Design optimal Fractional PID Controller for Inverted Pendulum with Genetic Algorithm

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Abstract— An intelligent optimization method for designing Fractional Order PID (FOPID) controller's base Genetic Algorithm (GA) ear presented in this paper. Fractional calculus can provide novel and higher performance extension for FOPID controllers. However, the difficulties of designing FOPID controllers increase, because FOPID controllers append derivative order and integral order in comparison with traditional PID controllers. To design the parameters of FOPID controllers. Experimental results show the proposed design method can design effectively the parameters of FOPID controllers.

Index Terms— Genetic Algorithm; Fractional order PID;fractional calculus; PID control; Inverted Pendulum

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1 INTRODUCTION

Fractional order control systems are described by fractional order differential equations. Fractional calculus allows the derivatives and integrals to be any real number. The FOPID controller is the expansion of the conventional PID controller based on fractional calculus. FOPID controllers' parameters designed have five, and the derivative and integral orders improve the design flexibility.

1.1 Fractional calculus

There are several definitions of fractional derivatives [1]. Grunwald-Letnikov definition is perhaps the best known one due to its most suitable for the realization of discrete control algorithms. The m order fractional derivative of continuous function $f(t)$ is given by:

$$D^m f(t) = \lim_{h \to 0} h^{-m} \sum_{j=0}^{[x]} (-1)^{-j} \binom{m}{j} f(t-jh) = \frac{d^m f(t)}{dt^m} \quad (1)$$

Where [x] is a truncation and $x = \frac{t-m}{h}$; $\binom{m}{j}$ is binomial coefficients,

$$\binom{m}{j} = \frac{m(m-1)\dots(m-j+1)}{j!} \quad (2)$$

$\binom{m}{j} = 1, (j = 0)$, it can be replaced by Gamma function,

$\binom{m}{j} = \frac{\Gamma(m+1)}{j!\Gamma(m-j+1)}$

The general calculus operator, including fractional order and integer, is defined as:

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$$D_t^\alpha = \begin{cases} \dfrac{d^\alpha}{dt} & R(\alpha) > 0 \\ 1 & R(\alpha) > 0 \\ \displaystyle\int_0^t (d\tau)^{-\alpha} & R(\alpha) > 0 \end{cases} \quad (3)$$

$$\mathcal{L}\{D^\alpha f(t)\} = s^\alpha F(s) - [D^{\alpha-1}f(t=0)] \quad (4)$$

Where $f(s)$ is the Laplace transform of $f(t)$ The Laplace transform of the fractional integral of $f(t)$ is given as follows:

$$\mathcal{L}\{D^{-\alpha}f(t)\} = s^{-\alpha}F(s) \quad (5)$$

1.2 Fractional order controllers

The differential equation of fractional order controller $PI^\alpha D^\beta$ is described by [2]:

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\delta e(t). \quad (6)$$

The continuous transfer function of FOPID is obtained through Laplace transform, which is given by:

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\delta e \quad (7)$$
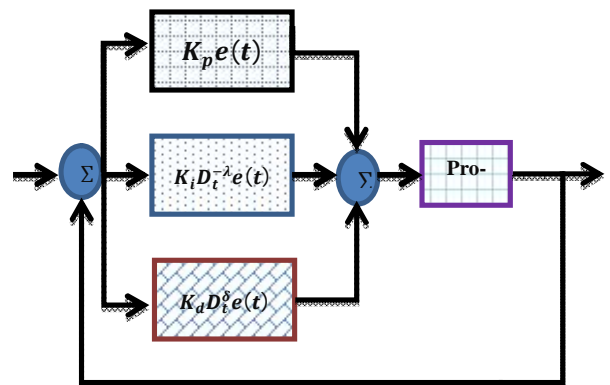$$G_c(s) = K_p + K_i s_t^{-\lambda} + K_d s_t^\delta \quad (8)$$

