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# Design of Controller for Inverted Pendulum System

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**Abstract**—In this paper PID Controller and LQR is designed for Cart- Inverted pendulum system to obtain optimal control. While calculating PID values here wind disturbance (Fw) is taken into account. The designed controller gives better set point tracking and disturbance rejection. Using Pole placement and linear quadratic regulator controller gain of PID's are calculated. Further this robust technique is verified using two different laws with disturbance input by using MATLAB and SIMULINK.

**Index Terms**—Disturbance input, Invereted pendulum system, optimal control, LQR, PID

## 1 Introduction

The inverted cart-pendulum system is an underactuated mechanical system. It has two degrees of freedom and one control input [1]. It is similar with non-linear dynamics presentin rocket launchers, missile guidance control applications.

The controller design is difficult for such a system. Theinvertedpendulum system has only one control input which is voltage applied to DC motor to control the force with which the cart is moved at desired position along its track [4]. The system has two outputs one is angular rotation of pendulum arm and other is horizontal movement of cartposition. In Inverted pendulum system the controller task is to stabilize the pendulum in an upright position whileachieving the desired cart position. The inverted pendulum has been used as a platform to study real world non-linear control problems .Many different control design techniques are being developed to balance the invereted pendulum in an upright position while moving the cart along its track.

A Proportional-Integral-Derivative controller (PID) is widely used in industrial control systems. It's a simple yet versatile controller. In the field of process control systems the PID controller have proved their usefulness by providing satisfactory control but in many situations PID controllers may not provide optimal control. There are many different types of tuning rules have been developed like Cohen-Coon method, Ziegler Nichole's method, GA and Optimal control technique but it is observed that these technique are not roboust [5].

In this paper the LQR with PID isused to control the nonlinear inverted pendulum system to obtained optimal control. The control strategy is designed by considering the disturbance input. The control parameters ofPID controllers are obtained using LQR design with the help of pole placement method.

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Two separate PID's are used one for control of cart position and another for control of Pendulum angle. The performance of the proposed controller is verified in MATLAB Simulink environment.

In [6] the two separate PID controllers designed for Inverted pendulum system .the controller parameters are obtained by LQR design using pole placement method. In [7] the optimal control technique for Nonlinear inverted pendulum using PID controller and LQR approach is presented,but the tuning of PID is done using Trial and Error method. In [9] the different control methods for Invertedpendulum systems are discussed like Fuzzy logic Controller, Expert system controller and Neural Network Controller. In [13] authors have used the Linear state feedback design method to balance the inverted pendulum in an upright position with restricted cart track length the controller stabilize the system with infinite gain margin. In [14] authors have presented simple and effective method for designing robust PI/PID controller for first order plus delay time process. Authors used Numerical Optimization Approachwhich gives monotonic response. This technique is compared with classical PID tuning technique such as Ziegler Nichole's method.

This paper is organized as, description of inverted cart pendulum system and its Modeling is explained in section 2. In section 3 the design of control methods are depicted .While section 4 shows simulation results of proposed PID with LQR Control and section 5 draws the conclusion from the results

Table1 Parameter of the system

| Symbol | Parameter | Value | Unit |
|---|---|---|---|
| $M$ | Mass of the cart | 2.4 | Kg |
| $m$ | Mass of the pendulum | 0.23 | Kg |
| $l$ | Length of the pendulum | 0.4 | M |
| $g$ | Gravitational constant | 9.8 | m/s2 |
| $u$ | Applied force to cart | ±24 | N |
| $b$ | Cart friction | 0.055 | Ns/m |
| $I$ | Moment of inertia | 0.099 | Kgm2 |
| $d$ | Damping coefficient | 0.005 | Nms/rad |

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## 2. Cart- Pendulum System And Modeling

Every control project starts with plant modeling .It is an important issue for stability and the controller design. Consider the cart Inverted pendulum system as shown in figure 1.The parameters [4] are as follows.

