

A MATHEMATICAL MODELING OF GROWTH & DECAY OF ELECTRIC CURRENT

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ABSTRACT: In this work, the mathematical governing equations were formulated from an electric circuit containing a resistor and an inductor usually called LR circuit using Kirchhoff's voltage law. Some basic differential and integral calculus principles were used to solve the differential equations formulated from the circuit. The solutions of the formulated problems represent the model for growth and decay current respectively. Some assumptions were made in analysing the models. Results from the analysis show an exponential increase and decrease in current values which represent the growth and decay of current respectively. This result was illustrated on graph using mat lab and the results were positive.

Keywords: Growth, Decay, Current, Active, Passive, Resistance, Inductance.



DEFINITIONS OF VARIABLE USED

- $i_{(t)}$ = Current at any given time t in amperes
- V = Voltage or electromotive force in volts
- R = Resistance in ohms (Ω)
- L = Inductance in Henry (H)
- $\frac{di}{dt}$ = Change in current with time
- V_L = Voltage across the inductor
- V_R = Voltage across resistance
- V_C = Voltage across capacitor
- τ = Time constant measured in seconds
- I = Maximum value of current in Amperes
- i = Growth and Decay current in Amperes
- e = Exponential
- R_{EQ} = Equivalent value of resistance in parallel connected circuit
- L_R = Resistance and inductive circuit
- AC = Alternating current
- DC = Direct current
- KVL = Kirchhoff's Voltage Law
- EMF = Electromotive Force
- RLC = Resistive, Inductive and Capacitive Circuit.

1.0 INTRODUCTION

A simple electric circuit is a closed connection of batteries and wires. The following quantities are measured in an electric circuit **(Eugene 1996)**

Circuit is classified into two major classes namely:

- (1) Active circuit element and (2) passive circuit element

Active circuit elements are those circuit components that provide electrical energy to the network e.g. Generators, Batteries, Thermocouple and Transistor.

They involve external energy source and provide the driving or forcing function in a network.

Passive circuit elements are those that either stores or dissipates energy obtain from the active circuit element. They do not involve external energy source but are characterised by predominantly exhibiting one of the three (3) basic electrical properties.

- (a) Resistance (b) Capacitor (c) Inductance

1.1 RESISTANCE

It is a measure of the ability of circuit element to oppose the flow of energy. A network containing source and resistance do not have transient properties. The physical device that exhibit resistances property is known as resistor. **(Hugh & Freedman 2005)**

1.2 CAPACITANCE

The property of an electrical component which enables it to store electric charges in an electrostatic field is known as capacitance. The physical devices which display predominantly the property of capacitance is called capacitor.

1.3 INDUCTANCE

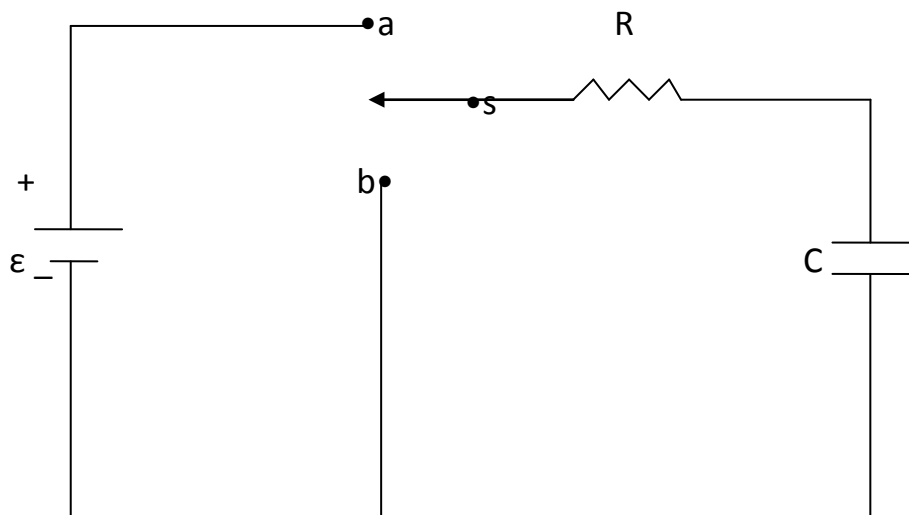
The property of a circuit element which enables it to store energy in the form of magnetic field is called inductance. An inductor is a physical element which is capable of storing energy by virtue of current flowing through it. Solenoid or a coil wire is an example of an inductor. **(Williams etal 1973)**. Inductor responds to transient property because of the self induced EMF and this now leads us to a phenomenon which is called transient.

