SENSORY DATA FUSION IN ROBOTICS THROUGH FUZZY AND INTERVAL MATHEMATICS

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Abstract- Traditionally, the domain of model-intensive data fusion of sensory systems depend largely on two vital aspects, namely, 'sensor type' and 'sensor layout'. Although a sound definition or problem identification about the sensor classification leads to a fruitful fused data at the end using deterministic calculations all through, the exercise turns futile when fuzziness exits in the sensor-data itself. Thus, in this article, we will focus on the paradigms of novel models for sensory data fusion using the concept of Interval Mathematics, exclusively for re-christening the time-varying data-clusters. This very concept of imbibing the alteration of data through re-representation in "interval" bounds gives enough flexibility in understanding the true phenomena of sensor fusion in real-time.

1. INTRODCTION

Multi-sensory data fusion has emerged as a true discipline of intensive research over the past decades, with its application manifold getting augmented with newer domains of engineering & science. Nevertheless, a substantial part of this research has been focused with somewhat traditional methodologies. This has left immense scope for augmenting parallel mathematical tools & techniques, like Interval Mathematics, that can be efficiently retrofitted for solving potential engineering applications. Robotics is one such domain where a vast majority of the problems may be tackled quite effectively using the principles of Interval Mathematics. The interesting lemma of Interval Mathematics which can be adopted with ease in practical problems are its two basic rules of computation, e.g. *addition* & *subtraction*. The rules for addition and subtraction are common sense-driven, and that's why those fit appropriately in almost all practical problems in engineering. However, the next two computational rule-bases, namely, *multiplication* & *division* do not enjoy such computational ease and the rule-bases for multiplication as well as division using interval algebra are quite varied. In past decades, researchers all over the world have thrived towards commonalizing multiplication rule-bases, specifically in engineering problems, but results are quite disperse. Similar complexity does arise in case of mathematical functions, involving divisions using interval algebra. With all theses, there is sufficient void available, so far as investigation with multiplication & division lemma is concerned towards solving practical problem in engineering.

In a similar line, Robotics research has also got direct influence by the principles of interval mathematics and we will highlight some niche areas in robotics wherein the concept of interval mathematics is quite apt.

Traditionally, use of interval mathematics was researched out in past decade in three major fields, namely: a] kinematic design of robots (mostly parallel manipulators); b] navigation of mobile robot(s) with odometry and c] controller design & algorithm with software specifications. Although these three domains, especially the first two, are vital in robotics, yet we can explore a number of other avenues of importance wherein interval mathematics can be sneaked in. One such emerging and promising look out is the domain of *sensory data fusion* in robotics, on which we will report in detail in this article.

Sensor data fusion, as applied in the field of robotics, using the formulation of interval mathematics is not addressed effectively by the researchers till date. It's by & large an *open* problem and demands in-depth research in devising suitable lemma / formulae in tacking the fusion of large number of individual sensor-data. A large agglomeration of sensor data, though very common in situations of aerospace engineering, is equally potent in robotic system too.

The generic paradigms of multi-sensor data fusion rely on the augmentation and assimilation of raw data (from various sources) in real-time. The three key facets of such fusion, namely, "augmentation", "assimilation" and "real-time" bear great significance; to be specific these three facets must go hand in hand in case of a fused data being oozed out of an engineering system. While the third paradigm, namely, "real-time" is by and large an engineering aspect, as it pertains to the actual functioning of the real-life application system involving (continuous) generation of raw data, the first two paradigms have scope for involving improvised mathematical models. Augmentation of raw data, emanating from a single source or multiple, irrespective of its characteristics (similar or dissimilar) essentially involve merging of suitable data in the form of *data-clouds* or *clustered data*. In such merged data system, clustering generally occurs in a selective way and in cases raw data is expressed in *intervals*, clustering needs to be done suitably, unlike the case where data is deterministic. In most of the practical situations, we come across a varied agglomeration of raw data, some are in clusters and some maintain individualistic stature even after the processing for fusion. It entirely depends on the source of such data, i.e. the application-domain. For example, for raw sensory data generated continuously from a chemical process plant will essentially be put under several *data-clouds*, representing the plant or its sub-groups in real-time. On the contrary, raw data emanating from a robotic sensor system can have a combination of both clustered data as well as individualistic data. The reason is also acceptable; because in case of robotic sensory system, data in more centric on the technical paradigms of the sensory-elements rather than the overall distribution of the by & large homogeneous sensor-cells (as appears in the process plant). Whatever the case may be, fusion of the raw data will be challenging when the base-data is expressed in *intervals.*

The other situation, which is equally valid in practice / application, is a conglomerate of data, not all are expressed in intervals. This is a very practical hypothesis, as in many situations, we need to dwell on variables that can only and must generate deterministic raw data. That means, for such variables data can't be thought of to be expressed in intervals at all. At the same time, we must consider these variables for the sake of getting fused data finally. Hence, we come across situations where we have to deal raw data expressed in either *intervals* or *non-interval* form. This is a real tricky issue and there is no way other than tackling it by proposing appropriate lemma, suitable for the application manifold. As a matter of fact, it may so happen that we may land up in having a partial fused data-set and un-fused data also, in case of heterogeneous data-clouds. The principles of interval mathematics will be applied in such cases only partially. Nonetheless, all fused data-sets must be *continuous* in nature and represent near real-time operation of the end-device, i.e. robotic sensor system.

With this perspective in mind, it can be summarized that the most vital facet of data-cloud based (sensory) fusion is to cater for raw data that are expressed in intervals. We need to appreciate the fact that in many situations we may find that it is not possible to get output data-clouds 'integrated' or '*union-ed*'; and we will have no option other than letting them retain their distinct

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locations, inside the data-cloud. This situation is very common in practical applications in robotic systems, involving heterogeneous sub-systems. In other words, for fusion of heterogeneous system, we can have partial fused and un-fused data.