The dynamical equations of this system by considering disturbance input can be derived as: *x(t)* is the horizontal displacement of cart along its track length and *Θ(t)* is the rotational angle of the pendulum arm with reference to vertically upward direction. Let $F_w$ be the horizontal wind force on the pendulum point mass.

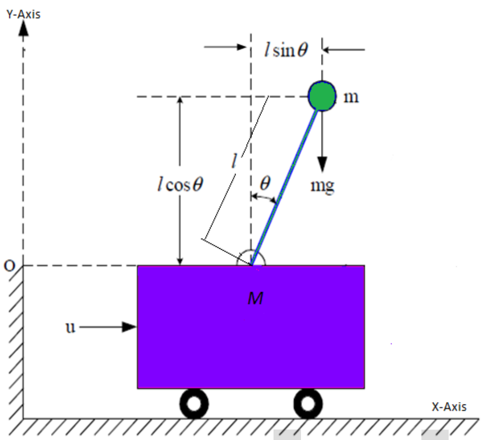


Fig.1 Cart Pendulum System

The force balance equations given by[7]

$$M\frac{d^2}{dt^2}x + m\frac{d^2}{dt^2}x_G = u + F_w \qquad (1)$$

And torque balance equation is given by [7]

$$(F_x\cos\theta)l - (F_y\sin\theta)l = (mg\sin\theta)l + (F_w\cos\theta)l \qquad (2)$$

Nonlinear equations (1) and (2) can be manipulated to have standard state space form as given by

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (M+mg)/Ml & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -mg/M & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ -1/Ml \\ 0 \\ 1/M \end{bmatrix} * u + \begin{bmatrix} 0 \\ -1/ml \\ 0 \\ 0 \end{bmatrix} F_w \qquad (3)$$

And

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \qquad (4)$$

## 3. Design of Control Method

To control the Invereted pendulum system following control methods are used.

### 3.1 PID

PID is the most common and most popular feedback controller used in Industrial Process.A PID controller calculates an error values as the difference between a measured processvariable and a desired set point. A PID controller is also known as three term control Proportional, Integral and Derivative.

The PID controller is defined by the relationship between the controller input e and the computed output u that is applied to the DC motor.

$$C_i = K_p e + K_i\int edt + K_d\frac{de}{dt} \qquad (5)$$

Taking Laplace transform of above equation we get

$$C(s) = K_p + \frac{K_i}{s} + K_d s \qquad (6)$$

Here two PID controllers are used one for cart position and another for Pendulum arm.

Let the two PID controllers be

$$C_p = \frac{(K_d^1 s^2 + K_p^1 s + K_i^1)}{s} \qquad (7)$$

and

$$C_\theta = \frac{(K_d^2 s^2 + K_p^2 s + K_i^2)}{s} \qquad (8)$$

Where,

$C_p$ - Controller for Cart Position

$C_\theta$ - Controller for Pendulum Arm

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### 3.2 LQR Design:

Linear Quadratic Regulator (LQR) is an optimal control method. The main objective of optimal control is to determine control signals that will cause a Process (Plant) to satisfy some physical constraints and at the same time extremize (maximize or minimize) a chosen performance criterion (performance index or cost function) [2].
The performance of the system is measured with a single scalar quantity- performance index (PI). A configuration of the controller is selected and free parameters of the controller that optimize the PI are determined. [1]
LQR is one of the most widely used static state feedback method.in this controller is designed in such a way that gives best possible performance of the system with respect to some given performance.
LQR can be designed for linear time varying plant given by equation

$$\dot{X}(t) = A(t)x(t) + B(t)u(t) \tag{9}$$

For LQR the quadratic Performance Index is given by

$$J = \frac{1}{2}\int_0^\infty [x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)]dt \tag{10}$$

