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Model Equation for Heat Transfer Coefficient of Air in a Batch Dryer

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Abstract— Heat transfer coefficients of dryers are useful tools for correlation formulation and performance evaluation of process design of dryers as well as derivation of analytical model for predicting drying rates. A model equation for predicting heat transfer coefficient of air in a batch dryer using BuckingHam Pi-theorem and dimensional analysis at various air velocities has been formulated. The model was validated by drying unripe plantain chips in a batch dryer at air velocities between 0.66 and 1.20m/s at corresponding temperatures between 42 and 66oC. Based on the analogy of heat and mass transfer rate equations for constant drying period, the prediction from the developed model agreed reasonably with the experimental data.

Index Terms—batch dryer, Buckinham pi-theorem, drying rate, heat transfer coefficient, model equation

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1 Introduction

Drying is a kinetic process that involves the removal of liquid, usually water from a moist material: solid, liquid or gas. The use of heat to remove liquid distinguishes drying from mechanical methods of removing water, such as: centrifugation, decantation, sedimentation and filtration in which no change in phase from liquid to vapour is experienced [1],[2].

The application of heat to remove moisture is widely used in the food industry to reduce moisture contents to levels considered safe for storage in order to prolong the life span of the food item [3],[4],[5],[6],[7],[8]. Also high moisture contents of moist food and agricultural materials constitute additional cost in bulk handling and transportation and must be removed in a manner that guarantees product quality [9],[10]. In the drying of solids to remove water a specialized device called dryer is used, and the desirable end products are in solid form. The final moisture contents of the dried solids are usually less than1%. The chemistry of drying a moist material can be represented as:

$$\text{Moist material} + \text{Heat} \longrightarrow \text{solid} + \text{vapour} \qquad (1)$$

When heat transfer by pure convection is used to dry a wet solid, the heat supplied is solely by sensible heat in the drying gas stream. A dynamic equilibrium exist between the rate of heat transfer to the material and the rate of vapour (mass) removal from the surface at instance, (that is, drying rate) and may be represented as follows:

$$\frac{dx}{dt} = hA^{|}\frac{\Delta T}{\lambda} \qquad (2)$$

The area of the heat and mass transfer may be assumed to be approximately equal [11].

The study of convective heat transfer is centered on ways and means of determining the heat transfer coefficient, h for various flow regions (laminar, transition or turbulent flow) and over various geometries and configurations. The local and average heat transfer coefficient may be correlated by (3) and (4) respectively:

$$Nu_x = f\,(x^*, Re_x, P_r) \qquad (3)$$

$$Nu_x = f\,(Re_x, P_r) \qquad (4)$$

where the subscript x emphasize the condition at a particular location on the surface.

The problem of convection involves how these functions are obtained, there are two approaches: theoretical and experimental. Theoretical approach involves solving the boundary layer equation for a particular geometry and equation such as (5)

$$Nu = \frac{hL}{K} = \pm\frac{\partial T^*}{\partial y^*}\Big|_{y^*=0} \qquad (5)$$

which is a dimensionless temperature gradient at the surface.

In the experimental approach, for a prescribed geometry in a parallel flow, if heated, convection heat transfer coefficient which is an average associated with the entire system could then be computed from Newton’s law of cooling. And from the knowledge of the characteristic length and the fluid properties, the Nusselt, Reynolds and Prandtl numbers could be computed from their definitions.

Meanwhile, the relevant dimensionless parameters for low-speed, forced convection boundary layer have been obtained by non-dimensionalizing the differential equation that describes the physical process occurring within the boundary layer. An alternative approach is the use of dimensional analysis in the form of Buckingham Pi theorem. The success of the theorem depends on the ability to select from intuition the various parameters that influ-

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ence the problem. Therefore, knowing before hand that

$$h = f(K, C_P, \rho, \mu, V, L) \quad (6)$$

One could use the Buckingham Pi theorem to obtain h, in (6) [12].

2 MODELLING AND EXPERIMENTAL VALIDATION

2.1 Model Formulation

Etebu and Josiah [13] suggested that in order to successfully create non-dimensional groups, each time a need arises, a set of rules must be followed; the Raleigh method and Buckingham's Pi-Theorems are reliable.

The Raleigh method is an elementary technique for finding a functional relationship between variables. Although very simple, the method does not provide any information concerning the number of dimensionless group that can be obtained. Another drawback of the method is that it can only be used for the determination of the expression for variables that depend on a maximum of three or four independent variables.

The BuckingHam's pi-theorem is an improvement over the Raleigh's method. Apart from its advantage of being able to handle large sets of variables, it gives a ready clue on how many dimensionless groups are designated by Pi.

In the determination of heat transfer coefficient therefore, it is necessary to note that

$$f\alpha(h, K_V, D, Re, Pr) = 0 \quad (7)$$

Where,

$$Re = \frac{\rho VD}{\mu} \quad (8)$$

$$Pr = \frac{\mu C_P}{K_V} \quad (9)$$

Therefore

$$f\{\alpha(h, K_V, D, \rho, V, \mu, C_p)\} = 0 \quad (10)$$

where α is a constant.