1.4 TRANSIENT

Transient is the response which occurs in a system leading to change or impulse and then finally dies down after a passage of time. The rising and falling of current in a circuit is called Growth and Decay of electric circuit respectively. The response of network containing only resistance and source has no transient properties. Transient occurs in a circuit containing Resistance and Inductance properties called RL circuit. **(John 2010)**

Theraja (2005) describes Transient as the response which occurs in a system leading to charge or impulse and then finally decays after a passage of time. The responses of network containing only resistances and sources have been shown by the Authors to be constant and time invariant. The response of network containing capacitance and inductance is time varying because of the necessary exchange of energy between capacitor and inductive elements. No model was developed by the Authors but they use Kirchhoff's laws and differential equations with constant coefficient to solve electric circuit problems containing resistances, inductances and capacitances.

Halliday, Resnick and Walter (2004) developed a model for charging and discharging of a capacitor. Their model was formulated from RC circuit shown below:



RC circuit.

The Authors used KVL to solve the above circuit and formulated a model as given below:

$$\frac{Rdq}{dt} + \frac{q}{c} = \epsilon \dots\dots\dots 2.1$$

The above equation was gotten from

$$\epsilon - iR - \frac{q}{c} = 0 \dots\dots\dots 2.2$$

Where ε = voltage source, R = Resistance, C = Capacitance and q = quantity of charge.

Their model was solved and later came up with current in a charging capacitor as

$$i = \frac{E}{R} e^{-\frac{t}{RC}} \dots\dots\dots 2.3$$

And discharging current in a capacitor as

$$i = \frac{E}{R} \left(1 - e^{-\frac{t}{RC}}\right) \dots\dots\dots 2.4$$

John (2010) presented a model for determining voltage for charging a capacitor as

$$Vc = \frac{q}{c} = E \left(1 - e^{-\frac{t}{RC}}\right)$$

Decay current as $i = \frac{E}{R} e^{-\frac{t}{RC}}$.

Gaul & Gupta (2012) worked on a model to determine decay on a capacitor. The governing equation for the decay formulated was given to

be $I = \frac{V}{R} - \frac{q}{RC}$. His model was left unsolved.

He defined V as the voltage, I = Current,

R = Resistance, C = Capacitance and q = Charge. This model was solved analytically by substituting V = 0 in the formulated problem (given). They came up with $q = Q_f \text{Exp}\left(\frac{-t}{CR}\right)$ as the model for discharge of capacitor.

George & Weber (2005) formulated a model for the growth of current in an inductive circuit. The solution to their model was given

to be $i_d = I_0 \text{Exp}\left(\frac{-Rt}{L}\right)$ $t > 0$ for the decay

and $i_g = I_0 \left(1 - \text{Exp}\left(\frac{-Rt}{L}\right)\right)$ for the growth

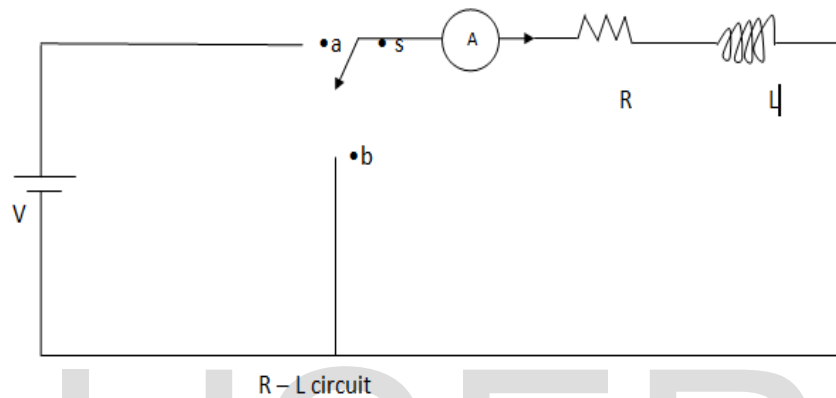
where $\tau = \frac{L}{R}$ in both cases and I_0 represents maximum current while i_d and i_g are the decay and growth current in the inductive current respectively.