Where Q is State weighted matrix and R is control weighted matrix.
The minimization of J is obtained by solving the algebraic Riccati Equation

$$\begin{aligned} &A^T(t)P(t) + P(t)A(t) - \\ &P(t)B(t)R^{-1}(t)B^T(t)P(t) + Q(t) = 0 \end{aligned} \tag{11}$$

And the optimal state feedback gain vector is given by

$$K = -R^{-1}(t)B^T(t)P(t) \tag{12}$$

By selecting state weighted matrix Q and Control weighted matrix R and by solving equation (11) and equation (12),
The Optimal state feedback gain vector obtained as

$$K = \begin{bmatrix} -137.7896 & -25.9783 & -22.3607 & -27.5768 \end{bmatrix}$$

### 3.3 PID with LQR Design:

**A. Control Law 1**: Fig.2show the PID and LQR controller with disturbance input. For this figure the control law is given by equation (13)the control parameters of both PID's are calculated using this equation and pole placement technique.

$$1 - P_1 C_P + P_2 C_\theta = 0 \tag{13}$$

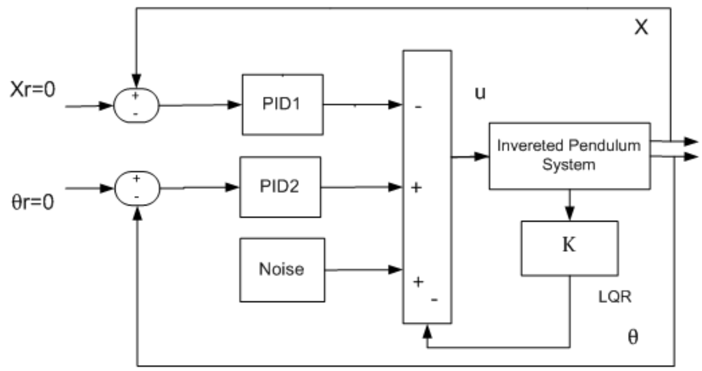


Fig.2 PID with LQR for Inverted Pendulum System with Disturbance input (Control Law 1)

By putting values in equation (13) and using pole placement (A-BK),we get gain values for position control, PID ($C_P$) tuning parameters are$K^1_p$=240.593,$K^1_i$= 178.527,$K^1_d$=-93.371 and for angle control,PID ($C_\Theta$)$K^2_p$=-356.7439,$K^2_i$ =-2247.60, $K^2_d$=10.

**B. Control Law 2**: fig shows the PID and LQR with disturbance input for inverted pendulum system. For this fig control law is given by equation (14)the control parameters of both PID's are calculated using control law equation

$$1 + P_1 C_P + P_2 C_\theta = 0 \tag{14}$$

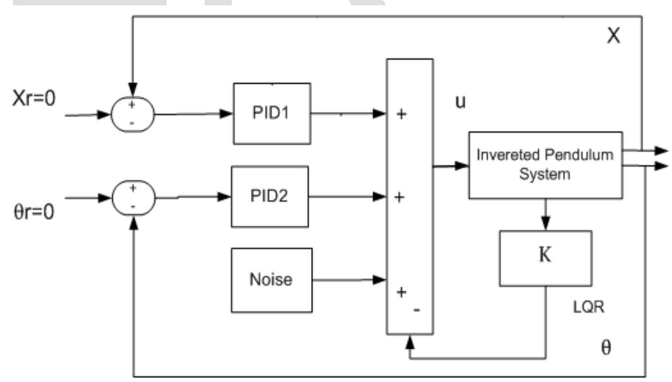


Fig 3.PID with LQR for Invereted Pendulum System Withdisturbance input. (Control Law2)

Using pole placement (A-BK) the values of control parameters for both PID can be calculated as $K^1_p$=-240.593,
$K^1_i$ = - 178.527, $K^1_d$= 93.371 and for angle control, PID ($C_\theta$)
$K^2_p$= -356.7439,$K^2_i$= - 2247.60,$K^2_d$=10.