Besides evolving fusion rule-base using the lemma of interval mathematics for a uni-dimensional data-space, we will highlight on more advanced and real-life example-matrix, comprising data with *higher dimensions*. Devising suitable clustering algorithm for higher-dimensional data space, by adopting the theory of interval mathematics is undoubtedly a challenging paradigm. This is more relevant for augmenting *multi-input not-all-similar* sensory system, especially in case of robotic application. In such fusion, we essentially need to know three facets, namely, a] total number of data-centres present/working in the whole (application) system; b] dimensions of the data & c] time-instants and/or intervals on which the output data is evaluated. However, it is to be noted that it is not mandatory that we will have 'data-clusters' all over the space; there can be a 'lone' or 'single' data also under one specific dimension. Clustering is being made with respect to similarity of the data (i.e. as per the dimension) and not corresponding to their *origin* (i.e. the data-centre) or time-instant. In that respect, for practical calculation, all *similar data*, i.e. having identical / equal power (exponent), will be clubbed together. This technique will be handy, especially when similar data are expressed in intervals; wherein simply addition formula (of interval algebra) can be applied.

It is to be noted that in all cases of sensory data fusion, "time" is an important attribute, as all practical application manifold essentially work in real-time. In fact, in all practical situations of sensor data fusion, expressed in 'intervals', there must be overlapping of time-spans. That's the reason, in interval-based data fusion we must always insist on the "time-scale"; all calculations using interval algebra will be re-repeated in time-scale / time-domain. The variables can have the same time-scale or different, depending upon the application manifold.

A new model for *multiplication* of variables using interval mathematics is being proposed in this paper. This theory uses the principles of "convex hull" and "B-rep" for the computational part of the model. The novelty of this model lies with the fact that all data are represented in time-scale. In that sense, this interval multiplication is essentially two-dimensional; i.e. geometric, where 'time-instant' is an *axis*. Unless time-domain concept is introduced, interval multiplication will be only uni-axial which is synonymous to the traditional way of performing the same.

It is needless to state that the pattern of fusion using the logic of Interval Mathematics does vary with the changes in sensor-type or the field of application. In other words, while we can have distinct gradation of type of sensors as *homogeneous* vs. *heterogeneous* (i.e. all or part thereof dissimilar), the other metric, viz. the layout of sensory placement can be either *uniform* (coherent) or *staggered* (non-coherent). It may be appreciated that both the metrics, stated above, are equally important for a fusion model to work efficiently, especially for a field-sensory system. In case of traditional ("deterministic") form of data fusion, we do get fused data, applying suitable rule-bases and models, depending upon the source as well as nature of raw data generated. This effort is fruitful, to the extent of the somewhat 'known' form of raw data and also the numerical values [1-5]. However, the methodology cripples as and when we try to introduce *time-dependency* factor in the system. In other words, when the raw data are varying significantly over a specified time-span of operation of the system against a specific data-source, the task starts getting difficult. The theories of Interval Mathematics can be of use in such situations, encompassing several data-clusters, having seamless generation of raw data over a time-period. Interval-data signifies variability of the data under different trial-runs / testing / batch-processing etc. and in all practical situations, we need to consider this inherent variability for further processing (e.g data fusion). However, we need to appreciate that this variability is not statistical in nature, it is a kind of natural /random variation (scatter) in the data, against a particular time-stamp.

To add fuel to the problem, in such practical situations, we need to consider additionally the effect of *dimensionality* of data. In practical application-specific scenarios, we often come across *higher dimensional data*, especially with field sensory systems. In field applications (e.g. robotic operation through pre-programmed cycle), we often come across output parameters that are simultaneously dependent on 'k' variables, where

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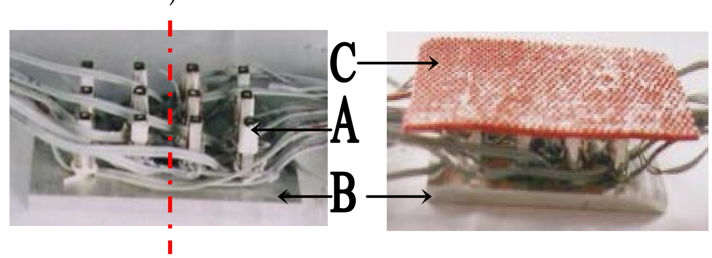
k=2,3,…,n. It is extremely difficult to tackle such situations involving higher-dimensional data space and the matter gets really computationally intensive in case data-sets are expressed in "*intervals*". We would like to address this sort of real-life cases of enhanced dimensionality in this article. The other issue related to higher-dimensional data-space which frequently causes hindrances in fusion model is the methodology of *clustering of data*. Truly speaking, efficient clustering of data-sets, expressed in 'interval' is instrumental in attaining a coherent fused output data. With this perspective, we will take up the lemma for data clustering algorithm for higher dimensional data-space in the article.

Apart from the situations when only data-set is being expressed in 'interval', we will highlight on some cases of cumulative evaluation, namely, *integration*, using the principles of interval mathematics. In such cases, we will derive the expression for fused data, considering the limits /bounds of such integration also in 'intervals'. This is a very unique case with lot of practicality involved and thus it is prudent to analyze such situations. The other important aspect of practical use is the *sensitivity analysis* of the input data-sets, emanating from various sensor-cells in the system. We will put forward an application-centric model for performing the sensitivity analysis for a sensory system, ideally heterogeneous, having all or part of the input data expressed in 'interval'.

