ISSN 2229-5518

Analysis of Heat Transfer for Varying Surface Fin

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Abstract- Study of one dimensional steady heat conduction equation along the length of the fin is conducted on varying cross-section. The heat transfer from the base of the fin is due to conduction and from the surface to the atmosphere is due to convection. The varying cross-sections are assumed to be triangular and parabolic. The internal heat generation for both the fin is zero. The final heat transfer equation is in the form of zeroth order Bessel equation. Heat transfer rate from the fin is along the length is calculated by using Bessel's function. The temperature profile is also calculated along the length of the fin as well as the effectiveness, efficiency. Assuming the same dimensions for the parabolic profile, all the above properties are calculated using simple differential equation without using Bessel function. The heat transfer rate increases along the length of the fin from the base in both the fins. Comparing the heat transfer between these two geometries, it is observed that the parabolic concave profile transfers more heat than that of triangular one. The efficiency and effectiveness of both fins are higher in case of triangular fin than the concave parabolic fin. All the results are plotted in the form of graphs.

Key Words – Steady, Conduction equation, Varying cross-section, Fin, Efficiency, Effectiveness.

1 INTRODUCTION

Heat transfer is a thermal energy which occurs due to temperature difference. The modes of heat transfer are conduction, convection and radiation. Extended surfaces have fins which is attached to the primary surface on one side of a two-fluid Fins can be of a variety of geometry—rectangular, triangular, parabolic, hyperbolic and can be attached to the inside, outside or to both sides of circular, flat plate. Fins are primarily used to increase the surface area (when the heat transfer coefficient on that fluid side is relatively low) and consequently to increase the total rate of heat transfer. In addition, enhanced fin geometries also increase the heat transfer coefficient compared to that for a plain fin. Fins are most commonly used in heat exchanging devices such as radiators in cars, computer CPU heat sinks, and heat exchangers in power plants.

In this paper the varying cross-sections are assumed to be triangular and parabolic. The internal heat generation for both the fin is zero. Assuming the dimensions for triangular fin, whose base length is 0.05m, width 0.03m, thickness 0.01m, thermal conductivity(k) 180w/mk, heat transfer coefficient(h) 15w/m^{2}k, base temperature 200^{0}c and ambient temperature 25^{0}c. The final heat transfer equation is in the form of zeroth order Bessel equation. Heat transfer rate from the fin is along the length is calculated by using Bessel's function. The temperature profile is also calculated along the length of the fin as well as the effectiveness, efficiency. Assuming the same dimensions for the parabolic profile, all the above properties are calculated using simple differential equation without using Bessel function.

2 Formulation-

2.1 Triangular fin

The Triangular fin as in Fig-1 with L as length of fin, ẟ as thickness of fin, bas width of fin and assuming heat transfer is unidirectional and it is along the length and heat transfer coefficient(h) on the surface of the fin is constant.

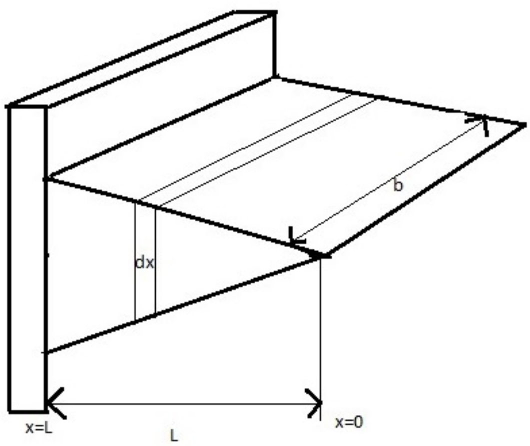

Fig-1 Triangular fin

Area of triangular fin is $A(x) = b\frac{\delta x}{L}$

Perimeter , $P(x) = 2b$

General governing equation of fin is,

$$\frac{d^2\theta}{dx^2} + \frac{1}{A(x)} \times \frac{dA(x)}{dx} \times \frac{d\theta(x)}{dx} - \frac{hP}{kA(x)}\theta(x) = 0 \quad \text{.........(1)}$$

Then,

$$\frac{d^2\theta}{dx^2} + \frac{1}{b\frac{\delta x}{L}} \times \frac{d}{dx} \times \frac{b\delta x}{L} \times \frac{d\theta(x)}{dx} - \frac{2hL}{k\delta x} \times \theta(x) = 0 \quad \text{.........(2)}$$

$$\text{Then, } x^2\frac{d^2\theta(x)}{dx} + x\frac{d\theta(x)}{dx} - \frac{2hL}{k\delta}x\theta(x) = 0 \quad \text{.........(3)}$$

Assume , $B^2 = \frac{2hL}{k\delta}$ = constant

Then equation(3) will be ,

$$x^2\frac{d^2\theta}{dx^2} + x\frac{d\theta}{dx} - B^2x\theta(x) = 0 \text{..................(4)}$$

