Application of Mathematics to Eclipse Analysis

Vanaja.V, Shailaja.M, S. BalachandraRao

Abstract:

Eclipses are astronomical aspects occurs in nature periodically. The two types of eclipses are lunar and solar eclipses. The computation of occurrence and periodicity of these eclipses are based on the positions (longitudes) of the celestial bodies, namely the Sun, the Moon and the Earth. Geometrically the 'conjunction' of the Sun and the Moon refers to solar eclipse and 'opposition' of the Sun and the Moon refers to lunar eclipse. To find the occurrence of eclipse different mathematical methods are used. In Indian classical *siddhāntic* texts eclipse computation is based on the true positions (longitudes) of the Sun, the Moon and the ascending or the descending node, in Indian parlance it is called *Rāhu* or *Ketu*. Modern computation of eclipse is based on the International Astronomical Union terms (IAU). Both Indian and Modern computation of eclipses are mathematically compared and analyzed.

In the present paper we discuss the mathematical procedure and algorithm for computation of lunar eclipse on the basis of *Improved Siddhāntic Procedures* which is highly improved version of *siddhāntic* procedures and one modern procedure. The timings of the eclipse are compared and it is mathematically analyzed.

Key words: **Eclipse, the Sun, the Moon, Moon's diameter, latitude, Saros, *Siddhāntic*.**

1. Introduction

The lunar eclipse occurs on a fullmoon day. In this particular day the Sun and the Moon are on the opposite sides of the earth. The light rays of the Sun falls on the earth facing the Sun, and a shadow will be cast on the other side. When the Moon enters the shadow of the earth, a lunar eclipse occurs. This happens when the Sun and the Moon are in *opposition* i.e., their longitudinal difference is 180^0. However, a lunar eclipse does not occur on every fullmoon day, because the plane of the Moon's orbit is inclined at about 5^0 to the ecliptic (the apparent path traced by the Sun). Generally, on a fullmoon day, the Moon will be either far above or far below the plane of the ecliptic and so does not pass through the shadow of the earth. But, on that fullmoon day, when the Moon does pass through the earth's shadow, a lunar eclipse occurs.

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1 Assistant professor, Department of Mathematics, Government First Grade college, Shidlaghatta-562105. Email ID: vanajaksr94@yahoo.com
2 Assistant professor, Department of Mathematics, Government First Grade college, Vijayanagar, Bengaluru-560104.
3 Honorary Director, Bhavan's Gandhi Centre for Science and Human Values, Bengaluru-560001.

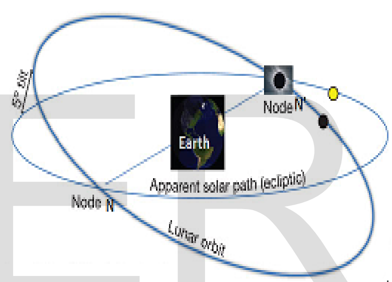

Fig. 1: Nodes of the Moon

A necessary condition for a lunar eclipse to occur, the Moon must come close to the ecliptic that means the Moon must be close to one of the nodes. In Fig. 1, roughly speaking, the orbit of the Moon intersects with the ecliptic at two points N and N'. These two points are referred to as the ascending and the descending nodes of the Moon. They are called *Rāhu* and *Ketu* in Indian astronomy.In Fig. 2, *S* and *E* represent the centres of the Sun and the earth, respectively. Draw a pair of direct tangents *AB* and *CD* to the surface of the Sun the earth, meeting *SE* in *V*. If these lines are imagined to revolve round *SE* as axis, they will generate a cone. There is, thus, a conical shadow *BVD*, with *V* as its vertex, across which no direct ray from the Sun can fall. This conical shadow is called *umbra*.