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## 4 Simulation Results:

The simulation is done by applying disturbance $F_w$ as band limit white noise input to system. The results obtained for cart position, pendulum angle and control signal all these are observed for both control laws.

### 4.1 Simulation result for control law 1

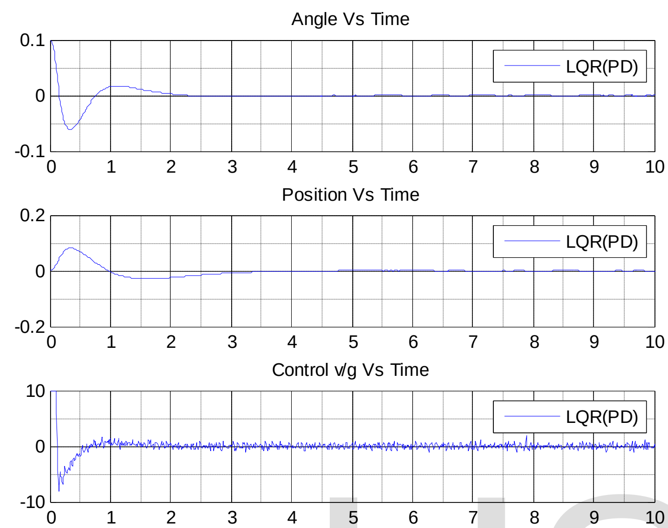


Fig.4 Response of system with Initial Condition 0.1 with Noise

From above fig4 it is observed that simulation result of proposed system forcontrol law 1 with initial condition of angle 0.1 with ref position isat zero.

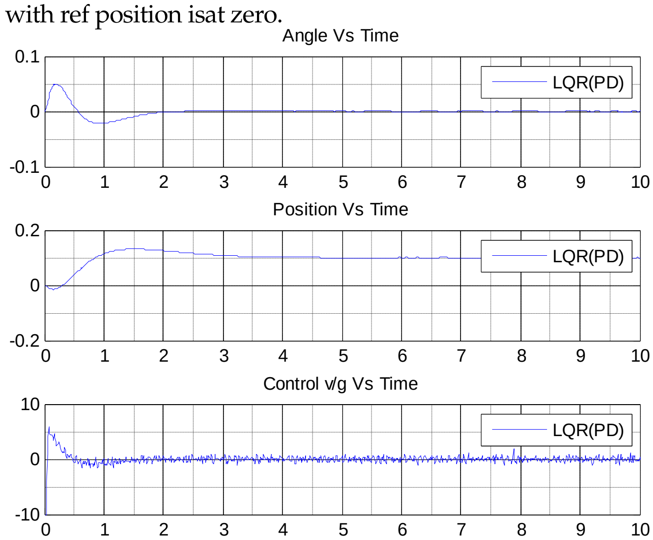


Fig.5 Response of system withReference position 0.1 With Noise

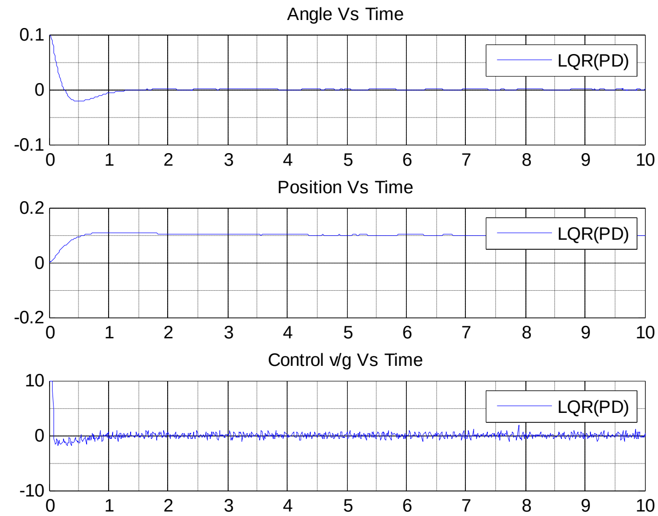


Fig 6 Response of system with initial condition 0.1 and reference position 0.1 with noise.

