}{2}\alpha a_1^2 = 0$$

$$6\beta c\lambda\mu a_2 + c\beta\lambda^2 a_1 + 2c\mu\beta a_1 + \alpha a_2 a_{-1} + \alpha a_0 a_1 - ca_1 + a_1 = 0$$

$$\frac{1}{2}\alpha a_0^2 - ca_0 + \beta c\lambda a_{-1} + \alpha a_{-2} a_2 + \alpha a_{-1} a_1 + 2\beta c a_{-2} + \beta c\lambda\mu a_1 + a_0 + 2c\mu^2\beta a_2 + K = 0$$

$$c\beta\lambda^2 a_{-1} + 2c\mu\beta a_{-1} + 6c\beta\lambda a_{-2} + \alpha a_{-2} a_1 + \alpha a_{-1} a_0 - ca_{-1} + a_{-1} = 0$$

$$3\beta c\mu\lambda a_{-1} + 4c\beta\lambda^2 a_{-2} + 8c\beta\mu a_{-2} + \alpha a_{-2} a_0 + \frac{1}{2}\alpha a_{-1}^2 - ca_{-2} + a_{-2} = 0$$

$$2c\mu^2\beta a_{-1} + 10c\mu\beta\lambda a_{-2} + \alpha a_{-2} a_{-1} = 0$$

$$6\beta c\mu^2 a_{-2} + \frac{1}{2}\alpha a_{-2}^2 = 0$$

Solving the above system of equations for $a_{-2}, a_{-1}, a_0, a_1, a_2, \lambda, \mu$ and K by Maple-17 software we have the following results:

Set 1.

$$a_0 = -\frac{c\beta\lambda^2 + 8c\mu\beta - c + 1}{\alpha},\ a_1 = -\frac{12\beta c\lambda}{\alpha},\ a_2 = -\frac{12\beta c}{\alpha},\ c = c \text{ and}$$
$$K = -\frac{c^2\beta^2\lambda^4 - 8c^2\mu\beta^2\lambda^2 + 16c^2\mu^2\beta^2 - c^2 + 2c - 1}{2\alpha}. \tag{24}$$

where λ and μ are free parameters and $\alpha \neq 0$.

Set 2.

$$a_{-1} = -\frac{12\beta c\mu\lambda}{\alpha},\ a_{-2} = -\frac{12\beta c\mu^2}{\alpha} a_0 = -\frac{c\beta\lambda^2 + 8c\mu\beta - c + 1}{\alpha}, a_1 = 0, a_2 = 0,\ c = c \text{ and}$$
$$K = -\frac{c^2\beta^2\lambda^4 - 8c^2\mu\beta^2\lambda^2 + 16c^2\mu^2\beta^2 - c^2 + 2c - 1}{2\alpha}. \tag{25}$$

where λ and μ are free parameters and $\alpha \neq 0$.

It is certain that the obtained results given by (24) and (10) are identical.

5 Results and Discussion

We have investigated the obtained solution by putting them back to the equation (8) using the Maple-17 software. It is notable that the family1 and family3 satisfy the equation (8) directly and family2 satisfies for a special condition as family3 which is not specifically found in previous literatures. Furthermore, by taking the specific values of $\lambda, \mu, \alpha, \beta$ and c the dynamics of acquired exact traveling waves are presented from figure1 to figure7 with the aid of computational software Maple-17.