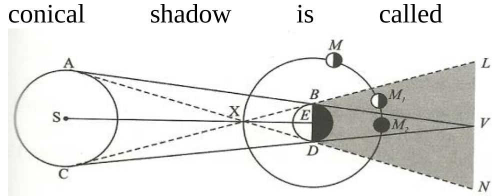

Fig. 2: Earth's shadow cone and the lunar eclipse

The spaces around the umbra, represented by *VBL* and *VDN*, form what is called penumbra, from which only a part of the Sun's light is excluded. It is to be noted that the passage of the Moon through the penumbra does not prompt an eclipse. It results only in diminution of the Moon's brightness. In Indian classical astronomy a penumbral eclipse is not considered as an eclipse. In Fig 4.2, the Moon is at M_1 it receives light from portions of the Sun next to *A*, but rays from the parts near *C* will not reach the Moon at M_1. Therefore, the brightness is diminished, the diminution growing greater as the Moon approaches the edge of the *umbra*. An *umbral*eclipse is considered as just commencing when the Moon enters the umbra or the shadow-cone. M_2 represents the Moon which is completely immersed in the shadow-cone or *umbra*.

The lunar eclipse is said to be total when the whole of the Moon passes through the shadow. The eclipse is partial when only a part of the Moon enters the shadow.

2.Half-Durations of Eclipse and of Totality

The next important step is to determine the instants of the beginning and the end of a lunar eclipse as also of the totality. For this, find the durations of the first half and the second half of the total duration of the eclipse. The configuration is shown in Fig 3.

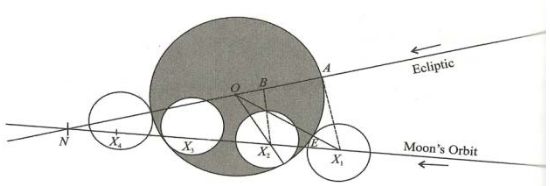

Fig 3: Half-duration of lunar eclipse

A half-duration is the time taken by the Moon, relative to the Sun, so that the point A in Fig.3 moves through OA. We have

$$OA^2 = OX_1{}^2 - AX_1{}^2 = (OE+EX_1)^2 - AX_1{}^2 = (d1+d2)^2 - \beta^2$$

Where $OE = d_1$ = semi-diameter of the shadow

$EX_1 = d_2$ = semi-diameter of the Moon

$\beta = AX_1$ = latitude of the Moon

When the Moon's centre is at X_1,

$$\text{Half-duration (HDUR)} = \frac{\sqrt{(d_1+d_2)^2-(\beta)^2}}{(Moon's\ daily\ motion - Sun's\ daily\ motion)}$$

Since the actual instants of the beginning and ending of the eclipse and hence the latitudes of the Moon at those instants are not known, the express for HDUR is taken as a first approximation.

By a similar analysis, the half-duration of maximum obscuration (totality or annularity as the case may be) is given by

$$\text{Half-duration of maximum obsuration (THDUR)} = \frac{\sqrt{(d_1-d_2)^2-(\beta)^2}}{(Moon's\ daily\ motion - Sun's\ daily\ motion)}$$

The thus obtained half-durations of the eclipse and the maximum obscuration are (i) subtracted from the instant of the opposition to the beginning moment; and (ii) added to the instant of opposition to obtain the ending moments. For these instants the latitudes of the Moon are determined and the same are used in the expressions for HDUR and THDUR. By the process of successive iteration the precise values for the half-duration as well as for the instants of the beginning, ending and totality are obtained. Finally, we have

$$\text{Magnitude of an eclipse} = \frac{Amount\ of\ obscuration\ (Grāsa)}{Angular\ diameter\ of\ the\ Moon}$$

Note: (1) If the magnitude is greater than or equal to 1, then the eclipse is total; otherwise, it is partial.

(2) If $d_1+d_2 < \beta$, there will be no eclipse.

3. Lunar eclipse according to *Improved siddhāntic Procedure (ISP)*

In this section we applied the algorithm of *improvedsiddhānticprocedure*, based on Indian classical *siddhāntic* texts with updated parameters.

The required parameters to compute an eclipse is given as below.

(i) Compute true Sun, true Moon and *Rāhu* for a convenient time on a full moon day.
(ii) True rates of daily motion of the Sun, the Moon and moon's node (*Rāhu*), the

notations are denoted as *SDM*, *MDM* and *RDM* respectively.

(iii) Moon's latitude 'β'.

(iv) Angular diameters of the Sun, the Moon and the earth's shadow cone.

(a) In the case of solar eclipse find the angular diameter (*Biṃba*) of the Sun and the Moon.

(b) For lunar eclipse find the angular diameter of the Moon and the earth's shadow cone.

(v) Moon's parallax (and also that of the Sun).

