Analysis and design of two span continuous prestressed beam

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behavior of continuous

Abstract: This paper discusses, analysis and design of medium span continuous beams of prestressed concrete, effects of prestressing continuous beams, its advantages and disadvantages and how to make cable profile concordant. For continuous post-tensioned girders, the analysis and research work are comparatively lesser than segmental girders. Apparently construction of continues girders seems to be difficult but it is more efficient for heavy construction practices as in continuous girders curved cables can be suitably positioned to resist span and support moments and there is a reduction in the number of anchorages in continuous beams as compared to simply supported beams. In this paper, two spans continuous prestressed beam of uniformly distributed load is design and analysed. The cable profile and stresses developed at support and span sections is presented in the results. Finally conclusions from the results are presented.

Key words : continuous prestressed beam, concordant cable profile, moment redistribution.

INTRODUCTION

Prestressed concrete is a recent major form of construction introduced in the structural engineering. Like ordinary reinforced concrete, prestressed concrete consists of concrete resisting compression and reinforcement carrying tension. Prestressing became essential in many applications in order to fully utilize the compressive strength of reinforced concrete and to eliminate or control cracking and deflection. Today, prestressing is used in buildings, underground structures, communication towers, floating storage and offshore structures, power stations, nuclear reactor vessels, and numerous types of bridge systems. In case of prestressed concrete beam bending moment are more evenly distributed between the centre of spans and support of members and also reduction in the size of members results in lighter structures. The prestressed continuous beam has higher ultimate load carrying capacity than that of RCC continuous beam. Segmental construction by using precast units connected by prestressed cables lead to continuous girders. Sergio et.al[3] described the concept of redistribution of moments in composite beam and it also gives a brief note on degree of redistribution of moments. Two point load was applied at mid span and ultimate load obtained from experimental data were compared with theoretical one in both plastic and elastic zone. Tiejiong et al.[2] presented flexural externally prestressed concrete beam by using finite element model and results were compared with experimental value.

The aim of this work is to analyses and design medium span continuous beams of prestressed concrete, effects of prestressing continuous beams, its advantages and disadvantages and how to make a cable profile concordant.

PROBLEM STATEMENT

In prestressed continuous beam the concordant cable profile studied by using Guyon's Theorem of linear Transformation. For analysis and design of two span continuous prestressed beam the following parameters are considered.

Span of beam = AB=BC=12m

uniformly distributed live load = 10KN/m.

Tensile stresses are not to permitted in concrete.

Compressive stress in concrete is not to exceed 13N/mm^2

METHODOLOGY

In addition to the basic assumptions such as, elastic behaviour of materials and linear strain distribution across the section, following assumptions are generally made for analysis of secondary moments in continuous prestress concrete beams:

1. The effect of change in the length of the member due to prestressing force and external loading is negligible.
2. The cable friction is considered to be negligible so that the prestressing force is constant at all points in the cable.

The most commonly used methods to compute the secondary moments in a secondary prestressed beam are:

1. Three moment theorem.
2. Consistent deformation/Flexibility method.
3. Tendon reaction/Equivalent load method.

THREE MOMENT THEOREM

The classical method of linear structural analysis, such as three moment theorem can be conveniently used to analyse the secondary moments developed in continuous prestressed beam. The equation for three moment theorem is given below:

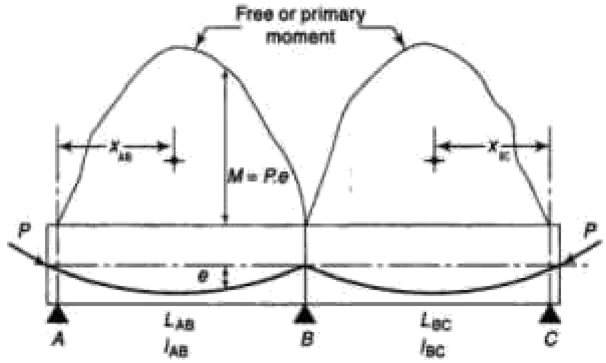

Fig 1 : Three moment diagram

$$M_A\left(\frac{L_{AB}}{I_{AB}}\right) + 2M_B\left(\frac{L_{AB}}{I_{AB}}+\frac{L_{BC}}{I_{BC}}\right) + M_C\left(\frac{L_{BC}}{I_{BC}}\right) = -6\left(\frac{A_{AB}\, x_{AB}}{L_{AB}\, I_{AB}} + \frac{A_{BC}\, x_{BC}}{L_{BC}\, I_{BC}}\right)$$

CONCORDANT CABLE PROFILE

Prestressing a statically indeterminate structure generally results in secondary moments due to the redundant reactions developed at the intermediate supports. However, it is possible to arrange the cable profile in such a way such that the structure does not deform at the supports or at any other points of restraint. In such a case, the redundant reactions and secondary moments are not induced by the prestressing cables.

Tendons which do not introduce secondary moments are referred to as concordant cable profiles. Any cable profile can be made concordant by linear transformation.