We propose a novel rule-base / methodology for achieving the final fused output from the gamut of sensory inputs, namely, *Multiplicative law of fusion*. The model is made customized, considering the fact that we are dealing with data-sets, expressed in 'intervals'. The thematic of this rule-base was successfully verified through deterministic data-sets [6-7], and the same was also tested for real-life experimentations [5-7]. In arriving at various intermediate lemma of this model we will use the concept of "Distance Function" (between two *data-clouds*), which essentially becomes an effective tool for judging the relative closeness between data-points, coming out from different sensory-sources. It may be stated here that the crux of the model lies with the *geometric interpretation* of the natural system, i.e. fusion, in real-time. We propose to analyze the physical system and its happenings in real-time through realization module that is sustainable over a period. Especially, the model is equipped for direct application in operational space, as the lemma is made compatible with geometric parlances, e.g. using the principles of *convex hull*.

We will investigate a specific case-study, pertaining to the indigenous developments of robotic sensory system involving similar and dissimilar sensory-cells for an *assessment* of the rule-based data fusion. The study is connected with the design of a small-sized tactile sensory unit, having all similar sensor-cells embedded, for use in robot gripper. Both the designs of the robotic sensory systems are well-documented and tested successfully [1],[2]. The prime idea behind the selection of these two categories of robotic sensory systems for the present analysis is the inherent variability of the individual sensor-cells. It may be noted that these sensor-cells can act independently, coherently and in real-time. All these three aspects are crucial for a successful data-fusion to take place in real-life situation. In our earlier publications [3], [4], [5],[6], we have demonstrated that both these sensory systems are adaptable to a certain range of design modifications, thereby making themselves generic in nature, so far as the modeling is concerned. This aspect of generalization is another theme-point for a good-level of data fusion, as the fusion rule-base and/or model must be *generic* in nature. It is obvious that we do not need to have detailed engineering information about the design & firmware of the sensory system in order to build-up the mathematical model for the data fusion, but a gross knowledge of the actual system should be a pre-requisite.

In this study, we will investigate a sensory system array having homogeneous tactile sensor-cells. The system is poised for augmenting to a robotic gripper (based on the physical external dimensions of the prototype) and thus, will act as 'gripper sensor' for all practical scenarios. Figure 1 shows a photographic view of the said (gripper) sensor, with two sub-grids differentiated (by vertical dotted line).



Index: A: Resistive-cell (with wires intermixed); B: Sensor base; C: Rubber pad.

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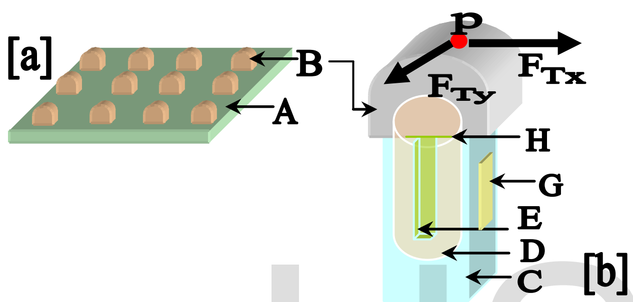
Fig. 1: Photographic view of the gripper sensor used for data fusion testing

The miniaturized gripper sensor has the semiconductor-based strain gauge fitted resistive cells embedded in a 4x3 matrix over a metallic (aluminium) base, with an overall external dimension: 45 mm. x 75 mm. x 16 mm. (height). Figure 6a presents an exploded schematic (not in scale) of the serrated rubber pad, while the details of the modular sub-assembly of the R-cell is illustrated in fig. 6b.



Index: A: Rubber pad; B: Serration(s); C: Strut; D: hole; E: Projecting pin; G: Strain gauge; H: Fixation between pin & pad; $F_{Tx,y}$: Slip force along x & y-axis; p: a generalized point on serration surface

Fig. 2: Schematic view of the [a] rubber pad & [b] R-cell module

An analytical model has been formulated towards evaluating the slip force, as and when an object is placed atop the slip sensory grid. In a way, the model is used to sense the external excitations on the sensory-grid, often operated remotely with an unknown loading. Based on the raw sensory signals from the R-cells, the model first evaluates tangential force on each of the taxels and thenafter, total tangential force or the *slip force* coming upon the grid. However, the *transformation of force*, e.g. external excitation (on the grid), is significant here and a correct *quantitative mapping* of *forcing-effect* is a pre-requisite for the model. Figure 3 presents this routing, wherein the basic tangential force impingement ($F_{T\text{-}Basic}$; *stage I*) is transmitted to the strut ($F_{T\text{-}Strut}$; *stage II*), which subsequently gets transformed into *induced* vibration force on the pin ($F^{V}_{T\text{-}Pin}$; *stage III*). Finally, we get oscillation of the pin inside the strut-hole, being arrested by the strain gauges ($F_{T\text{-}SG}$; *stage IV*).

As a direct consequence of the force transformation metric explained above, we shall now explore the actual displacement or the slippage of the object. Figure 4 illustrates the vectorial mapping of the (resultant) slip force (F_{TR}), so generated inside the taxels, and overall slippage of the object, atop the sensor. The measure of *planar slip*, thus occurred, is computed from the *slip triangle*, having components in (x,y). The *direction cosines* of F_{TR} are assumed to be largely uniform throughout the grid and numerically equal to 0.7072 (for $\theta = 45^0$).

Index: A: Serration; B: Strut; C: Pin; D: Strain Gauges; $Y_{i,j}$: Readings at the strain gauges $\delta\varphi$: Angle of swing of the pin.