Assume, $Z = 2B\sqrt{x}$

Then, $\frac{dZ}{dx} = \frac{B}{\sqrt{x}}$

Then, $\frac{d\theta}{dx} = \frac{B}{\sqrt{x}} \times \frac{d\theta}{dZ}$

Then, $\frac{d^2\theta}{dx^2} = \frac{B^2}{x}\frac{d^2\theta}{dx^2} - \frac{B}{2x^{-1.5}}\frac{d\theta}{dZ}$

Then equation (4) will be

$$\frac{d^2\theta}{dZ^2} + \frac{1}{Z}\frac{d\theta}{dZ} - \theta(x) = 0 \text{............................(5)}$$

This equation represent the zeroth order Bessl's equation. The general solution of the equation is

$$\theta = C_1 I_0(Z) + C_2 K_0(Z) \text{..........................(6)}$$

where, $C_1 and C_2$ are constant

I_0 (Z) = zeroth order Bessel's equation of 1st kind

$K_0(Z)$ = zeroth order Bessel's equation of 2nd kind

Applying boundary conditions –

(i) At $x = 0, \theta(x) = finite$

(ii) At $x = L, \theta(x) = \theta_0$

So the temperature distribution in the fin is

$$\frac{\theta}{\theta_0} = \frac{I_0(2B\sqrt{x})}{I_0(2B\sqrt{L})} \quad \text{.........(7)}$$

Total heat transfer rate from the surface is

Where, b= width of fin in m.

θ_0 = temperature difference

k = thermal conductivity in w/m.k

h= heat transfer coefficient in w/m²k

δ = thickness of fin in m.

L = length of fin in m.

B = fin parameter

I_1 = Bessel's function of 1st kind

I_0 = Bessel's function of 1st kind

$$\text{Efficiency of fin } (\eta) = \frac{b\theta_0\sqrt{2hk\delta}\,\frac{I_1(2B\sqrt{L})}{I_0(2B\sqrt{L})}}{hLb\theta_0}$$

Effectiveness of fin

$$(\varepsilon) = \frac{b\theta_0\sqrt{2hk\delta}\dfrac{I_1(2B\sqrt{L})}{I_0(2B\sqrt{L})}}{h\delta b\theta_0}$$

2.2 PARABOLIC FIN

For a parabolic fin representing length of fin L, thickness δ , width of fn b and assuming heat flow is unidirectional and it is along the length and heat transfer coefficient(h) on the surface of the fin is constant and internal heat generation is neglected.

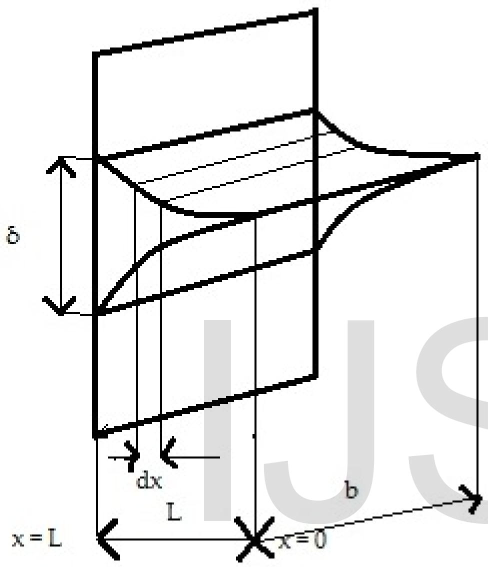

Fig-2 Parabolic fin

Equation of parabolic profile is $Y = Cx^2$

Applying boundary condition

(i) At $x = 0, Y = 0$

(ii) At $x = L, Y = \frac{\delta}{2}$

The general governing equation of fin is

$$\frac{d^2\theta(x)}{dx^2} + \frac{1}{A(x)}\frac{dA(x)}{dx}\frac{d\theta(x)}{dx} - \frac{hP(x)}{kA(x)}\theta(x) = 0 \quad \text{.........(8)}$$

So the equation of parabolic profile will be $Y = \frac{\delta x^2}{2L^2}$

Surface area of small section is , $dA = P(x)dx$

Cross section area of fin at any distance is $A(x) = 2Yb$

Putting these value in equation (8), then the equation will be

$$x^2\frac{d\theta(x)}{dx^2} + 2x\frac{d\theta(x)}{dx} - m^2L^2\theta(x) = 0 \quad \text{................(9)}$$

Where, $m^2 = \frac{2h}{k\delta} = constant$

Assume , $\log_e x = Z$

Then, $\frac{dZ}{dx} = \frac{1}{x}$

Then, $\frac{d\theta}{dx} = \frac{1}{x}\frac{d\theta}{dZ}$

Then, $\frac{d^2\theta}{dx^2} = \frac{1}{x^2}\frac{d^2\theta}{dZ^2} - \frac{1}{x^2}\frac{d\theta}{dZ}$