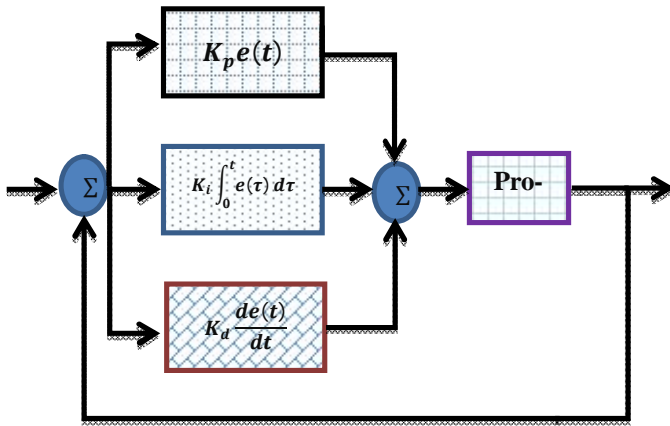Fig. 1. Generic closed loop control system With a FOPID controller

Fig. 2. Generic closed loop control system With a PID controller

Fig. 4.The structure of an Inverted Pendulum

It is obvious that the FOPID controller not only need design three parameters, K_p, K_i and, K_d but also design two orders, λ and δ of integral and derivative controllers. The orders λ ,δ are not necessarily integer, but any real numbers. As shown in Fig.3 the FOPID controller generalizes the conventional integer order PID controller and expands it from point to plane. This expansion could provide much more flexibility in PID control design. [3]
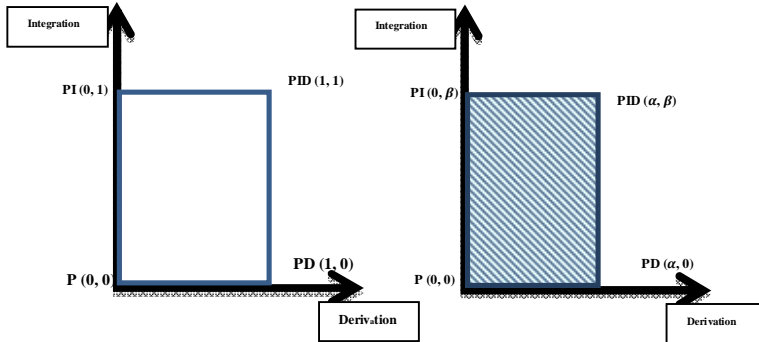
TABLE 1. PHYSICAL PARAMETERS OF INVERTED PENDULUM

M	mass of the cart	0.5 kg
m	mass of the pendulum	0.2 kg
b	friction of the cart	0.1 N/m/sec
l	length to pendulum center of mass	0.3 m
I	inertia of the pendulum	0.006 kg*m^2
F	force applied to the cart	
x	cart position coordinate	
theta	pendulum angle from vertical	

Fig. 3. PID controllers with fractional orders

This system is tricky to model in Simulink because of the physical constraint (the pin joint) between the cart and pendulum which reduces the degrees of freedom in the system. Both the cart and the pendulum have one degree of freedom (X and theta, respectively). We will then model Newton's equation for these two degrees of freedom.

2 MODELING AN INVERTED PENDULUM

The cart with an inverted pendulum, shown below, is "bumped" with an impulse force, F. Determine the dynamic equations of motion for the system, and linearize about the pendulum's angle, theta = 0 (in other words, assume that pendulum does not move more than a few degrees away from the vertical, chosen to be at an angle of 0). Find a controller to satisfy all of the design requirements given below.For this example, let's assume that.
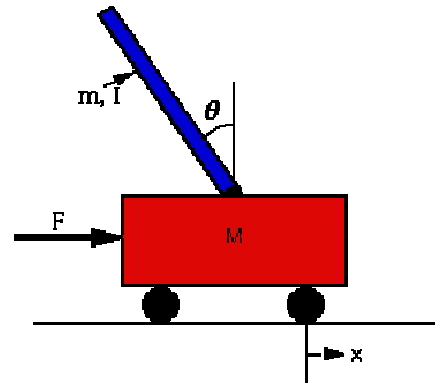
$$\frac{d^2x}{dt^2} = \frac{1}{M}\sum_{cart} F_x = \frac{1}{M}\left(F - N - b\frac{dx}{dt}\right) \qquad (9)$$

$$\frac{d^2\theta}{dt^2} = \frac{1}{I}\sum_{pend}\tau = \frac{1}{I}(NL\cos(\theta) + PL\sin(\theta)) \qquad (10)$$