The present model is an analytical model formulated from the principle of charging and discharging of capacitor given by **Halliday, Resnick and Walter (2004)**. RL circuit was used in this model instead of RC and the mathematical model was formulated

using principle of Kirchoff's voltage law to form a differential equation and solved using some basic differential and integral calculus principles which now produced the desired growth current and decay current respectively. The exponential increase and falling of current confirmed the efficiency of the models.

1.5 RESISTANCE & INDUCTANCE CIRCUIT (RL CIRCUIT)

Considering a circuit, where resistance (R) is connected in series with a coil of inductance (L) and the two are connected across a battery when switch (s) is connected to terminate (a) and is short circuited when (s) is connected to (b). The inductive coil is assumed to be resistance less. It's actual small resistance being included in (R).



When switch (s) is closed at time (t) = 0, the current does not reach its maximum value instantaneously but takes some time and this is because of the production of self induced EMF by the inductor (L) which opposes the flow of current. In the same way, when the switch is

connected to b, the source voltage is removed and the current does not return to zero (0) instantly but dies down with the passage of time. **Charles & Sadiku, (2004)**. This phenomenon describes growth and decay of current in an inductive circuit.

2.0 MODEL FORMULATION

Considering an Inductive circuit below (A circuit containing resistance R and Inductance

L) connected in series combination through a battery of a constant Electromotive force (EMF) with a two way switch S.

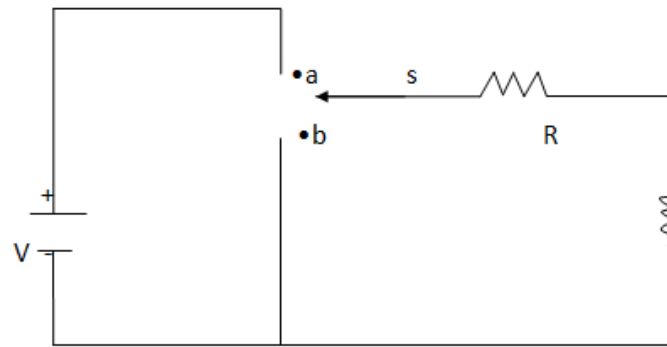


Fig A. An RL circuit with switch at a position

An analogous rise (or fall) of current occurs if we introduce an EMF (V) into (or remove from) a single – loop circuit containing a resistor R and an Inductor L . When the switch S in (figure A) above is closed on a , for example the current would rise rapidly to a steady value $\frac{V}{R}$. Because of the inductor, however, a self induced emf V_L appears in the circuit; this induced emf opposes the rise of the current, which means that it opposes the battery emf V in polarity. Thus current in the resistor response to the difference between two EMFs, a constant EMF (V) due to the battery and a variable EMF (V_L) = $(-L \frac{di}{dt})$ due to self- induction. As long as V_L is

present, the current in the resistor will be less than $\frac{V}{R}$.

As time goes on, the rate at which the current increases becomes less rapid and the magnitude of the self – induced EMF, which is proportional to $\frac{di}{dt}$ becomes smaller. Thus the current in the circuit approaches $\frac{V}{R}$ asymptotically. Initially, an inductor (L) acts to oppose changes in the current through it. A long time later, it acts like ordinary connecting wire.

Now let us analyse the situation quantitatively. With the switch S in the Figure above is thrown to (a) , the circuit is equivalent to that of figure B below:

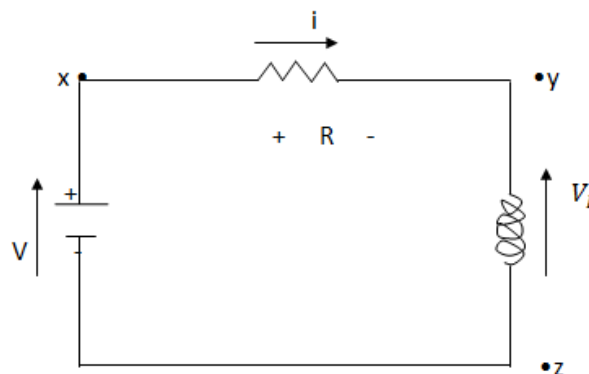


Fig B. A closed circuit with switch (S) on (a)

Applying Kirchoff's voltage law (KVL) to the circuit above:

$$-iR - L \frac{di}{dt} = -V \dots \dots \dots 3.1$$

Voltage across the battery is given by V ,

$$iR + L \frac{di}{dt} = V \dots \dots \dots 3.2$$

Voltage across the resistor = iR , because, as we move through the resistor in the direction of current i , the electric potential decreases by iR . That is, there is a decrease of potential.

$$\therefore V = Ri + L \frac{di}{dt} \dots \dots \dots 3.3$$

Voltage across the inductor = $-L \frac{di}{dt}$.