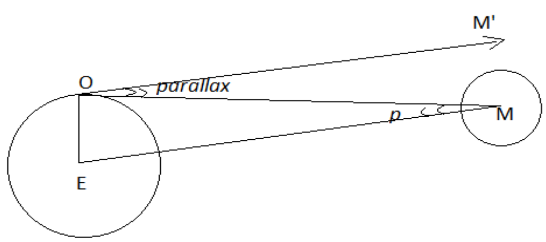

Fig 4 : Moon's parallax.

In Fig 4, consider an observer at 'O' on the surface of the earth centred at 'E'. The calculation of positions of the Moon and the planets is for the centre of the earth 'E'. Therefore for an observer on the earth's surface there is an apparent shift in the positions of the Moon and other bodies. It is maximum in the case of the Moon (because it is the nearest body to the earth). Join EM, OM and EO, where M is the centre of the Moon. When observed from 'E' the direction of the Moon is EM. For an observer at 'O', the direction corresponds to OM' (Parallel EM), but the actual direction of M for the observer at 'O' is OM. Thus there is a shift in the direction of M equivalent to angle M$\hat{O}$M' = E$\hat{M}$O. This shift is called the Moon's parallax. Thus it is the angle E$\hat{M}$O subtended by the radius EO of the earth at M. In computations of solar eclipse we have to consider the parallaxes of the Moon and the Sun. However the Sun's parallax being about 9" is negligible compared to the Moon's parallax which is about 1^0 (i.e., 3600"). Thus the Moon's parallax is about 3600/9 = 400 times greater than the Sun's parallax.

Here we considered one example to explain the computations of lunar eclipse according to ISP with explanatory comments.

Example: Lunar eclipse on 4th April 2015, Saturday.

Instant of opposition is 17^h 36^m (IST).

At the instant of opposition
True Sun: $350^0 20'$; True Moon: $170^0 20'$; *Rāhu*: $165^0 56'$
Sun's daily motion, *SDM*: 59' 08''
Moon's daily motion, *MDM*: 714' 54''.

(i) Moon's latitude $=\beta$ = 308' ×sin(M-R) = $0^0 23'37''$.

(ii) Moon's angular diameter (*Candra*biṃba) =
MDIA = $2\frac{[939.6+(61.1)\cos GM]}{60}$ in minutes of arc
where *GM* is the Moon's anomaly (*mandakendra*) measured from its perigee and it is given by
GM=$134^0.9633964 + 13^0.06499295T+ \ldots$
where *T* be the number of days completed since the epoch 2000 January 1, noon (*GM*T) i.e., 17^h 30^m (IST) . From the tables of *ahargaṇa*, *JD* for this particular date i.e., 4th April 2015 is 2457117.
JD for 6^m = (17^h $30^m - 17^h$ 36^m) $=\frac{6^m}{24^h}$ = 0.00416 days. ∴the days from the epoch (2000 Jan1, noon *GM*) is
T = *JD* for 4th April 2015 – *JD* for 1st Jan 2000.
T= 2457117.00416 – 2451545 = 5572.00416. Using the value of T in *GM* it obtains the value
GM=$134^0.9633964 + 13^0.06499295T+ \ldots = 213^0.051858$.
∴ *MDIA* =29'.614662.
GS = The Sun's mean anomaly from its perigee
= $357^0.529092 + 0^0.985600231$ T = $89^0.297679$.

(iii) Diameters of the earth's shadow (*chāyābiṃba*):
SHDIA = $2\frac{[2545.4+228.9\cos GM-(16.4)\cos GS]}{60}$ in minutes of arc
SHDIA = 78'.451230.

(iv) True daily motions of the Sun and the Moon:
Sun's daily motion, *SDM*: 59'08''; Moon's daily motion, *MDM*: 714' 54''.
Vyarkendusphuṭanāḍīgati, *VRKSN* = (*MDM*- *SDM*) per *nāḍī*
i.e., *VRKSN* $=\frac{(MDM-SDM)}{60}$= 10'.929444.
Note: One day = 60 *nāḍī*s; 1*nāḍī*= 60 *vināḍīs* = 24 minutes.

(v) *Biṃbayogārdham* = $D = \frac{(MDIA+SHDIA)}{2}$ = 54'.032946.