Choosing M, L, T, and K as fundamental dimension for mass, length, time and temperature, implies that in (10), the number of fundamental dimension, m, is 4, while the number of quantities, n = 7 as shown in Table 1

Therefore number of π groups is, $N\boldsymbol{\pi} = 7 - 4 = 3$

Since m is 4, there will be four repeating quantities: Geometric property (D). Flow property (V), fluid property (ρ) and heat property (C_P). A $\boldsymbol{\pi}$ group is a function of the repeating variables and one of the remaining variables. Thus the 3π-terms as functions of the repeating variables are as follows:

π-terms

$$\pi_1 = D^{w_1}V^{x_1}\rho^{y_1}C_P^{Z_1}K_v \quad (11)$$

$$\pi_2 = D^{w_2}V^{x_2}\rho^{y_2}C_P^{Z_2}\mu \quad (12)$$

$$\pi_3 = D^{w_3}V^{x_3}\rho^{y_3}C_P^{Z_3}h \quad (13)$$

TABLE 1
DIMENSION OF QUANTITIES

S	Quantity	Sym	Unit	Dimension
1	Diameter	D	M	L
2	Density	ϱ	Kgm^{-3}	ML^{-3}
3	Velocity	V	m/s	LT^{-1}
4	Viscosity (absolute)	μ	$kgm^{-1}s^{-1}$	$ML^{-1}T^{-1}$
5	Thermal Conductivity	K_V	$kgms^{3}K^{-1}$	$MLT^{-3}K^{-1}$
6	Heat Capacity	C_P	$m^2s^{-2}K^{-1}$	$L^2T^{-2}K$
7	Heat Transfer Coefficient	H	$kgs^{-3}K^{-1}$	$MT^{-3}K^{-1}$

Setting up and solving equations for π groups, using the dimensions in Table 1, we obtain

$$\pi_1 = D^{-1}V^{-1}\rho^{-1}C_P{}^{-1}K \quad (14)$$

$$\text{Or } \pi_1 = \frac{K_v}{\rho VDC_P} \quad (15)$$

Using similar procedures for (12) and (13), we obtain values for π_2 and π_3 as in (16) and (17) respectively

$$\pi_2 = \frac{\mu}{\rho VD} \quad (16)$$

$$\pi_3 = \frac{h}{\rho VC_P} \quad (17)$$

Combining (15), (16) and (17), we have

$$f\left\{\alpha\left(\frac{k_v}{\rho VDC_P}, \frac{\mu}{\rho VD}, \frac{h}{\rho VC_P}\right)\right\} = 0 \quad (18)$$

Hence

$$\frac{h}{\rho VC_P} = \alpha\left(\frac{\mu}{\rho VD}, \frac{k_v}{\rho VDC_P}\right) \quad (19)$$

From (15), (16) and (17)

$$\pi_3 = \alpha(\pi_2, \pi_1) \quad (20)$$

Substituting the numerical values of the quantities in (19) from Table 3, the numerical values of π_1, π_2 and π_3 are obtained as shown in Table 4.

Plotting the (π_3) against (π_1), (π_2), in (20), we obtain graphs which when regressed yields the theoretical heat transfer coefficient (h_B).

The regression gives the equations as:

$$\pi_3 = \alpha\{6.37(\pi_1) + 0.029\} \quad (21)$$

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$$\pi_3 = \alpha\{68.88(\pi_2) + 0.030\} \quad (22)$$

Substituting for π values from (15), (16) and (17) we obtain expressions for h_{B1} and h_{B2} as:

$$h_{B1} = \alpha\{6.37\frac{k_v}{D} + 0.029RePrk_v/D\} \quad (23)$$

$$h_{B2} = \alpha\{68.88\frac{Prk_v}{D} + 0.030RePrk_v/D\} \quad (24)$$

$$\sum h_B = h_{B1} + h_{B2}$$

$$\sum h_B = \left\{6.37\frac{k_v}{D} + \frac{0.029RePrk_v}{D} + 68.88\frac{Prk_v}{D} + 0.030RePr\frac{k_v}{D}\right\}$$

$$\sum h_B = \alpha\frac{k_v}{D}\{6.37 + Pr(68.88 + 0.059Re)\} \quad (25)$$

The values of α can be obtained as follows using the thermo-physical properties of air from Table 3.
To calculate the heat transfer coefficients at various air flow velocities using Dittus –Boelter equation [14] ($h_D = \frac{K_v}{D}0.023Re_D^{0.8}Pr^{0.4}$):

1) For V_1 = 0.66m/s
$25.5297 = \alpha_1 691.0651$, $\alpha_1 = 0.03694$
2) For V_3 = 0.92m/s
$34.4787 = \alpha_3 970.6760$, $\alpha_3 = 0.03552$
3) For V_5 = 1.20m/s
$44.6411 = \alpha_5\ 1312.1077$, $\alpha_5 = 0.03402$