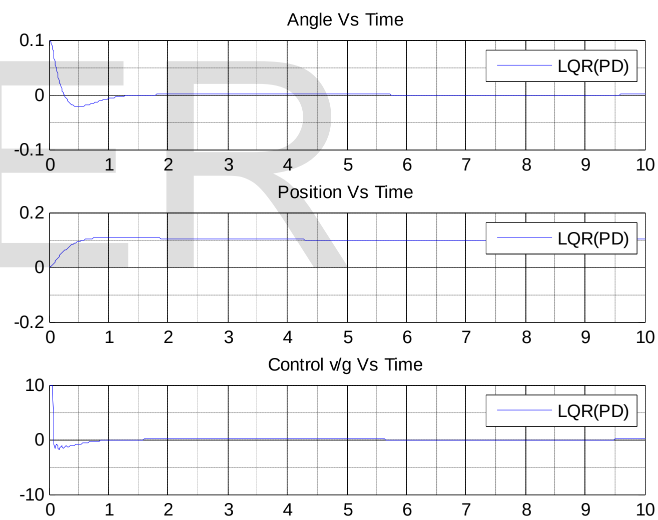


Fig 7 Response of system with initial condition 0.1 and reference position 0.1 without Noise

It is seen from Fig.5 that for cart reference position 0.1 with no initial condition of pendulum arm,presence of noise also pendulum get stabilize and cart achieves desired location.

Fig. 6 and Fig. 7shows response of systemfor reference position of cart 0.1 and forinitial condition of pendulum arm is 0.1 With noise and without noise the system get stable in less time.

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## 4.2 Simulation Result for Control Law 2

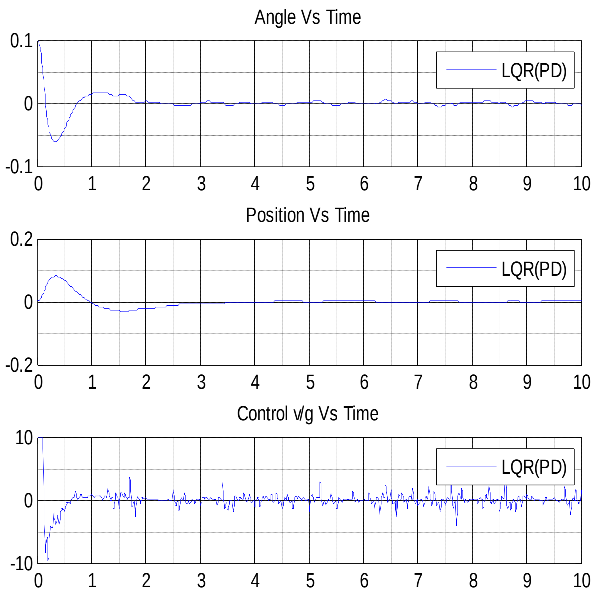


Fig 8.Response of system for Initial condition of angle 0.1 With noise

Above fig .shows that for the initial condition of pendulum arm 0.1, system gets stable but the response is oscillatory.