(vi) *Biṃbaviyogārdham* = D' = $\frac{(SHDIA-MDIA)}{2}$ = 24'.418284.

(vii) *Sphuṭaśara* = $\beta' = \beta \times \left(1-\frac{1}{205}\right) = \frac{23'.61666\times 204}{205}$ =23'.514195.

where β is the Moon's latitude from step (i) above.

(viii) *MDOT*, $\dot{m}$ = $VRKSN \times \left(1+\frac{1}{205}\right)$ = $10'.9327 \times \frac{206}{205}$ = 10'.982759.

(ix) If $|\beta'|<D$, then lunar eclipse occurs. If $|\beta'|<D'$, then the eclipse is total.
In this case $|\beta'|<D'$, i.e., 23'.5141 < 24'.4182. Hence eclipse is total.

(x) Vī*RāhuCandra*, *VRCH*=(True Moon – True *Rāhu*) = $4^0 24'$.

(xi) Calculate: COR = $\frac{|\beta'|\times 59}{10\times \dot{m}}$ *vināḍīs*.

(a) If *VRCH* is in an odd quadrant (i.e., I or III), then subtract the above value COR from the instant of opposition to get the instant of the middle of the eclipse.

(b) If *VRCH* is in an even quadrant (i.e., II or IV), then add the above value COR to the instant of opposition to get the instant of the middle of the eclipse.

In the current example, COR = $\frac{|\beta'|\times 59}{10\times \dot{m}}$ = $\frac{23'.5141\times 59}{10\times 10'.982759}$ = 12'.63190 *vināḍīs*

COR = $12'.63190 \times \frac{2}{5} \approx 0^h\ 5^m\ 3^s$.

Now, *VRCH* = $4^0 24'$. Since *VRCH*<900, i.e., *VRCH* is in 1 quadrant (odd), the above value is subtractive from the instant of opposition.

∴ Middle of the eclipse = Instant of opposition – COR =$17^h\ 36^m - 0^h\ 5^m\ 3^s = 17^h\ 30^m\ 57^s$.

(xii) Half-duration of the eclipse

HDUR = $\frac{\sqrt{D^2-(\beta')^2}}{\dot{m}}$ = $\frac{\sqrt{(54'.032946)^2-(23'.514195)^2}}{10'.982759}$ = 4.429501 *nāḍī*s

= $4.429501\times \frac{2}{5} = 1^h\ 46^m\ 18^s$.

(xiii) Half-duration of totality

THDUR = $\frac{\sqrt{(D')^2-(\beta')^2}}{\dot{m}}$ = $\frac{\sqrt{(24'.418284)^2-(23'.514195)^2}}{10'.982759}$ = 0.599389 *nāḍī*s

= $0.599389 \times \frac{2}{5} = 0^h\ 14^m\ 23^s$.

(xiv) Magnitude = $\frac{D-|\beta'|}{MDIA}$ = $\frac{54'.032946-23'.514195}{29'.614662}$ =1.030528 =Total Eclipse

Summary of the eclipse :

IST

(1) Beginning of the eclipse =Middle –*HDUR* = $17^h30^m57^s - 1^h46^m18^s = 15^h44^m39^s$

(2) Beginning of totality = Middle –*THDUR*= $17^h30^m57^s - 0^h14^m23^s = 17^h21^m37^s$

(3) Middle = Instant of fullmoon−COR = $17^h\ 36^m - 0^h5^m3^s$ = **$17^h\ 30^m\ 57^s$**

(4) End of totality = Middle +*THDUR* =$17^h30^m57^s + 0^h14^m23^s$ = $17^h45^m20^s$

(5) End of the eclipse = Middle +*HDUR*= $17^h30^m57^s + 1^h46^m18^s$ = $19^h17^m15^s$.

4. Lunar eclipse according to modern procedure:

In this section we discuss the procedure of lunar eclipse in modern terms. The following expressions are based on the International Astronomical Union terms. This includes periodical terms and gives more accurate result.