Fig. 3: Schematic of the force transformation inside the sensor-cell

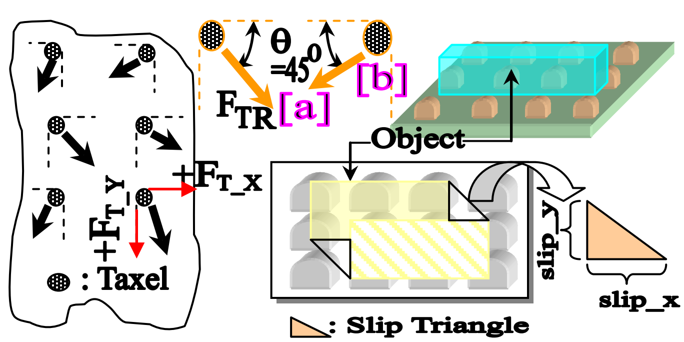


Fig. 4: Vectorial map of slip forces at the taxels causing slippage of the object

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It may be mentioned here that algorithmic data fusion for modular sensory elements (taxels) involves many intricacies, due to its assembly from constituent electro-mechanical members. In fact, quantifying the output response in such modular tactile cells becomes fuzzy, because of the force discretization issues therein. Although homogeneous, fusion metric in such tactile sensory grid turns even critical when we try with a finite number of taxels, as against traditional theories catering to somewhat denser agglomeration of sensor-cells. The fusion model to be adopted in this case must necessarily be dependent on the inter-taxel geometric parlances, which will be represented through the new rule-bases involving Interval Mathematics.

The foremost attribute of multi-sensory sensor data fusion for a practical application manifold in robotic manipulation is the time-sensitiveness of the raw / field data. As explained before, irrespective of the application-domains, i.e. field robotic sensory system or robotic gripper sensor, we will receive data in specific time-instants only. It is to be noted that in all such cases of sensory data fusion, "time" is an important attribute. Hence, all the pertinent variables in the data fusion process must be addressed with reference to *time-scale*. For example, if our sensory system is comprised of two independent input variables {X} & {Y}, which is running for a time-period of "T" $\subset$ (T_1, T_2,…..,T_N), a representative plot of the raw-data over the entire time-span can be pictorially presented, as shown in fig. 5.

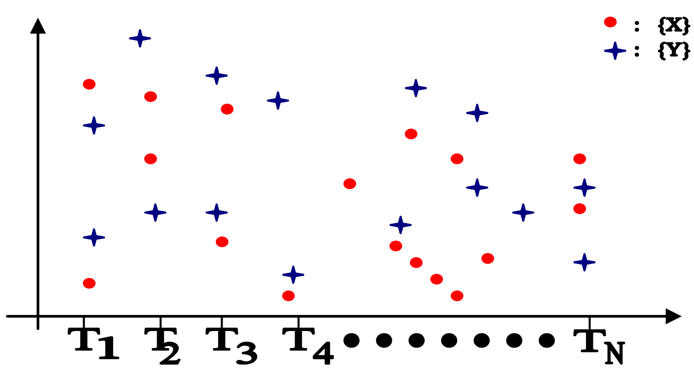

Fig. 5: Representative plot of time-varying raw data from two independent input variables

It may be noted here that the above representation is generic; as we can have multiple time-varying variables $\{K\}_T$, which follow the norm as stated below,

$$\{K\}_T^{\Phi} = \left[\{X\},\{Y\},.......\{M\},\{N\}\right]_{\{T=T_j\}}^{\Phi} \subseteq \left(\{K\}_{\min},\{K\}_{\max}\right) \quad (1)$$

Where, {K}: generalized time-varying variable pertaining to the robotic sensory system; T: generalized time-instant, i.e. $T_{1,2,\ldots,N}$; Φ: generalized sensor ensemble, i.e. rank of the sensor in the whole system under a multi-sensory system; {X}, {Y}.., {N}: time-varying system variables under a sensory system or a sensor-unit; T_j: specific value of a time-instant of interest / investigation; $\{K\}_{min}$ & $\{K\}_{max}$: minimum and maximum values of the generalized variable in real-time.

With this fundamental attribute defined, we need to also appreciate that in all practical situations of sensor data fusion in 'intervals', there must be *overlapping* of time-spans. In other words, $\{K\}_T^{\Phi}$ will be represented as a *data-cloud*, with sufficiently 'defined' boundaries, spanning between the 'minimum' and 'maximum' values. Researchers have debated about defining the near-perfect shape of such data-clouds, under time variability; but, till date there is no universally accepted graphical representation. As a matter of fact, the shape-manifold largely depends on the type of mathematics / analytical tools that have been employed to solve the fusion problem [7]. Nonetheless, it was agreed in principle by the research-community to use geometrically-exploitable shapes for near-perfect representation of the data-clouds, so that analytical formulation for interval-based algebra gets computationally simplified. Figure 6 schematically shows the representation of the data-cloud, for a single variable used in the robotic sensory system. As we can see from fig. 6, the easiest geometric mapping of data-cloud is *circle*, because of its defined analytics, namely, equation in 2D. We can approximate the gamut of raw data from the sensor(s), in the form of scatter diagram, by mere consideration of the 'centre' and 'radius' of the scatter diagram, so formed. This helps in analyzing the system, as the subsequent formulation for the operations (like multiplication & division; as well as integration) in interval mathematics become easier.