Then equation (9) will be,

$$\frac{d^2\theta(x)}{dZ^2} + \frac{d\theta(x)}{dZ} - m^2L^2\theta(x) = 0 \quad \text{..............(10)}$$

This equation is in the form of 2nd order linear homogeneous equation. The characteristics equation is

$$D^2 + D - m^2l^2 = 0 \quad \text{..................................(11)}$$

Solution of equation (11) is

$$D = \frac{-1 \pm \sqrt{1+4m^2L^2}}{2} = D_1 or D_2$$

Then the solution of equation (10) will be

$$\theta(x) = C_1x^{D_1} + C_2x^{D_2} \quad \text{...................................(12)}$$

Applying boundary condition

(i) At $x = 0, \theta(x) = finite$

(ii) At $x = L, \theta(x) = \theta_0$

After applying the boundary condition the temperature distribution will be

ISSN 2229-5518

$$\theta(x) = \theta_0 (\frac{x}{L})^{D_1} \text{ ....(13)}$$

So the total heat transfer will be

$$Q = \frac{kb\delta\theta_0}{2L}(-1 + \sqrt{1 + 4m^2L^2})$$

Where, k= thermal conductivity w/mk

b= width of fin in m.

δ = thickness of fin in m

L= length of fin in m.

θ_0 = temperature difference

m^2 = fin parameter

Efficiency of fin is

$$(\eta) = \frac{\frac{kb\delta\theta_0(-1 + \sqrt{1 + 4m^2L^2})}{2L}}{2hLb\theta_0}$$

Effectiveness of fin is

$$(\varepsilon) = \frac{\frac{kb\delta\theta_0(-1 + \sqrt{1 + 4m^2L^2})}{2L}}{2h\delta b\theta_0}$$

3 RESULTS AND DISCUSSION-

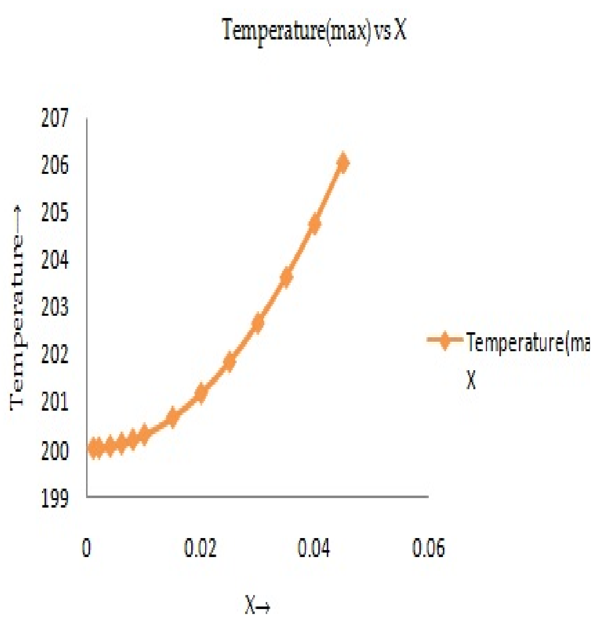

Fig-3 Variation of temperature of triangular fin with length (L)

Fig-3 shows the variation of temperature through the triangular fin to the length of fin. It is noted that as length of fin increased, temperature increases.

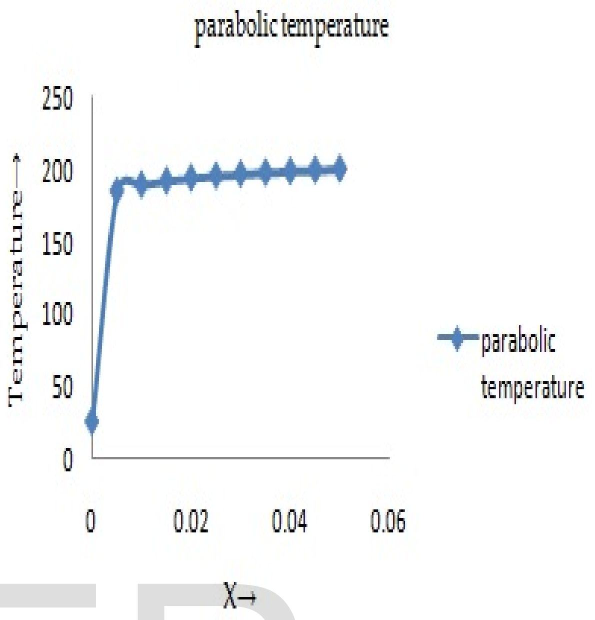

Fig-4 Variation of temperature of parabolic fin with length(L)

Fig-4 shows the variation of temperature of parabolic fin to the length of fin. It is noted that as length of fin increased, temperature increases.