Astronomical algorithms to compute lunar eclipse:

The times of the mean phases of the Moon, already affected by the Sun's aberration and by the Moon's light-time, are given in terms of *JDE*. i.e.,

(i) *JDE* (Julian Ephemeris days) = $2451550.09766 + 29.530588861k + 0.00015437T^2 - 0.000000150T^3 + 0.00000000073T^4$.
where the value of k =0 corresponds to the newmoon of 2000 January 6. If k is an integer value gives a newmoon, an

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integer increased by 0.5 gives a fullmoon. This is calculated by

$$k \approx (year - 2000) \times 12.3685.$$

where the 'year' should be taken with decimals, for example 2015.25 for the end of March 2015 and T is the time in Julian centuries since the epoch 2000 Jan 1, is given by

$$T = \frac{k}{1236.85}$$

(ii) Calculate angles of M, M', F and Ω in degrees.

Sun's mean anomaly at time *JDE*:

(a) $M = 2.5534 + 29.10535670k - 0.0000014T^2 - 0.000000058T^3$

Moon's mean anomaly:

(b) $M' = 201.5643 + 385.816935k + 0.0107582T^2 + 0.00001238T^3 - 0.000000058T^2.$

Moon's argument of latitude:

(c) $F = 160.7108 + 390.67050284k - 0.0016118T^2 - 0.00000227T^3 + 0.000000011T^4.$

Longitude of the ascending node of the lunar orbit:

(d) $\Omega = 124.7746 - 1.56375588k + 0.0020672T^2 + 0.00000215T^3.$

Eccentricity of the Earth's orbit around the Sun:

(e) $E = 1 - 0.002516T - 0.0000074T^2.$

Note: (1) The occurrence of the eclipse depends on the value of F. If F differs from the nearest multiple of 180^0 by less than $13^0.9$, then there is certainly an eclipse, if the difference is greater than 21^0 then there is no eclipse.

(2) If F is near 0^0 or 360^0, the eclipse occurs near the Moon's ascending node. If F is near 180^0, the eclipse takes place near the descending node of the Moon's orbit.

(iii) Calculate $F_1 = F - 0^0.02665 \sin\Omega$;

$A_1 = 299^0.77 + 0^0.107408k - 0^0.009173T^2$;

COR $=[(-0.4065 \times sinM') + (0.1727 \times E \times sinM) + (0.0161 \times sin2M') - (0.0097 \times sin2F_1) + (0.0073 \times E \times \sin(M' - M)) - (0.0050 \times E \times \sin(M' + M)) - (0.0023 \times \sin(M' - 2F_1)) + (0.0021 \times E \times sin2M) + (0.0012 \times \sin(M' + 2F_1)) + (0.0006 \times E \times \sin(2M' + M)) - (0.0004 \times sin3M') - (0.0003 \times E \times \sin(M + 2F_1)) + (0.0003 \times sinA_1) - (0.0002 \times E \times \sin(M - 2F_1)) - (0.0002 \times E \times \sin(2M' - M)) - 0.0002 \times sin\Omega]$

Time of maximum eclipse $=JDE+$ COR. (In Indian texts it is called as instant of opposition).

(iv) Calculate

P= $[(0.2070 \times E \times sinM) + (0.0024 \times E \times \sin 2M) - (0.0392 \times \sin M') + (0.0116 \times \sin 2M') - (0.0073 \times E \times \sin(M' + M)) + (0.0067 \times E \times \sin(M' - M)) + (0.0118 \times sin2F_1)]$.

Q=$[5.2207 - (0.0048 \times E \times cosM) + (0.0020 \times E \times \cos 2M) - (0.3200 \times \cos M') - (0.0060 \times E \times \cos(M' + M)) + (0.0041 \times E \times \cos(M' - M))]$.

$W=|cosF_1|$.

$\gamma = (PcosF_1 + QsinF_1) \times (1 - 0.0048W)$.

where, γ is the least distance from the centre of the Moon to the axis of the Earth's shadow. The value of γ gives the Moon's centre passes through north or south of the axis of the shadow if it is negative or positive respectively.

$$u = [0.0059 + 0.0046EcosM - 0.0182cosM' + 0.0004cos2M' - 0.0005\cos(M + M')].$$

(v) Magnitude of eclipse = $\frac{1.0157-u-|\gamma|}{0.5450}$

(vi) The *semidurations* of the partial and total phases in the umbra can be found as follows.
Calculate, $p = 1.0157 - u; t = 0.4707 - u; n = 0.5458 + 0.0400 \cos M'$;
Then the *semidurations* in minutes are
Partial phase (*HDUR*) $= \frac{60}{n}\sqrt{p^2 - \gamma^2}$;
Total phase (*THDUR*) $= \frac{60}{n}\sqrt{t^2 - \gamma^2}$;

Using these above steps we calculated the lunar eclipse on 4th April 2015, Saturday.