$V = Ri + L \frac{di}{dt}$ is the formulated problem for the growth of current in the circuit.

This is because the induced EMF, V_L opposes the battery current (i), thus we move from point y to point z , opposite the direction of V_L , we encounter a potential change of $-L \frac{di}{dt}$.

Now, to formulate the mathematical problem for the current decay, let the switch (S) in figure A is to be thrown to (b) with equivalent circuit shown below:

From KVL, which state that the algebraic sum of voltage in a circuit is equal to zero (0).

$$\Rightarrow V - iR - L \frac{di}{dt} = 0 \dots \dots \dots 3.0$$

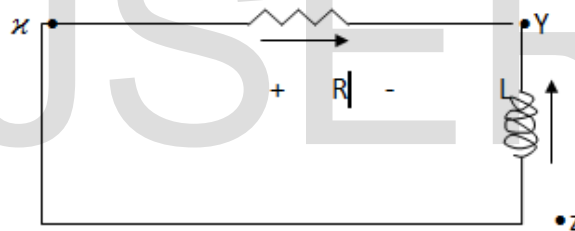


Fig C. A closed circuit with switch (S) on (b) [No source]

With battery source gone, the current through the resistor will decrease; however, it cannot drop immediately to zero but must decay to zero (0) over time. The differential equation that governs the decay can be found by putting $V = 0$ and applying Kirchoff's voltage law again to circuit of (fig C) we have

$L \frac{di}{dt} + iR = 0$ is the formulated problem for the decay of current in the circuit.

$$L \frac{di}{dt} + iR = 0 \dots \dots \dots 3.4$$

The differential equations that govern the growth and decay of electric current in a circuit to be solved are given below respectively

$$V = Ri + L \frac{di}{dt} \quad \text{and} \quad L \frac{di}{dt} + iR = 0$$

3.0 SOLUTION OF THE MODEL

The two models are solved analytically and we assumed that the solution of this differential equations satisfy an initial condition $i_{(0)} = \frac{V}{R}$

i_0 Represent current at time $t = 0$. In this case, that happened to be $\frac{V}{R}$ but it could be any other initial value.

$$V = Ri - L \frac{di}{dt} \dots\dots\dots 3.1$$

Dividing equation 3.1 by R, we have

$$\frac{V}{R} = \frac{Ri}{R} - \frac{L}{R} \frac{di}{dt}$$

$$\frac{V}{R} = i - \frac{L}{R} \frac{di}{dt}$$

But $\frac{V}{R} = I$, then this becomes

$$I = i - \frac{L}{R} \frac{di}{dt} \dots\dots\dots 3.2$$

$$(I - i) = - \frac{L}{R} \frac{di}{dt} \dots\dots\dots 3.3$$

$$\int \frac{1}{(I - i)} di = - \int \frac{R}{L} dt$$

$$\Rightarrow \ln(I - i) = - \frac{Rt}{L} + K \dots\dots\dots 3.4$$

Where K = constant of integration

Now from initial condition stated above:

$i_{(t)} = i_{(0)} = i_0$, that is $t = 0$, because current is at minimum before the growth $\Rightarrow \ln(I - 0) = - \frac{R(0)}{L} + K$

$$\ln I = K$$

Now substituting the value of the constant $K = \ln I$ into equation 3.4 gives

$$\ln(I - i) = - \frac{Rt}{L} + \ln I$$

Solving the first model which represents the growth of current analytically:

$$V = Ri + L \frac{di}{dt} \dots\dots\dots 3.0$$

Since the induced Emf in coil (L) is opposite to the battery voltage (V), equation 3.0 becomes