GUYON'S THEOREM

In a statically indeterminate prestressed concrete beams, it is possible to make simple modifications to a predetermined tendon profile without altering the pressure line in the members. This property was first stated by Guyon as follows: In a continuous prestressed beam, if the tendon profile is displaced vertically at any of the intermediate supports by any amount, but without altering its intrinsic shape between the supports, the resultant line of thrust is unchanged.

The operation of displacing the cable at intermediate supports while holding the position of the end anchorages constant without changing the intrinsic shape is also referred to as linear transformation of the cable.

In the design of continuous prestressed concrete beams, it is often necessary to determine a cable profile lying in the zone of limiting thrust and also satisfying the conditions of concordancy. A method based on the principle of virtual work is used. For the cable to be concordant, the cable must satisfy a condition

$$\sum \boldsymbol{Kme} = \mathbf{0}$$

. where K = dx/I

dx= length of a segment of the beam
I= Moment of inertia at that section

m= Coefficient of unit moment diagram at that section e= Eccentricity of the cable at that section

DESIGN APPROACH

The design of statically indeterminate prestressed concrete beams involves the computations of maximum and minimum moments at various cross sections of the members so as to obtain the range of moments that determines the cross sectional dimensions of the members. The steps involved in the design of continuous prestressed beams are given below:

1. Calculate the maximum positive and negative moments due to live and dead load at various sections of the member and compute the range of moments, which is the difference of maximum and minimum moments at a cross section.
   $$M_r = M_{max} - M_{min}$$
2. The overall cross-sectional dimensions are fixed using the permissible compressive stress in concrete, f_c, by the equation, assuming b=0.4-0.5h
   $$Z = \frac{bh^2}{6} = \frac{M_r}{f_c}$$
3. The minimum prestressing force is estimated by the expression,
   $$P = M_r/\left(\frac{h}{3}\right)$$
4. The limiting zone for thrust is obtained by plotting ($M_{ma}x/P$) and (M_{min}/P) at each section measured from the upper and lower kern respectively.
5. The profile of a cable lying within the limiting zone and suitable for a concordant profile is determined.
6. The stresses developed at transfer and working conditions are checked at important sections.
7. The cable profile, if necessary, may be linearly transformed to reduce the slopes at supports with regard to cover requirements.

1. Maximum positive and negative moments under different combinations of live load.

The various loading combinations to be considered is shown below

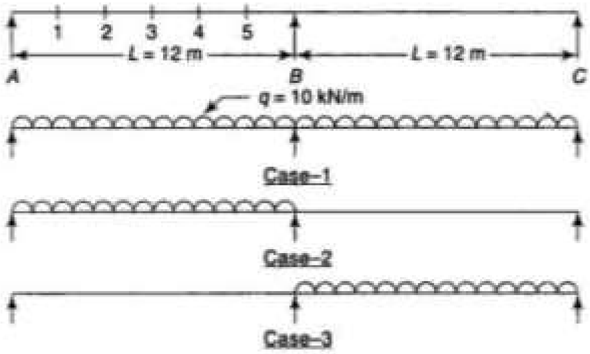

Fig 2 : Loading pattern

The maximum range of live load moment at support section B,

Mqr= $0.125qL^2 = 0.125 \times 10 \times 12^2 = 180$kNm

Assume b=250mm, the overall depth is obtained by,

$$Z = \frac{250 \times h^2}{6} = \frac{M_{qr}}{f_c} = \frac{180 \times 10^6}{13}$$

$$h = \sqrt{\frac{6 \times 180 \times 10^6}{13 \times 250}} = 570mm$$

Adopt a section 250mm wide and 600mm deep.

Self –weight of beam = (0.25 X 0.6 X 24) = 3.6kN/m

The moments at each point are expressed as $M=kw_d L^2$ where *k* is the moment coefficient obtained from Reynold's handbook and are shown in the table below. The value in the brackets represents the coefficient values.

2. Limiting zone for thrust.

The range of moments and the position of the limits of line of thrust are determined in the table below.