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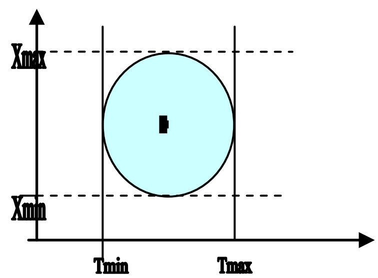
It is needless to say that if we approximate the data-cloud with 'rectangle' (as shown in dotted lines in fig. 6), instead of 'circle', computationally we will be in better position. However, we may include many undesired points, which will be out of the zone of scatter diagram, ideally. But, with 'circle' approximation, chances of including undesired points are minimum. Thus, 'circle' is the best approximation, so far as the simplicity of geometry is concerned. We can appreciate also that unless time-domain concept is introduced, the fusion / interval-multiplication/ interval-based integration will remain uni-axial.



Considering 'circle' approximation, let us now take the situation of fusing data-clouds from two variables, namely, {X} & {Y}, as depicted in fig. 6. In all practical situations of sensor data fusion in 'intervals', there must be *overlapping* of time-spans and, obviously, the fused outcome / data will also be characterized in the same time-scale. In other words, all calculations using interval algebra will be re-represented in time-scale / time-domain. It may be noted here that {X} and {Y} can be of same time-scale; in that case 'centre' will lie on the same data-axis. But, in any case, {X} & {Y} can't be totally disjointed either, as they have intrinsic relationship in terms of sensory performance. After plotting the raw data for both {X} & {Y} in real-time, we can build the respective 'circles' and note down the 'centre' location in 2D. Likewise, the process can be extended to more variables, say, adding {Z}, as shown in fig. 7. As seen from the figure, time-span for {X} is 'T_a', while the maximum possible time-span for the system is '[T_a +T_b]', subsuming the data-clouds of {Y} & {Z}.

Contrary to the 'circle' approximation, another option for representing data-cloud is to use the concept of "*B-rep*", which is a true '*convex hull*'. It is to be noted that minimum & maximum values of all variables (e.g. X_{min} & X_{max}) are achieved for the respective process-parameter, after repeated trials /experiments of the shop-floor data. However, the problem of using arbitrary shaped convex hull does have difficulty in computation.

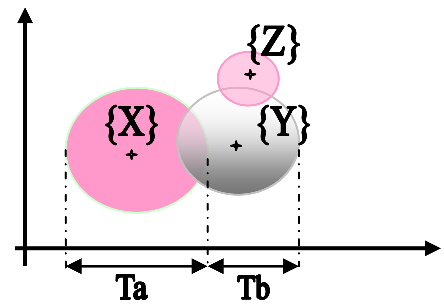


Fig. 7: Schematic of the overlapping data-clouds for three variables in real-time

Apart from time variance of the raw data, another important aspect of sensory data inside a data-cloud, which plays significant role in interval computation, is its '*dimensionality*'. A time-varying data, say, F(t), against the sensor-variable $\{K\}_T^{\Phi}$ can be in *clusters*, having data in different dimensions. Generally, a cluster has sufficiently large data-points, say 'n' (n>0); but, it can even have single data. We will adhere to a novel *model*, conceptualized indigenously, in order to take care of dimensionality aspect of raw data. As part of the model as well as with a generic representation, if a system has got 'β' data-clouds, and the cumulative data-points got an ensemble of several clusters, then we can view the following as an index-table, where 'DC': Data-cloud (1,2,…,β); 'D': Dimension of the output data (1,2,….,α); 'C': Cluster of data (refer Table 1).

Table 1: Generalized trade-off between data-cloud, cluster & dimension of data

	D_1	**D_2**	**D_3**	**D_4**	**……..**	**……..**	**D□**
DC 1	C_{11}	C_{12}	C_{13}	C_{14}	……..	…..	…..
DC 2	C_{21}	……	……..	……	……	……	$C_{2\alpha}$
….	……	……	……	…..	…….	…..	…….
DC β	$C_{\beta 1}$	$C_{\beta 2}$	$C_{\beta 3}$	…….	………	……..	$C_{\beta\alpha}$

As an elaboration of the concept put forward in Table 1, we can observe that raw output data from the sensor system can have several dimensions; and, as a generalization, we can say that a particular data-cloud possesses data with 'D_α', i.e.

'$\alpha^{th.}$ dimensional space. As a *lemma* of the mathematical function, we can state the following:

1^{st} order Dimensional Space (D-Sapce)

$$y = f(x_1) \quad \ldots\ldots\ldots \quad (2a)$$

$2^{nd.}$ order D-space

$$y = f(x_1, x_2) \quad \ldots\ldots\ldots \quad (2b)$$

$3^{rd.}$ Order D-space

$$y = f(x_1, x_2, x_3) \quad \ldots\ldots \quad (2c)$$

Likewise, □$^{th.}$ D-space

$$y = f(x_1, x_2, \ldots\ldots, x_{□})$$

□□□□□□□□□□□□ □□ d□□

where x_1, x_2, ……, $x_{□}$□□are all independent variables, as explained before□□

We may note that while defining various 'D-spaces', we are encountering three non-correlated facets, namely, a] *Number of Data-Centres* (DCs) participating in the sensory system; b] *Time-instants / Intervals* (t_1, t_2, …….., t_n) for the real-time run of the sensory system and c] *Dimension* of the raw data. In-line with the conceptual framework shown in table 1 "Cluster C_{11}" signifies all data-sets, generated out of "Data-cloud # 1" (DC_1), from t_1 to t_n (i.e. during entire time-span), having first-order dimension. In other words, the maximum possible clusters under a particular data-cloud, encompassing various 'dimensions' will be the summation of all the clusters, pertinent to that data-cloud. It may also be noted here that as per the proposed model, we can even make spatiation of data with respect to the dimensions only amongst various DCs. In that case, the clubbing of all $1^{st.}$ order D-space data under various DCs in the entire time-span will be summation of C_{11}, C_{21}, C_{31},……$C_{\beta 1}$ etc. Of course, we can deduce other statistics too by using the mathematical properties using the entries under table 1, but, all such evolutions may not have specific practical significance. Let us take an algebraic example to clarify the model.