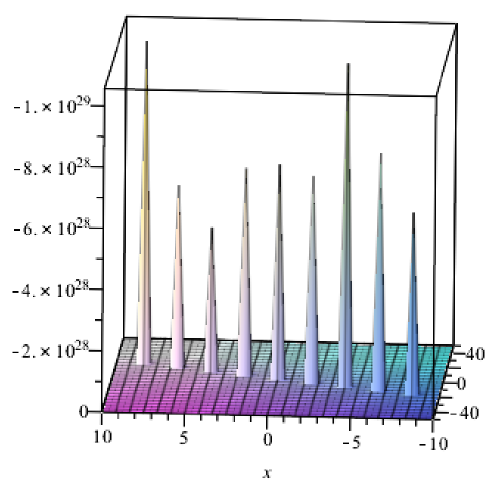

Fig1. $\lambda = 3, \mu = 1, \alpha = 2, \beta = 1$ and $c = .5$

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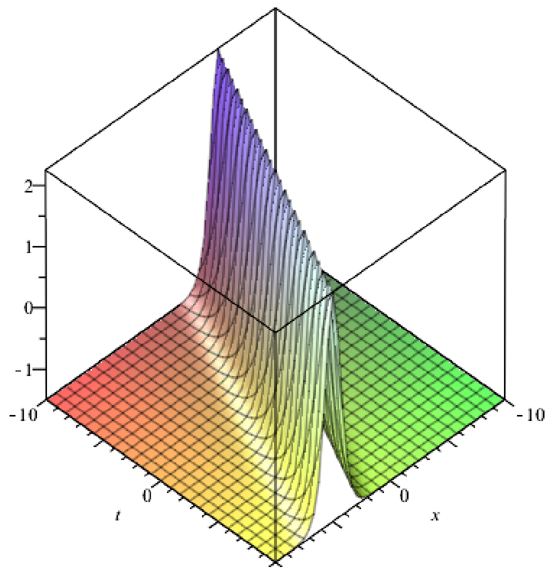