$$P = \frac{M_{max} - M_{min}}{h/3} = \frac{180 \times 10^3}{600/3} = 900kN$$

3. Determination of concordant cable profile.

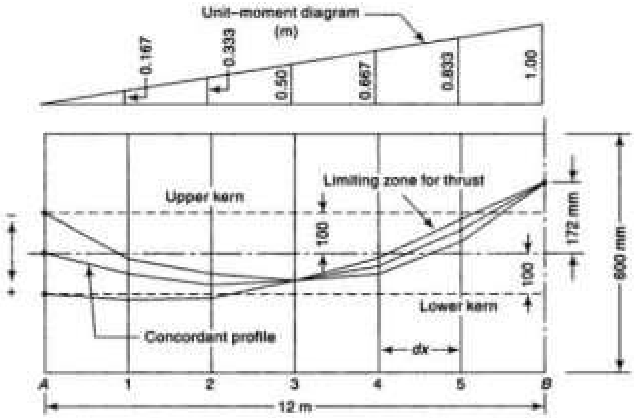

Fig 3 : Unit moment diagram and Limiting zone for Thrust and Concordant profile

The calculations for the cable profile are shown below**.**

Table 1 : Computation of maximum positive and negative moments

Location	Loading Pattern			Maximum Moments due to live load, kN/m		Dead load moments, kN/m
	Case 1	Case 2	Case 3	Mlp	Mln	Mg
A	(0) 0	(0) 0	(0) 0	- 0	- 0	- 0
1	(0.048) 69.0	(0.059) 85.0	-(0.01) 14.4	85	-14.4	25.0
2	(0.07) 101	(0.091) 130	-(0.02) 30.2	130	-30.2	36
3	(0.062) 90.5	(0.093) 135	-(0.03) 45.5	135	-45	32
4	(0.033) 47.5	(0.07) 101	-(0.04) 60.5	101	-60.5	17.4
5	-(0.03) 52	(0.017) 24.5	-(0.05) 75	24.5	-75	-18.8
B	-(0.12) 180	-(0.06) 90.5	-(0.06) 90	0	-180	-65

Table 2 : Determination of limiting zone for thrust line

Location point	Mlp kNm	Mln kNm	Mg kNm	Maximum moments M_2=Mlp+Mg kNm	Maximum moments M_1=Mln+Mg kNm	Range of moment M_2-M_1 kNm	Eccentricity E_2= (M_2/P)-100	Eccentricity E_1= (M_1/P)-100
A	0	0	0	0	0	0	-100	100
1	85	-14.4	25.0	110	10.6	99.4	22	111.8
2	130	-30.2	36	166	5.8	160.2	84	106.4
3	135	-45	32	167	-13	180.2	86	86
4	101	-60.5	17.4	118.4	-43.1	161.5	32	52
5	24.5	-75	-18.8	5.7	-93.8	99.5	-93.7	-4
B	0	-180	-65	-65	-245	180	-172	-172

Table 3 : Determination of Concordant profile

Point	I	Simpson's rule Coefficient q	$D_x/3$	K=q $D_x/3I$	Unit moment diagram m	Km	First Trial e_1 cm	Kme	Second Trial e_2 cm	Kme
A	1	1	2/3	0.67	0	0	0	0	0	0
1	1	4	2/3	2.67	0.167	0.44	6	2.64	6.7	2.97
2	1	2	2/3	1.34	0.333	0.44	9	3.96	9.7	4.30
3	1	4	2/3	2.67	0.5	1.33	8.6	10.56	8.6	11.50
4	1	2	2/3	1.34	0.67	0.88	4	3.52	4.0	3.52
5	1	4	2/3	2.67	0.834	2.3	-5	-11.15	-4.8	-10.70
B	1	1	2/3	0.6	1	0.6	-17.2	-11.50	-17.2	-11.50
								Σ Km e_1= -1.87		Σ Km e_2= 0.09

4. Check for stresses at section 3 and B.

Section 3

P = 900kN A = 15×10^4mm^2 P/A = 6 N/mm^2
Z = 15×10^6mm^3 e = 86mm Pe/A = 5.16 N/mm^2
Mmax = 167 kNm Mmin = -13 kNm

Section B

P = 900kN A = 15×10^4mm^2 P/A = 6 N/mm^2
Z = 15×10^6mm^3 e = 172 mm Pe/A = 10.32 N/mm^2
Mmax = -65 kNm Mmin = -245 kNm

Table 4 : Stresses at Section 3.

Stresses in N/mm^2		+ Compression		- Tension	
Fibre	Prestress	Max. load stress	Min. load stress	Max. stress	Min. stress
Top	6-5.16=0.84	$167\times10^6/15\times10^6$= 11.2	$-13\times10^6/15\times10^6$= -0.87	12.04	-0.03
Bottom	6+5.16=11.16	-11.2	0.87	-0.04	12.03

Table 5 : Stresses at Section B.

Stresses in N/mm^2		+ Compression		- Tension	
Fibre	Prestress	Max. load Stress	Min. load stress	Max. stress	Min. stress
Top	6+10.32=16.32	$-65\times10^6/15\times10^6$= -4.35	$-245\times10^6/15\times10^6$ = -16.32	11.97	0
Bottom	6-10.32= -4.32	4.45	16.32	0.03	12

CONCLUTIONS

The above work has shown the different methods of analysis of prestressed continuous beams. It has also shown the method of design of a continuous prestressed beam with a solved example. It has shown a method of making a cable profile concordant with an example.

From the above work we can conclude that continuous prestressed beams are more advantageous than simply supported beams due to moment redistribution thereby leading to lighter sections, smaller deflections and higher ultimate load carrying capacity. Also precast beams can be made continuous by providing cables at suitable locations thereby providing for segmental construction. There is a reduction in cost by reducing the number of anchorages as compared to simply supported beams.

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