Separating the variables, that is multiply equation (3.3) by $\frac{1}{di}$ we have

$$(I - i) \frac{1}{di} = - \frac{L}{R} \frac{di}{dt} \times \frac{1}{di}$$

$$(I - i) \frac{1}{di} = - \frac{L}{R} \frac{1}{dt}$$

The equation above can also be expressed as

$$\frac{di}{(I - i)} = - \frac{R}{L} dt$$

Integrating both sides gives

$$\ln(I - i) - \ln I = - \frac{Rt}{L}$$

$$\ln \left[\frac{(I - i)}{I} \right] = - \frac{Rt}{L}$$

Taking the natural logarithm of the above equation gives

$$\frac{(I - i)}{I} = e^{-\frac{Rt}{L}}$$

Cross multiply

$$\Rightarrow (I - i) = I e^{-\frac{Rt}{L}}$$

$$I - I e^{-\frac{Rt}{L}} = i$$

$$\therefore i = I \left(1 - e^{-\frac{Rt}{L}} \right)$$

But Inductive time constant is given by $\tau = \frac{L}{R}$

$$\Rightarrow i = I \left(1 - e^{-\frac{t}{\tau}} \right) \dots \dots \dots 3.5$$

$i = I \left(1 - e^{-\frac{t}{\tau}} \right)$ Is the mathematical model for the growth of current in an Inductive circuit.

The physical significant of the time constant is seen below:

From the growth model $i = I \left(1 - e^{-\frac{t}{\tau}} \right)$, when the switch is closed at time $t=0$ and for a time long after the switch is closed ($t \rightarrow \infty$). If we substitute $t = 0$ into the model $i = I \left(1 - e^{-\frac{t}{\tau}} \right)$, the exponential

Becomes

$$e^{-0} = 1$$

$$\Rightarrow i = I(1 - e^{-0})$$

$$= i = I(1 - 1) = 0$$

$$\therefore i = 0$$

This tells us that the current is initially $i = 0$ as we expected.

$$L \frac{di}{dt} + Ri = 0 \dots \dots \dots 3.6$$

Dividing both sides by R

$$\frac{L}{R} \frac{di}{dt} + \frac{Ri}{R} = 0$$

$$\frac{L}{R} \frac{di}{dt} + i = 0$$

$$\frac{L}{R} \frac{di}{dt} = -i$$

Multiply both sides of the equation by $\frac{i}{di}$ to separate variables

Next, if we let t goes to ∞ , then $e^{-\infty} = 0$

Thus our model $i = I \left(1 - e^{-\frac{t}{\tau}} \right)$ becomes

$i = I(1 - 0)$ Which means $i = I$ and it tells us that the current goes to its equilibrium value $I = \frac{V}{R}$.

Now the physical significance of the time constant follows from the model, if we put $t = \tau$, then the model becomes

$$i = I \left(1 - e^{-\frac{\tau}{\tau}} \right)$$

$$i = I(1 - e^{-1})$$

$$i = I(1 - 0.3679)$$

$$i = 0.63I$$

Thus, the time constant (τ) is the time it takes the current in the circuit to reach about 63% of its final equilibrium value $\frac{V}{R} = I$.

Solving the second model, this represents the decay of current analytically:

$$\frac{L}{R} \frac{1}{dt} = -i \times \frac{1}{di}$$

$$\frac{L}{Rdt} = \frac{-i}{di}$$

$$\Rightarrow \frac{R}{L} dt = \frac{di}{-i}$$

$$\frac{di}{i} = -\frac{R}{L} dt$$

Integrating the above equation gives

$$\int \frac{di}{i} = \int \frac{-R}{L} dt$$

$$\Rightarrow \ln i = \frac{-Rt}{L} + K$$

K.....3.7

Where K = constant of integration.

Now from initial condition stated above:

$$i_{(t)} = i_{(0)} = i_0, \quad i = I$$

$$\Rightarrow t = 0$$

$$\therefore \ln i = \frac{-Rt}{L} + K$$

$$\ln i = \frac{-R(0)}{L} + K$$

$$\Rightarrow K = \ln i = \ln I$$

Putting K into equation 4.7

$$\ln i = \frac{-Rt}{L} + \ln I$$

$$\ln i - \ln I = \frac{-Rt}{L}$$

$$\ln \left(\frac{i}{I}\right) = -\frac{Rt}{L}$$

Taking the natural logarithm of both sides

$$\frac{i}{I} = e^{-\frac{Rt}{L}}$$

Cross multiply

$$i = I e^{-\frac{Rt}{L}}$$

But $\frac{L}{R} = \tau$ (time constant)

$$\therefore i = I e^{-\frac{t}{\tau}} \dots \dots \dots 3.8$$

$i = I e^{-\frac{t}{\tau}}$ Is the mathematical model for the decay of current in an Inductive circuit.