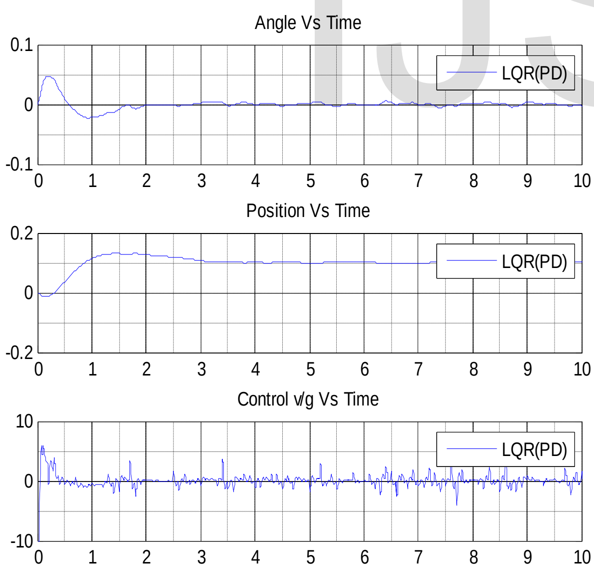


Fig 9.Response of system for cart Reference Position 0.1With noise

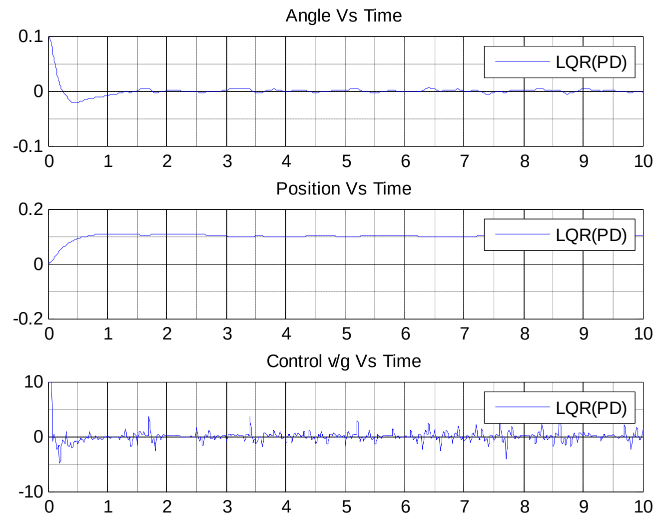


Fig 10.Response of system for initial condition 0.1 and Reference position 0.1 with noise

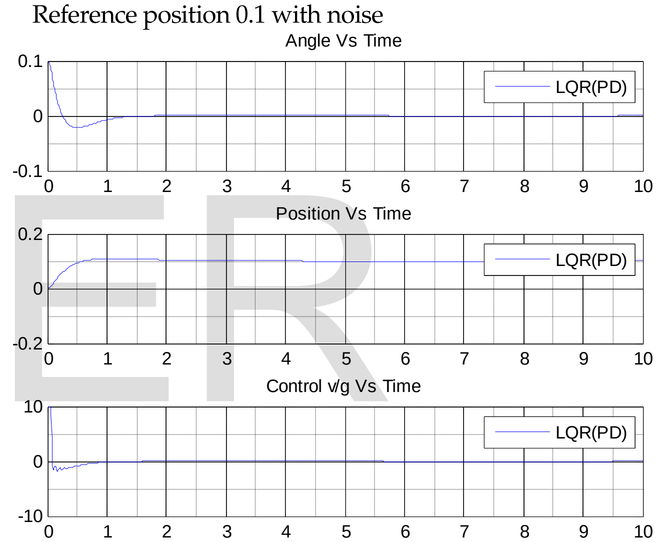


Fig 11.Response of system for initialcondition of angle 0.1 and reference position of cart 0.1 without Noise

Fig.9 shows that cart achieves its reference position that is 0.1 in presence of noise signal but the settling time islarge enough.

It is analyzed from Fig10 andFig.11 that Pendulum gets stabilized for two different conditions that are of cart position and pendulum arm in presence of noise signal and without noise signal respectively. Also the system gets stable with minor oscillations.

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## 5 Conclusion:

It is observe that recently proposed optimal tuning method for PID using LQR pole placement is robust. In this paper, we have verified to stabilize the invereted pendulum in upright positionby Matlab simulation for two different control law as mentioned above in presence ot disturbance is used. Here we found pendulum stabilizes with minor oscillations and cart is approaches to desired position even in presence of disturbance Fw. By observing simulation plots, we can conclude that there exist marginal oscillations for control law 2 whereas the control law 1 yields satisfactory response.

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