Example: say, output of DC_1:

{ x_1^2; $2x_1+x_2$; $5x_1^3$; $4x_1^2+ x_2^2+x_3$; $3x_1$; ……………}

In this case, x_1^2 ➔ $1^{st.}$ order dimension at $t=t_1$; $(2x_1+x_2)$ ➔ $2^{nd.}$ order dimension at $t=t_2$; $5x_1^3$ ➔ $1^{st.}$ order dimension at $t=t_3$; $(4x_1^2+ x_2^2+x_3)$ ➔ $3^{rd.}$ order dimension at $t=t_4$ etc.

Similarly, output of DC_2 : *{*y_1^3*;* $2y_1+3y_2$*;* $4y_1^4$*;* $3y_1^2+4y_2+5y_3^3$*; ……..}* can also be analyzed at different time-instants for grouping the data under different dimensions.

It may be noted that there can be 'lone', i.e. only one / single data under one specific dimension. It is also not mandatory that we will have "clusters" all over the data-space.

Having defined the time variance as well as dimensionality of the output data from the data-clouds under the sensory system, we will now revisit to the standard formulae of interval algebra so far as the two prime-most mathematical operations are concerned, namely, '*Addition*' and '*Subtraction*'. Notwithstanding the criticalities of the dimensionality of the data, we can adopt the standard formulae of addition & subtraction to the fusion problem. However, we need to keep the fact in mind that the standard formulae of addition & subtraction in interval algebra do not account for *weighted average* of the data. In other words, if, for some reason, a particular data-set has got 'weights' or 'bias' associated with it, then traditional formulae of interval algebra will not be helpful. Hence, if discrete addition is allowed, then pure numerics can be joined together, while intervals can be combined separately. Thus, we can write the generalized lemma as,

$$[X, Y] \pm [P, Q] = [X \pm P, Y \pm Q] \ldots.. \quad (3a)$$

$$[X, Y] \pm \{Z\} = [X \pm Z, Y \pm Z] \ldots\ldots\ldots \quad (3b)$$

where [X,Y] &[P,Q] are two data-sets, in intervals while {Z} is a definite numeral. By using (3a) & (3b), we can have the following as a numerical example:

[2,7] + {8} + [4,9] = [6,16] +{8} = [14,24]. If successive addition is followed, then:

[2,7] + {3} + {5} + [4,9] = [5,10] +{5} +[4,9] = [10,15] +[4,9] = [14,24]

Let us now take the real-life case of augmentation of data in matrix form. If data-points / *taxels* are homogeneous, then the consolidated matrix [A] can be written as,

$$[A] = \{a_{ij} \,|\, t \,|\} \, [a_1 \;\; a_2 \;\; a_3 \;\; \ldots\ldots \; a_n] \qquad \ldots\ldots(4)$$

In the contrary, the matrix [Ã], constituted by heterogeneous or *unfused* data, can be written as:

$$[\tilde{A}] = \begin{bmatrix} a_{11} & a_{12} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & \ldots\ldots & a_{2n} \\ \ldots & \ldots & \ldots\ldots & \ldots \\ a_{m1} & a_{m2} & \ldots\ldots & a_{mn} \end{bmatrix} \qquad \ldots\ldots(5)$$

where the rows represent the data against a specific time-instant, e.g. $\{a_{11}, a_{12}, \ldots, a_{1n})$ corresponds to time-instant 't_1' etc.

Finally, the expanded form of the matrix [A♠], constituted by the homogeneous or fused data, will look like,

$$[A^\spadesuit] = \begin{bmatrix} t_1 \cdots a_{11} & \ldots FU & SED & DATA \\ t_2 \cdots a_{21} & a_{22} & a_{23} & \ldots FUSED \ldots DATA \\ t_3 \cdots a_{31} & \ldots FU & SED & DATA \\ t_4 \cdots FU & SED & DATA & \ldots\ldots \end{bmatrix} \qquad \ldots\ldots(6)$$

where the rows have a combination of 'fused data' (spanning through the columns as shown) and some 'un-fused data', e.g. $\{a_{11}, \ldots$FUSED DATA} corresponds to time-instant 't_1', signifying the data from first data-cloud (a_{11}) as *unfused* while the data from the rest data-clouds are *fused*.

The matrix-representations, vide (4), (5) & (6) amply highlight the scenarios of output data in three possible facets, viz. a] time-invariant homogeneous / fused data [refer (4)]; b] time-variant heterogeneous /unfused data [refer (5)] and c] time-variant homogeneous / fused data [refer (6)].For Division, using Interval Mathematics, we know: [X] / [Y] = [X]. [1/Y], which will be applied in general, except for integration. In jist, we need to take into account that in sensory data fusion, all calculations, using interval algebra, will be re-represented in time-scale / time-domain.

Lemma:1"fusion" or "non-fusion" can't be retraced back. That means, we can't have a_{21}, a_{22}, a_{23} and then $\{a_{fused}$! for i=4 to n} again a_{2p} (where p<n).

1. If it is of fused nature then it has to be continuous over the full gamut of data-cloud.
2. While number of rows will be governed by the 'time-instants', number of columns will be decided by the span of 'unfused data'.

Once done with addition & subtraction, we will adopt the new Multiplicative Law of Fusion (Successive Fusion), as detailed below:

$$\prod x_{ij} = x_{(i-1)j} \left[1 + \frac{\left| x_{ij} - x_{(i-1)j} \right|}{x_{ij} + x_{(i-1)j}} \right] \qquad \ldots(7)$$

Lemma:

1. Perform fusion for x_{21} & x_{11} first .
2. Then do the same for x_{31} & the fused (x_{21}, x_{11}).
3. Proceed likewise.