The physical significant of the time constant is shown below:

From our model, if $t = \tau$, we have

$$i = I e^{-\frac{t}{\tau}} = I e^{-\frac{\tau}{\tau}}$$

$$i = I e^{-1}$$

$$i = I \times 0.3679$$

$$i = 0.37I$$

\Rightarrow It takes the current in the circuit to decrease to about 37% of its equilibrium value I

4.0 ANALYSIS OF RESULT

To analyse these results we make certain assumptions that will help us to realise curves indicating growth and decay of current respectively:

Let $V = 100V, R = 10\Omega, L = 40H$

So that $I = \frac{V}{R} = \frac{100}{10} = 10A$

$$\tau = \frac{L}{R} = \frac{40}{10} = 4seconds$$

Also let time (t) be taken for every 1seconds to 15seconds.

Now solving for the growth of current which is given by

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right)$$

for time (t), where $t = 0, 1, \dots, 15secs$

When $t = 0$

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{0}{4}}\right) = 0.00A$$

When $t = 1$

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{1}{4}}\right) = 2.21A$$

When $t = 2$

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{2}{4}}\right) =$$

3.93A

When t = 3

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{3}{4}}\right) =$$

5.28A

When t = 4

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{4}{4}}\right) =$$

6.32A

When t = 5

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{5}{4}}\right) =$$

7.13A

When t = 6

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{6}{4}}\right) =$$

7.77A

When t = 7

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{7}{4}}\right) =$$

8.26A

When t = 8

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{8}{4}}\right) =$$

8.65A

When t = 9

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{9}{4}}\right) =$$

8.95A

When t = 10

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{10}{4}}\right) =$$

9.18A

When t = 11

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{11}{4}}\right) =$$

9.36A

When t = 12

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{12}{4}}\right) =$$

9.50A

When t = 13

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{13}{4}}\right) =$$

9.61A

When t = 14

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{14}{4}}\right) =$$

9.70A

When t = 15

$$i = I \left(1 - e^{-\frac{t}{\tau}}\right) = 10 \left(1 - e^{-\frac{15}{4}}\right) =$$

9.76A

The maximum value of the current is 10A, hence we can stop here and see the

t(sec)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
i(A)	0	2.21	3.93	5.28	6.32	7.13	7.77	8.26	8.65	8.95	9.18	9.36	9.50	9.61	9.70	9.76

nature of current growth graphically using mat lab:

Table 1: tabulate results for current growth

$$\text{Decay current } i = Ie^{-\frac{t}{\tau}}$$

Also solving for the decay of current using the same time interval $0 \leq t \leq 15$:

When t = 0

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{0}{4}} = 10A$$

When $t = 1$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{1}{4}} = 7.79A$$

When $t = 2$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{2}{4}} = 6.10A$$

When $t = 3$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{3}{4}} = 4.72A$$

When $t = 4$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{4}{4}} = 3.70A$$

When $t = 5$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{5}{4}} = 2.87A$$

When $t = 6$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{6}{4}} = 2.2A$$

When $t = 7$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{7}{4}} = 1.74A$$

When $t = 8$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{8}{4}} = 1.40A$$

When $t = 9$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{9}{4}} = 1.05A$$

When $t = 10$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{10}{4}} = 0.82A$$

When $t = 11$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{11}{4}} = 0.64A$$

When $t = 12$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{12}{4}} = 0.50A$$

When $t = 13$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{13}{4}} = 0.39A$$