Fig2. $\lambda = 3, \mu = 1, \alpha = 2, \beta = 1$ and $c = .5$

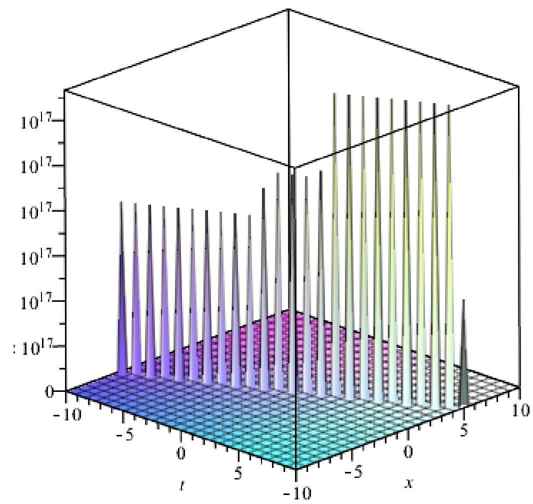

Fig4. $\lambda = 3, \mu = 1, \alpha = 2, \beta = 1$ and $c = .5$

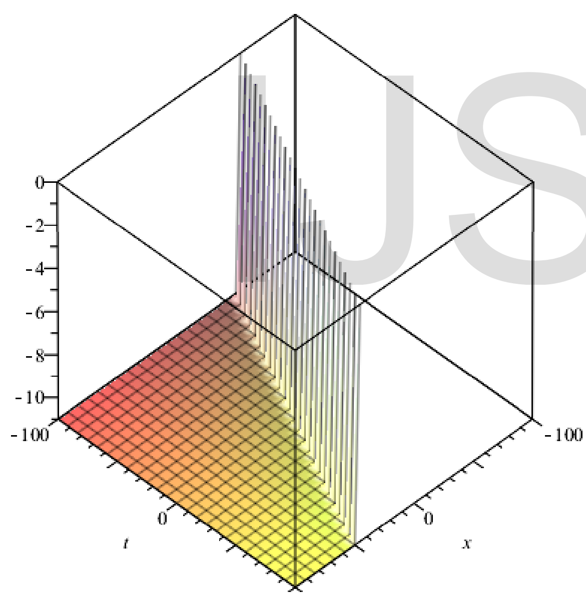

Fig3. $\lambda = 3, \mu = 1, \alpha = 2, \beta = 1$ and $c = .5$

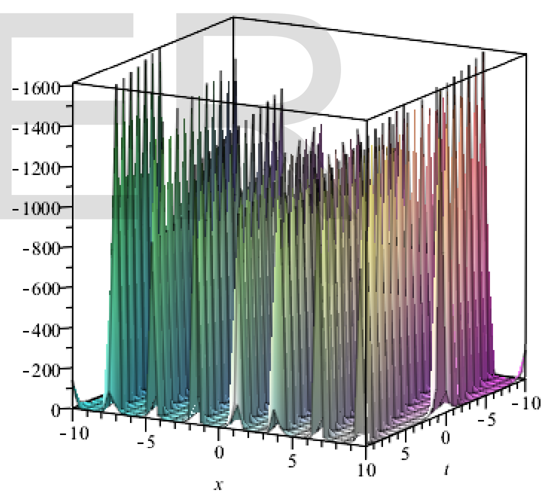

Fig5. $\lambda = 3, \mu = 1, \alpha = 2, \beta = 1$ and $c = .5$

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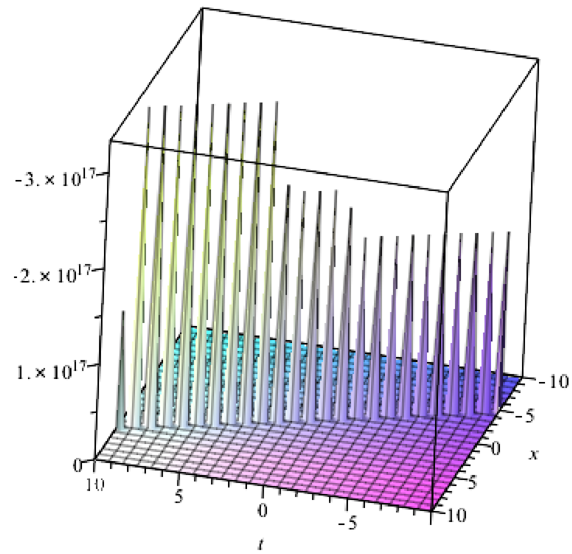

Fig6. $\lambda = 4, \mu = 1, \alpha = 1, \beta = 1$ and $c = .5$

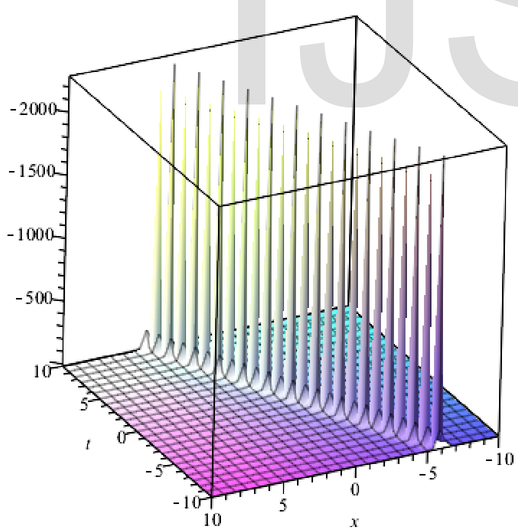

Fig7. $\lambda = 6, \mu = 2, \alpha = 3, \beta = 2$ and $c = .5$

6 Conclusion

In this article, the RLW equation is inquired and successfully explored some new exact traveling wave solutions by using the $\frac{G'}{G}$- expansion method. The acquired solutions include hyperbolic function solutions, trigonometric function solutions as well as rational function solutions with arbitrary constants where the trigonometric function solutions are not the explicit solutions. The constants provide enough freedom to construct exact travelling wave solutions which may be used to study real structure of the considered physical problem. In addition, the new solution structures and physical phenomena of the considered NLEE are convenient to understand from the provided graphs.

References

[1] M. Wang, X. Li and J. Zhang, The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics, Physics Letters A, 372 (2008) 417-423.

[2] Constructing of new exact solutions to the GKdV–mKdV equation with any-order nonlinear terms by (G'/G)-expansion method, Applied Mathematics and Computation, 217(2010):1398-1403.

[3] S. Zhang, L. Dong, J. Ba and Y. Sun, The (G'/G)-expansion method for nonlinear differential-difference equations, Physics Letters A, 373 (2009): 905-910.

[4] E.M.E. Zayed, K.A. Gepreel, The (G'/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics, J. Math. Phys. 50 (2009): 013502–013513

[5] A. Malik, F. Chand, H. Kumar and S.C. Mishra, Exact solutions of some physical models using the (G'/G)-expansion method, Pramana-Journal of Physics, 78 (2012) 513-529

[6] A Bekir, Application of the (G'/G)-expansion method for nonlinear evolution equations, Physics Letters A 372 (2008) 3400–3406

[7] Cole, J.D., On a quasi-linear parabolic equation occurring in aerodynamics. Quart. Appl. Math., 9(1951):225-236.

[8] Hopf, E., The partial differential equation $u_t + uu_x = \mu u_{xx}$. Commun. Pure Appl. Math., 3(1950):201-230.