Additive Rule of fusion will be applicable for Data Clouds which generate approximate sequential data, both thematically & numerically. Multiplicative /Successive Rule of fusion will be applicable when various Data Clouds have implicit weightage. These weightages are proportionate, by & large, but can vary within a specified range.

Example:

If the data is deterministic; say x_{11}=3; x_{21}=5, then Π x_{21} = 3 [1+(2/8)] = 15/4 = 3.75. For x_{31} = 8 and Π x_{21} = 3.75 ; we get, Π x_{31} = 3.75 [1+ (8-3.75)/(8+3.75)]= 3.75 [1+(4.25/11.75)] = 5.11

Say $x_{(i-1)j}$ = [3,2] ; x_{ij} = [5,7]

Then Π x_{ij} = [3,2] . [1+ {[5,7] ~[3,2]}/([5,7] +[3,2])] = [3,2] . [1+([2,5])/([8,9])] = [3,2] . [1+ [2,5] . [1/9, 1/8]]

=[3,2] . [1+[min {2.(1/9), 2.(1/8), 5.(1/9), 5.(1/8)}, max {2/9, 2/8, 5/9, 5/8}] =[3,2] . [1+ [2/9, 5/8]]

= [3,2] . [11/9, 13/8] = [min {2.(11/9), 2. (13/8), 3. (11/9), 3. (13/8)}, max {…….}] = [22/9, 39/8]

In case of Interval Integrals, such as

$$\int_{[x_1]}^{[x_2]} [y] \; dx$$

We note that both limits and the function are expressed in Interval Mathematics. In other words, for a function e.g. [y] = f ([x]) = $[x]^2$ + 2 [x], we can arrive at three options, namely: a] only limits are expressed in interval; b] Function is expressed in interval but limits are definite and c] Function is in interval and also the limits, as expressed below:

$$\int_{[x_1]}^{[x_2]} [y] \; dx \equiv \left[\int_{x_1^{max}}^{x_2^{min}} [y_{min}] \; dx, \quad \int_{x_1^{min}}^{x_2^{max}} [y_{max}] \; dx \right] \quad \text{……..(8)}$$

Paradigm of data fusion for a robotic sensory system having *modular* sensor-cells embedded is still an *open-end* research problem. In a way, data fusion of the taxels, assumed homogeneous, working in a grid-type layout, becomes instrumental in evaluating the final quantitative output of the sensory system. The classical theory of optimal sensor signal processing is based on statistical estimation and hypothesis testing methods. The theory is based on 'Decentralized Testing & Augmentation' of sensory signals and thereby generating a kind of *unified signal* as output. This logically driven coherent output is used for processing of control system signals of the robotic sensory system. Unlike most of the decentralized control problems, hypothesis-testing paradigm can be solved in a relatively straightforward way. This is due principally to the fact that since the decisions made do not get looped back into the system dynamics, those do not affect the information of other decision makers either. However, even in the case of independent observations, several types of unusual behaviour can occur. For example, the threshold computations can yield locally optimal thresholds, drifted by globally optimal values.

The paradigm of decentralized sensor fusion has hitherto been attributed largely by Bayesian Theory, which deals quite robustly the situations involving probabilistic hypothesis testing but fails to address the cases where fuzziness is involved in the main process itself. On the contrary, Dempster-Shafer (D-S) Theory only tackles those problems where system caters for fuzzy concepts. Unfortunately both of the theories are inadequate so far as the data fusion in mechatronic system is concerned. To add to it, the problem gets even more critical in situations where the system is comprised of a finite/ limited number of elemental tactile sensor-cells, as deciphering the fused outcome is very subtle as the taxel-volume is less. Traditional theories (Bayesian or D-S) do not face this trouble, as quantification of outcome is easier there due to the presence of large number of identical sensor-cells. Moreover, we do not get any reprieve towards an authentic *grasp signature*, i.e. estimation of grip force and incipient slippage, from the traditional theories, except for the detection of object's presence /absence. Those apart, traditional theories are silent about the *weightage* of the individual taxel-outcome, considering the fact that outcome of one taxel may 'influence' the succeeding taxel(s) and/or get 'influenced' by the presence of other taxels in the proximity. In light of the above queries, the present paper proposes a new fusion theory, which will take care of the *relative dependency* of the finite-numbered taxel(s). Also, we advocate for a *dynamic threshold-band* in the model, that can be suitably adapted depending upon the end-application during fusion. Unlike dedicated threshold mark, here we will consider a distinct fuzzy-zone, equaling the width of the threshold-band, in order to compensate for the finite number of sensory signals involved in the fusion process. This fuzzy-zone will, no doubt, generate an *in-decision* regarding the object's presence /absence, but, can be tackled by our model. The threshold estimation for the present work has been based on using the *variable limits*, exploiting the metrics of Type I error (i.e. rejecting the Alternative Hypothesis when true). The present work is thus concentrated on three vital aspects, viz. the model should be able to i] *cater limited number of homogeneous* but *modular sensor-cells*, ii] *sense the presence of tiny 'point-objects'* on the surface of the sensory grid, iii] *grade the sensor-cells by their relative dependencies*, following *successive recursion (SR)* pattern and iv] evaluate the *zonal influence metric (ZIM)* along the plane of the slip, over the sensory grid surface. It may be mentioned that all these paradigms were overlooked in the researches hitherto and thus, the existing fusion cum hypothesis testing models are unsuitable to real-life applications in robotics. In the contrary, our model of data fusion and statistical hypothesis testing with new threshold-zone thematic will ensure reliable measure towards overall *qunatization* of the object(s) (e.g. its presence, size & contour) avilable in the vicinity of the sensor.