When $t = 14$

$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{14}{4}} = 0.30A$$

When $t = 15$

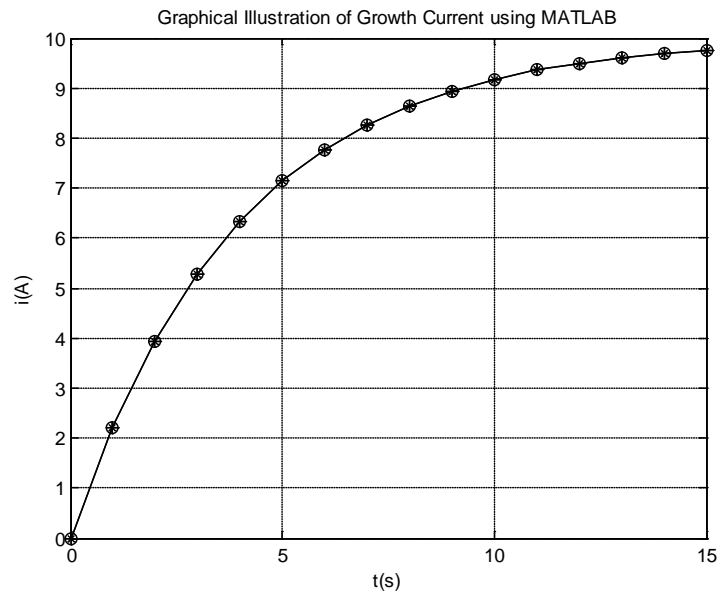
$$i = Ie^{-\frac{t}{\tau}} = 10e^{-\frac{15}{4}} = 0.24A$$

Since the minimum value of current when a switch is open is zero(0), hence we stop here as we discover that current i tends to zero(0) as t increases.

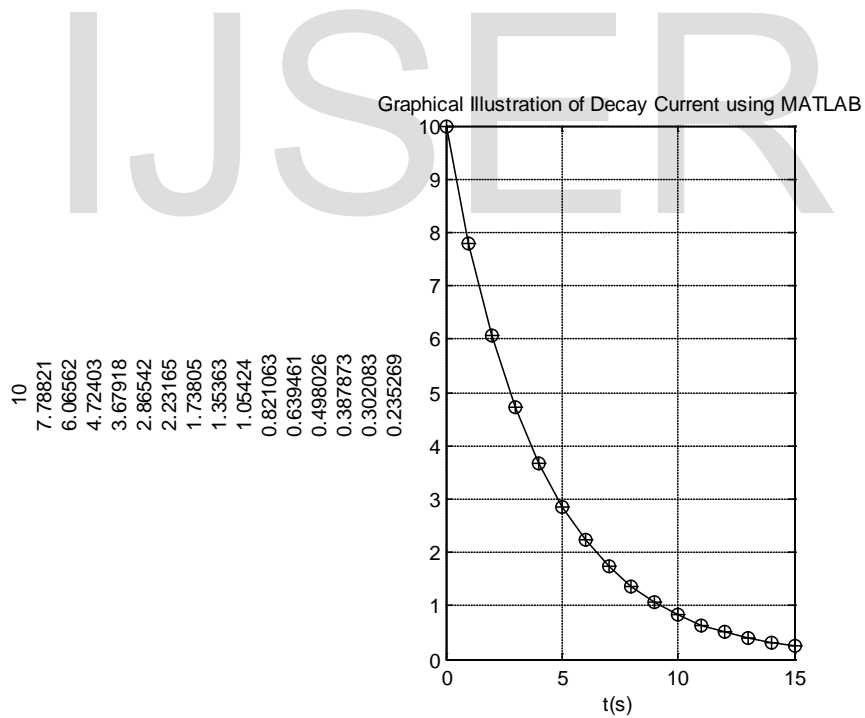
t(s)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
i(A)	10	7.79	6.10	4.72	3.7	2.87	2.2	1.72	1.4	1.05	0.82	0.64	0.5	0.39	0.3	0.244

Table 2: Tabulated results for Decay current

Graphical representation for growth of current is shown below:



Graphical representation for decay of current is shown below:



CONCLUSION

From the analysis, it is seen that the graph of current against time (t) increases exponentially until it reaches the maximum equilibrium current of 10A and this shows the build up of current in a circuit and it is termed **Growth**. Similarly, the second graph shows an exponential decrease of current from the maximum equilibrium value of 10A until it reaches the minimum equilibrium of 0A which represent decline of current in a circuit which is also called **Decay**.

In a real life, this phenomenon occurs in our home electronic appliances when power

comes into our appliances, charges or current build up exponentially before full operation begins and in the same way, when power goes off, our system or appliance does not goes off instantly but the power dies down until it is gone finally. The rising and the falling of this current can cause a big surge of voltage which damages appliances, to this end, it is recommended for us to use a system or appliance that has an inbuilt 3 – 5 minutes delay buffer system that waves the surge of such power and protect the home appliances or use a **voltage regulator** also known as **stabilizer** for protection.

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