[9] He.J. H and Wu. X. H. Construction of solitary solution and compactionlike solution by variational iteration method. Chaos, Solitons & Fractals, 29 (2006): 108-113.

[10] E.J. Parkes, B.R. Duffy, An automated Tanh-function method for finding solitary wave solution to non-linear evolution equations, Comput. Phys.Commun. 98 (1996): 288–300.

[11] Malfiet. W. The Tanh method: I. Exact solutions of nonlinear evolution and wave equations. Phys Scripta, 54 (1996):563-568

[12] E.G. Fan, J. Zhang, Applications of the Jaccobi elliptic function method to special-type nonlinear equations, Phys. Lett. A 305 (2002): 383–392

[13] Zhang. H. New exact Jacobi elliptic function solutions for some nonlinear evolution equations. Chaos, Solitons & Fractals, 32 (2007):653-660.

[14] J.H. He, X.H. Wu, Exp-function method for nonlinear wave equations, Chaos Soliton Fract. 30 (2006):700–708

[15] M. L. Wang, Y. B. Zhou, and Z. B. Li, Application of a homogeneous balance method to exact solutions of nonlinear equations in mathematical physics, Physics Letters A, 216(1996): 67–75,.

[16] R Hirota, The direct method in soliton theory (Cambridge University press, Cambridge,2004)

[17] Sirendaoreji. Auxiliary equation method and new solutions of KleinGordon equations. Chaos, Solitons & Fractals, 31 (2007) :943-950.

[18] Zhang. S. Further improved F-expansion method and new exact solutions of Kadomstev-Petviashvili equation. Chaos, Solitons & Fractals, 32 (2007): 1375-1383.

[19] Abdou, M.A.,The extended F-expansion methodand its application for a class of nonlinearevolution equations. Chaos Soliton. Fract., 31(2007):95-104.

[20] E.M.E. Zayed, New traveling wave solutions for higher dimensional nonlinear evolution equations using a generalized G'/G-expansion method, J. Phys. A: Math. Theor. 42 (2009): 195202–195215.

[21] E.M.E. Zayed, The G'/G-expansion method and its applications to some nonlinear evolution equations in the mathematical physics, J. Appl. Math. Comput. 30 (2009): 89–103

[22] Zhou Y. Bin, L. Chao, Application of modified G'/G-expansion method to traveling wave solutions for Whitham–Broer–Kaup-like equations, Commun. Theor. Phys. 51 (2009): 664–670.

[23] E.M.E. Zayed, K.A. Gepreel, Some applications of the G'/G-expansion method to non-linear partial differential equations, Appl. Math. Comput. 212 (2009): 1–13.

[24] E.M.E. Zayed, K.A. Gepreel, The G'/G-expansion method for finding traveling wave

solutions of nonlinear partial differential equations in mathematical physics,

J. Math. Phys. 50 (2009): 013502–013513.

[25] E.M.E. Zayed, S. Al-Joudi, Applications of an Improved G'/G-expansion method to nonlinear PDEs in mathematical physics, AIP Conf. Proc. Am. Inst. Phys.1168 (2009):371–376

[26] Jishe Feng, Wanjun Li, Qiaoling Wan, Using G'/G -expansion method to seek the traveling wave solution of Kolmogorov–Petrovskii-Piskunov equation, Applied Mathematics and Computation, 217 (2011) 5860-5865.

[27] D. H. Peregrine, Calculation of the Development of an Undular Bore, Journal of Fluid Mechanics, Vol. 25, No. 2, 1966, pp. 321-330. doi:10.1017/S0022112066001678

[28] Hasibun Naher and Farah Aini Abdullah, New Approach of (G'/G)-expansion Method for RLW Equation. Res. J. App. Sci. Eng. Technol., 7(2014): 4864-4871

[29] E. M. E. Zayed and M. A. S. EL-Malky, Exact Solutions of Nonlinear Partial Differential Equations in Mathematical Physics Using the G'/G-Expansion Method. Adv. Theor. Appl. Mech.,4 (2011):91 -100

[30] Xiaohua Liu,Weiguo Zhang and Zhengming Li, Application of Improved (G'/G)–Expansion Method to Traveling Wave Solutions of Two Nonlinear Evolution Equations. Adv. Appl. Math. Mech. 4 (2012):122-130