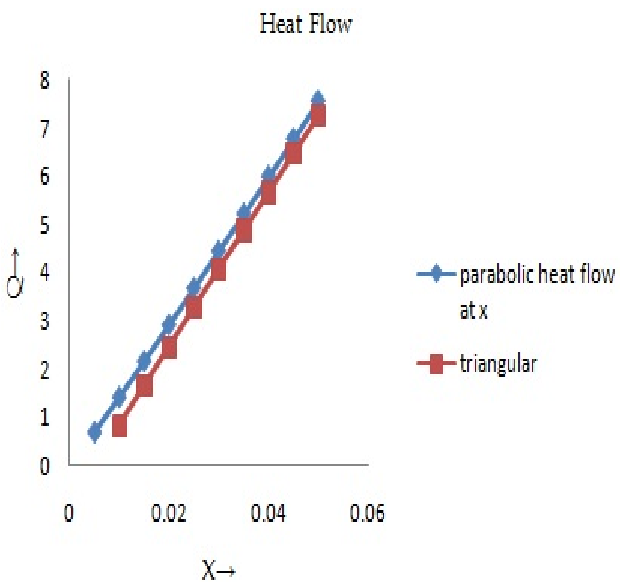

Fig-5 Variation of heat flow (Q) of the fin with length(L)

Fig-5 shows a variation between rate of heat flow to the length of fin. It is observed that heat flow of parabolic fin is more when compared to a triangular fin at any point of length.

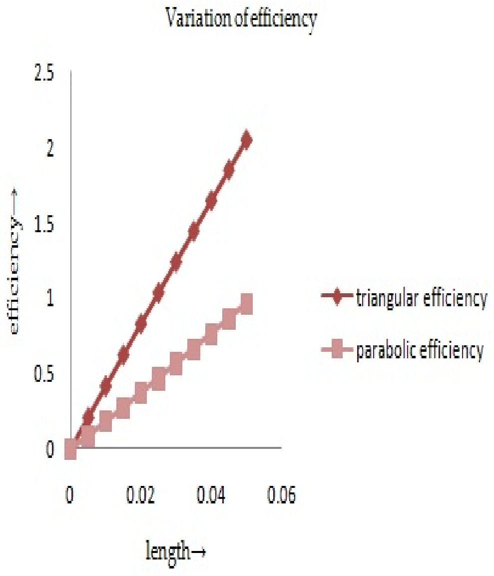

Fig-6 Variation of efficiency(η) of fin with length of fin(L)

Fig-6 shows the variation of efficiency of fin. The efficiency of fin increases with increase in length of fins. It is observed that triangular fin exhibits more efficiency than parabolic fin.

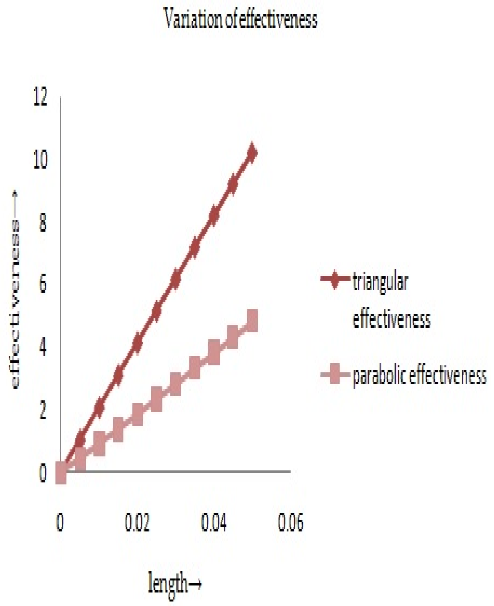

Fig-7 Variation of effectiveness (ε) of fin with length of fin (L)

Fig-7 shows the variation of effectiveness of fin. The effectiveness of fin increases with increase in length of fin. It is noted that triangular fins give more effectiveness than the parabolic fins.

TABLE 1 Nomenclature

Symbol	quantity	Units
A	Cross sectional area of fin	M^2
L	Length of fin	M
B	Width of fin	M
δ	Thickness of fin	M
B	Fin parameter	dimensionless
T_0	Base temperature	^0C
K	Thermal conductivity	w/mk
H	Heat transfer coefficient	w/m^2k
$T\infty$	Ambient temperature	^0C
M	Parabolic fin parameter	dimensionless
I_0	Bessel function of 1st kind	
I_1	Bessel's function of 1st kind	

4 CONCLUSION

Heat transfer in a triangular and parabolic fin is derived. Heat transfer rate is calculated by using Bessel's function which will help to get heat transfer in complicated geometry like triangular fin. Heat transfer rate of parabolic fin is calculated by using simple differential equation.

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