It is necessary, however, to include the interaction forces N and P between the cart and the pendulum in order to model the dynamics. The inclusion of these forces requires modeling the x and y dynamics of the pendulum in addition to its theta dynamics. Generally, we would like to exploit the modeling power of Simulink and let the simulation take care of the algebra. Therefore, we will model the additional x and y equations for the pendulum.

$$m\frac{d^2x_p}{dt^2} = \sum_{pend} F_x = N \tag{11}$$

$$\Rightarrow N = m\frac{d^2x_p}{dt^2} \tag{12}$$

$$m\frac{d^2y_p}{dt^2} = P - mg \tag{13}$$

$$\Rightarrow P = m\left(\frac{d^2y_p}{dt^2} + g\right) \tag{14}$$

However, xp and yp are exact functions of theta. Therefore, we can represent their derivatives in terms of the derivatives of theta.

$$x_p = x - L\sin(\theta) \tag{15}$$

$$\frac{dx_p}{dt} = \frac{dx}{dt} - L\cos(\theta)\frac{d^2\theta}{dt^2} \tag{16}$$

$$\frac{d^2x_p}{dt^2} = \frac{d^2x}{dt^2} + L\sin(\theta)\left(\frac{d\theta}{dt}\right) - L\cos(\theta)\frac{d^2\theta}{dt^2} \tag{17}$$

$$y_p = L\cos(\theta)(10)$$

$$\frac{dy_p}{dt} = -L\sin(\theta)\frac{d\theta}{dt} \tag{18}$$

$$\frac{d^2y_p}{dt^2} = -L\cos(\theta)\left(\frac{d\theta}{dt}\right)^2 - L\sin(\theta)\frac{d^2\theta}{dt^2} \tag{19}$$

These expressions can then be substituted into the expressions for N and P. Rather than continuing with algebra here, we will simply represent these equations in Simulink. Simulink can work directly with nonlinear equations.
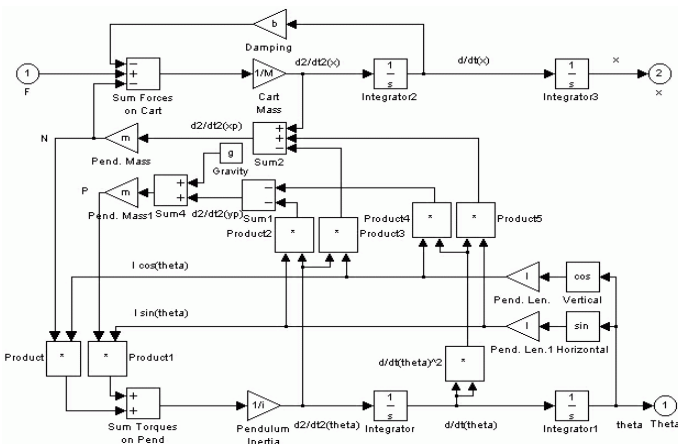

Fig. 5.The block diagram of an Inverted Pendulum

3 GA BASED TUNING OF THE $PI^{-\lambda}D^{\delta}$ CONTROLLER GAINS

3.1 Introduction to Genetic Algorithm $PI^{-\lambda}D^{\delta}$

In 1975, GA was proposed firstly by Holland [3]. It is an optimization algorithm and applied to various fields, including business, science, and engineering. Based on the survival-of-the-fittest strategy proposed by Darwin, this algorithm will eliminate unfit components to select the fittest component by Man-made fitness functions generation by generation.

A. Initialization

In the initialization, the first thing to do is to decide the coding structure. Coding for a solution, termed a chromosome in GA literature, is usually described as string of symbols from {0, 1}. These components of the chromosome are then labeled as genes. The number of bits that must be used to describe the parameters is problem dependent.

B. Selection

GA uses proportional selection; the population of next generation is determined by n independent random experiments.

C. Crossover

Cross over is an important random operator in GA and the function of this operator is to generate a new 'child 'chromosome from two 'parents 'chromosomes by combining the information extracted from the parents.

D. Mutation

Mutation is another important component in GA, though it is usually conceived as a background operator. It operates independently on each individual by probabilistically perturbing each bit string. A usual way of mutation used in GA is to generate a random number between zero and one and then make a random change in the v-th element of the string with probability p_m belonging to (0, 1).