The novel fusion rule-base, namely, *Zonal Influence Metric Successive Recursion (ZIMSR)* has been formulated in order to reveal the inter-cell relationship of the matrix layout of the slip sensory grid. In other words, this rule-base has been devised to represent the exact way the taxels are 'reacting' with the succeeding taxel and in what way these taxels are 'influencing' the neighbouring taxels and/or getting influenced by those. The model takes care of this relative influence, under zonal attributes, which is christened as *zonal influence metric* (ZIM). Figure 8 schematically explains the taxel-level interaction and formation of *micro-slip* between the succeeding taxels inside the sensory grid thereby. Taxel-level progression of slip inside the sensory grid (refer fig. 8a) is manifested by the omni-directional micro-slip (δs) produced inside the ZIM (Ω), as a result of inter-taxel interaction for the first *sector* (i.e. for u_1 & u_2), in general, for the ij$^{th.}$ sector (refer fig. 8b). The territory of taxel-influence is limited by the segmented partition-line, 'L_{ij}'.

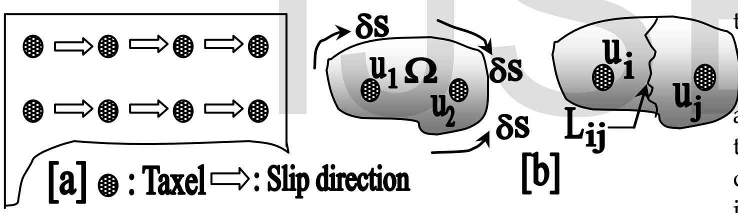


Fig. 8: Schematic of (a) slip progress in grid & (b) inter-taxel slip formation

After processing for taxel-wise outcome paradigm, we will finally have a logical *unified* output from the system controller using the rule-base, considering the set of 'u_i' as $\{u_i\}$=TFN[a,b,c]. The model culminates in a non-zero value of U_G for $\{u_i\}$=TFN[a,b,c] and the evaluation is *unbiased* so far as the object-size is concerned.

The Successive Recursion model defines 'U_G' as,

$$U_G = \prod_{i=1,\, j=i+1}^{i=M,\, j=M-1} \left[(x)u_i^{\,p} + (y)(1+u_j)^q \right] \quad \ldots\ldots\ldots\ldots\ldots\ (9)$$

where, $\{u_i\}$: localized decision for the i$^{th.}$ taxel, $\forall i$ =1,2,….M; 'M': total number of taxels activated in the slip sensory grid; x,y: positional attributes for the i$^{th.}$ and j$^{th.}$ taxels respectively, where numerically, (x)=i/M & (y)=j/M; p: relative weightage of the i$^{th.}$ taxel and q: relative weightage of the succeeding taxel, i.e. (i+1)$^{th.}$ cell, where $0 \le p,q \le 2$. We also assume that in eqn. 9, $\{u_{i+1}\}_{|i=M} = 0$. Our model doesn't consider backtracking of taxels, i.e. it only considers taxel(s) that are *ahead* of the specific taxel. The numeral '1' in eqn 9 has been added deliberately over 'u_j' to show the relative importance of the succeeding taxel during slip formation. Likewise, the conjugate parameter (x,y) has been augmented based on the logic that slip computed for a specific (i-j) segment of the grid is over and above what has occurred already in all preceding (i-j) segments. In other words, say for (i=3,j=4) segment, U_G considers all the previous *slip-segments* by default, viz. that for (i=2,j=3) & (i=1,j=2) segments. Also, micro-slip so produced at (i=3,j=4) segment is numerically larger than that at say (i=2,j=3) or (i=1,j=2) segment. Mathematically, we can write,

$$\delta s\,|_{\,i=k,j=k+1} > \{\delta s\,|_{\,i=k-p,j=k-p+1}\},\ \forall p=1,2,3\ldots.,(k-1)$$

It may be noted that this model is essentially specific to taxel location (i.e. zone), wherein cumulative effect of the taxels are reflected only. In contrast, the *relative dependency* of one taxel over the preceeding ones is getting priority in the present model. Hence, we have christened this model as *'successive recursion'*, as the effect of adjoining taxels can be taken into consideration towards computing global fused data (U_G), depending upon their relative influence / importance, pronounced under ZIM. Further, this model, by definition, is best suited for taxels arranged in a row or column-wise fashion, i.e. applied for *row /column matrix*. Also, taxels may or may not be equally likely; nonetheless, we are unsure about the outcome of a specific taxel in the *grid*. It may be mentioned additionally here that the effect of relative dependency of the taxels could also be considered by another *sister-model* of U_G, viz. $U_G = \Pi[\, x u_i^{\,p} - y(1+ u_{i+1})^q]$, but it would have been rather difficult to be interpreted graphically.

Now, the *fused* decision regarding the selection of test hypothesis will be ruled by the evaluation paradigm, decided a-priori. In our model, we use the *dynamic threshold band* and the numerical value of the *mean threshold* ($\lambda_{\text{Threshold-mean}}$) as the evaluation metric. We define the evaluation metric as: if $U_G \ge \lambda_{\text{Th-mean}}$, then accept H_1, otherwise reject H_1. But, alongwith discrete acceptance or rejection, we will also encounter one *fuzzy-zone*, signifying *in-decision* regarding the acceptance or rejection of H_1. Numerically, this in-decision zone will be directly proportional to the width of the threshold-band.

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