In this example the date 4-04-2015 stands for the day in this year is (94/365=0.257342). To find the value of k substitute year =2015.257342 in above equation *k*.

$k \approx$(year-2000)× 12.3685 =(2015.257342-2000)× 12.3685 = 188.71043.

It is a fullmoon day, hence consider value of *k* as 188.5 and it yields *t* =0.1524032825.

JDE=2457116.613664.

The angles of *M*, *M'*, *F* and Ω are found as

M = 88^0.913137; M' = 208^0.056547; F=2^0.1005421; Ω=-169^0.9933354; E=0^0.99996163;

F_1=2^0.10518633; A_1=320^0.01619.

Here, the value of *F* is near to 0^0, this eclipse occurs near the Moon's ascending node.

Correction for *JDE*, COR =0.387842.

∴ Corrected *JDE* =*JDE* + COR = 2457117.001506.

The decimal part of corrected *JDE* gives the instant of opposition timings.

Instant of opposition = $17^h32^m10^s$.

P=0.248261; *Q*=5.496289; *W*=0.999325;

γ=0.447836; *u*=0.022045;
Magnitude=1.0015>1. The eclipse is total.

The *semidurations* of the partial and total phases in the umbra:

Calculate, $p = 1.0157 - u = 0.993655;$ $t = 0.4707 - u = 0.448655; n = 0.5458 + 0.0400 \cos M' = 0.510501;$

Then the *semidurations* in minutes are

Partial phase (*HDUR*)=$\frac{60}{n}\sqrt{p^2-\gamma^2}$ =104'.252225 = $1^h\,44^m\,15^s$ Total phase(*THDUR*) = $\frac{60}{n}\sqrt{t^2-\gamma^2}$ = 3'.185116 = $0^h\,3^m\,11^s$.

Summary of the eclipse in IST;

(1) First contact with the umbra = Max.ofecl–*HDUR* = $17^h32^m10^s$–$1^h44^m15^s$ = $15^h47^m55^s$
(2) Beginning of total phase = Max.ofecl–*THDUR*= $17^h32^m10^s$–$0^h03^m11^s$ = $17^h28^m59^s$
(3) Maximum of the eclipse = Instant of opposition = **$17^h32^m10^s$**
(4) End of total phase = Max.ofecl+*THDUR* = $17^h32^m10^s$+$0^h03^m11^s$ = $17^h35^m21^s$
(5) Last contact with the umbra = Max.ofecl+*HDUR* = $17^h32^m10^s$+$1^h45^m36^s$ = $19^h16^m25^s$

In the following Table 1 we can observe the timings of the lunar eclipse of the day 4th April 2015. The traditional way of calculation gives nearly 5 minutes of difference as compare to *ISP*, Modern producer and NASA calculations, this is due the parameter they considered in those times. The *GL* procedure for the computation of lunar eclipse is comparatively reasonable even this text dispense with the trigonometric terms. The durations of the *ISP* tally well with those of modern and NASA data with maximum of one or two minutes. This is usually occurs in our traditional way of calculation.

Table 1: Durations of lunar eclipse on 4th April 2015

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Contacts	*Sūryasiddhānta*	*Grahalāghava*	*ISP*	Modern computation	NASA
Beginning	**$15^h 55^m$**	**$15^h 53^m 32^s$**	**$15^h 44^m 39^s$**	**$15^h 47^m 55^s$**	**$15^h 45^m 45^s$**
Begg(Totality)	**$17^h 22^m$**	**$17^h 27^m 1.68^s$**	**$17^h 21^m 37^s$**	**$17^h 28^m 59^s$**	**$17^h 27^m 54^s$**
Middle	**$17^h 36^m$**	**$17^h 36^m$**	**$17^h 30^m 57^s$**	**$17^h 32^m 10^s$**	**$17^h 32^m 37^s$**
End(Totality)	**$17^h 50^m$**	**$17^h 37^m 56^s$**	**$17^h 45^m 20^s$**	**$17^h 35^m 21^s$**	-------------
End	**$19^h 17^m$**	**$19^h 11^m 25^s$**	**$19^h 17^m 15^s$**	**$19^h 16^m 25^s$**	**$19^h 14^m 46^s$**

The timings calculated in the above Table 1 is relate to the Earth as whole and not to any particular locality. This eclipse is a total lunar eclipse, but it is a partial lunar eclipse for Indians. World in general lunar eclipse picture is shown in Fig 5. This Fig 4.5 with timings in UT (Universal time) 12 noon GMT is compared the eclipse computation with our Indian astronomical texts in IST.