E. Encoding & Decoding

The design variables are mapped onto a fixed-length binary Digit string, which are constructed over the binary alphabet {0,1}, and is concatenated head-to-tail to form one long string referred as a chromosome. That is, every string contains all design variables. The physical values of the design variables are obtained by decoding the string.

F. Fitness Function

In GA, the value of fitness represents the performance, which is used to rank the string, and the ranking is used to determine how to allocate reproductive opportunities. This means that individuals with higher fitness value will have higher probability of selection as a parent. Fitness thus is some measure of goodness to be optimized. The fitness function is essentially the objective function for the problem.
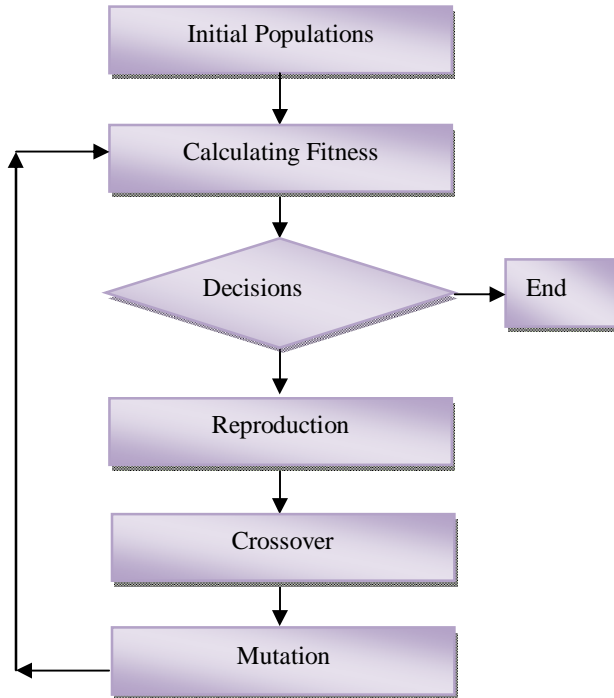
Fig. 6. illustrates the block structure of the FOPID controller optimizing process with GA
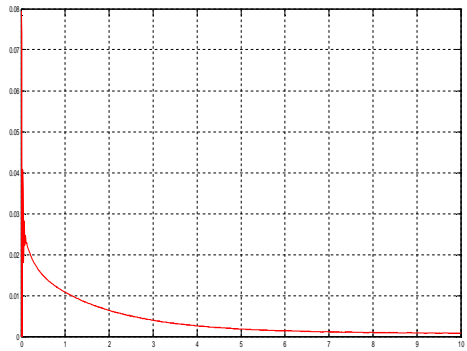

Fig. 7. Tuning process of the FOPID controller parameters with GA

4 SIMULATION RESULT

Fig. 8. PID controller

Fig. 9. FOPID controller

TABLE 2. PARAMETR OF CONTROLLER

	Kp	Ki	Kd	α	β	FITNES	ESS	US
PID	100.328	17.049	100.50	-	-	0.424	0.01	0.02
FOPID2	90.236	81.443	87.817	0.784	0.97	0.052	0.01	0

5 CONCLUSION

It has been demonstrated that the parameters optimization of fractional order controller based on modified GA is highly effective. According to optimization target, the proposed method can search the best global solution for FOPID controllers' parameters and guarantee the objective solution space in defined search space. Based on improved GA, the design and application of FOPID will be appeared in various fields.

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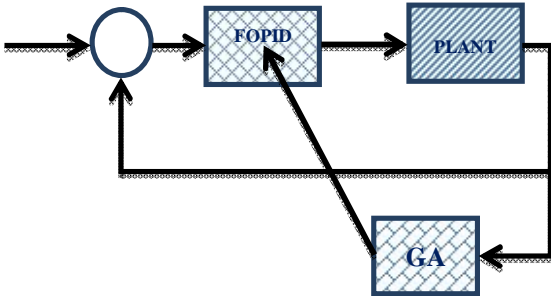[3] . J. Holland, "Adaptation in Natural and Artificial Systems", University of Michigan Press, Ann Arbor, Michigan, 1976.