Note: 12 noon GMT is nothing but $12+5^h 30^m = 17^h 30^m$ IST

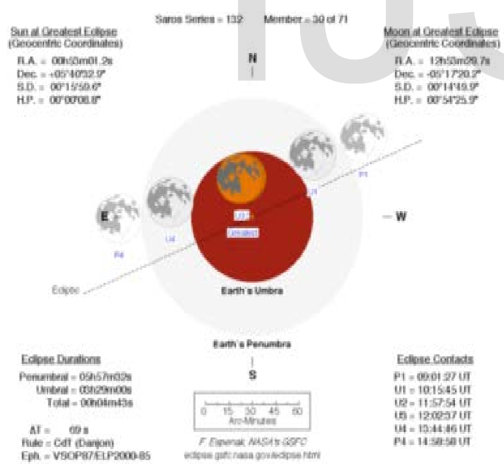

Fig. 5: Total Lunar Eclipse on 4th April 2015.

In Indian classical astronomy a penumbral eclipse is not considered as an eclipse and procedure for its computation is not given. In the considered example computation of an eclipse start with umbral region. That means, the timings of the Moon entering into the umbral region is the beginning of the eclipse. Similarly the Moon coming out from the umbral region is the end of the eclipse.

5. Saros Cycle of Lunar eclipses

Astronomers in ancient civilizations, based on their long term observations, discovered some cylces of periodicity of repetitions of eclipses, lunations known as Saros cycles. The period of 6585.32 days is called saros and is equivalent to 18 years 11 days and 8 hours. Eclipses of the same type are generally repeated once in a saros period. The mean length of a synodic month is 29.5306 days and there will be 223 lunations and 19 revolutions of the Sun with respect to the Moon's node i.e the interval between two successive passages of the Sun through Moon's node is about 346.62 days. Thus we have the relation: 223 lunations≈ 19 revolutions of the Sun w.r.t Moon's node.

The above discussed example of lunar eclipse lies in 132 series of saros cycle as 30th eclipse. This cycle consists of 71 eclipses and all these eclipses occur at the Moon's ascending node and the Moon moves southward with each eclipse. The beginning of this series is start from 12th May 1492 and ends on 26th June 2754 this is due to the longitude of successive eclipses changes by the angle through which the earth rotates about its axis. As on effect of this, every third eclipse in a particular saros series is about 10^0 west of that occurring about 54 (3×18) years earlier. A saros series does not continue endlessly.

Table 2: 19th to 21nd Century Eclipses in 132 series of Saros cycle

Series Sequence No.	Date	Magnitude	Middle of the Eclipse IST
21	26 Dec 1852	0.6815	18h 33m
22	07 Jan 1871	0.6893	2h 46m
23	17 Jan 1889	0.6972	10h 59m
24	09 Jan 1907	0.7110	19h 08m
25	09 Feb 1925	0.7304	2h 12m
26	20 Feb 1943	0.7616	11h 08m
27	02 Mar 1961	0.8006	18h 58m
28	14 Mar 1979	0.8538	2h 39m
29	24 March 1997	0.9195	5h 10m
30	04 April 2015	1.0008	17h 30m
31	15 April 2033	1.0944	00h 43m
32	26 April 2051	1.2022	5h 46m
33	06 May 2069	1.3229	14h 40m
34	17 May 2087	1.4554	21h 25m

7. Conclusion

In the present paper we have discussed the occurrence of the lunar eclipse and its computations on the basis of Indian classical astronomy ie*Improved Siddhāntic Procedure* and modern astronomy with an example of 4^{th} April 2015. Analyzed the result of this eclipse mathematically in two systems of computation and the concept of saros cycles are worked out very well for the period of 19^{th} to 21^{st} century.

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