Hence the average values of α is;

$$\alpha = \frac{\alpha_1 + \alpha_3 + \alpha_5}{3} = 0.03550$$

Substituting **α** into (24), we have the model equation for calculating the coefficient of heat transfer as:

$$\sum h_B = 0.03550\frac{K_v}{D}\{6.37 + Pr(68.88 + 0.059Re)\}$$

2.20

For V_1 = 0.66m/s, $h_{B1} = 24.5295 Wm^{-2}K^{-1}$
For V_3 = 0.92m/s, $h_{B3} = 34.4543 Wm^{-2}K^{-1}$
For V_5 = 1.20m/s, $h_{B5} = 45.5735 Wm^{-2}K^{-1}$

Hence the modeled mean heat transfer coefficient is:

$$h_B = (h_{B1} + h_{B3} + h_{B5})/3 \quad (26)$$

Therefore,

$$h_B = 34.8524 Wm^{-2}K^{-1}$$

2.2.1 Experimental Materials and Method

Unripe plantains chips were used in the experiment. The moisture content of the plantain before drying was 26.76g dry base. The moisture content was determined by periodically weighing of the sample at 3 minutes interval, for three hours to generate 61 data points.

In evaporative heating based on heat and mass transfer analogy, as the gas flow over the moist material evaporation occurs from the surface, and the energy associated with the phase change is the latent heat of vaporization of the liquid.

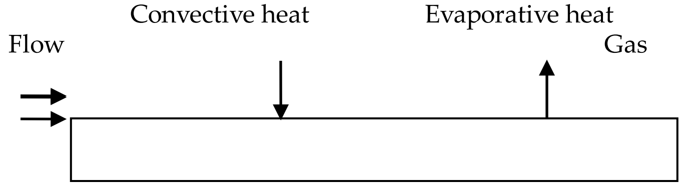

Fig 1: Heat and Mass Transfer Analogy

Applying conservation of energy to a control surface about the material, we have

$$q''_{convective} + q''_{added} = q''_{evaporation} \quad (27)$$

Since no heat is added, and for constant drying

$$q''_{convective} = q''_{evaporation} \quad (28)$$

Where $q''_{evaporation}$ may be approximated as the product of the mass (moisture loss, dw) and the latent heat of vaporization.

$$q''_{evaporation} = n''_A h_{fg} = \frac{dw}{dt}\lambda \quad (29)$$

$$q''_{convective} = hA(T_{hf} - T_S) \quad (30)$$

Therefore

$$hA(T_{hf} - T_S) = hA(\Delta T) = \frac{dw}{dt}\lambda \quad (31)$$

Where $\frac{dw}{dt}\lambda$ represents the constant drying rate
For surface temperature, T_s = 28⁰C
and the heating fluid is at T_5, $T_{hf} = T_5$

$$T_5 - T_s = \Delta T_5 \quad (32)$$

Therefore $(h_E A\Delta T)_5 = \lambda\frac{dw}{dt}_{(5)}$ (33)

$$h_{E(5)} = \left(\frac{\lambda\, dw}{A(\Delta T)dt}\right)_5 \quad (34)$$

When heating fluid is at T_3, $T_{hf} = T_3$

$$h_{E(3)} = \left(\frac{\lambda\, dw}{A(\Delta T)dt}\right)_3 \quad (35)$$

When heating fluid is at T_1, $T_{hf} = T_1$

$$h_{E(1)} = \left(\frac{\lambda\, dw}{A(\Delta T)dt}\right)_1 \quad (36)$$

Total Area of Plantain Chips $A = \left(\frac{\pi D^2}{4}\right)$ (37)

D = diameter of Plantain Chips = 0.033m
For the 6 pieces of plantain = 0.198m

Hence $A = \left(\frac{\pi\, 0.198^2}{4}\right) = 0.030795m^2$

From (34), (35) and (36), the mean experimental heat transfer coefficient

$$h_E = \frac{\left\{\frac{\lambda}{A}\left(\frac{dw}{(\Delta T)dt}\right)_1 + \left(\frac{dw}{(\Delta T)dt}\right)_3 + \left(\frac{dw}{(\Delta T)dt}\right)_5\right\}}{3} \quad (38)$$

3 RESULTS AND DISCUSSION

The results obtained from this work are presented in Table 2-4 and Figure 2-3.

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Where V and T are the average values. Substituting the thermo-physical properties and V_{Bcal} based on the calibration, into the Dittus-Boelter equation ($h_D = \frac{K_V}{D} 0.023 Re_D^{0.8} Pr^{0.4}$) we obtain values of heat transfer coefficient shown in Table 3.

TABLE 2
CALIBRATION OF THE BATCH DRYER

S/NO	Velocity (m/s)				Temperature (°C)		
	V_1	V_2	V_A	V_{Bcal}	T1	T_2	T
1	3.60	3.40	3.50	0.66	66.00	66.00	66,00
2	4.20	4.20	4.20	0.79	60.15	60.05	60.10
3	5.00	4.80	4.90	0.92	54.50	54.40	54.45
4	5.65	5.75	5.70	1.07	48.30	48.20	48.25
5	6.40	6.40	6.40	1.20	42.00	42.00	42.00

TABLE 3
THERMO-PHYSICAL PROPERTIES AND CALCULATED VALUES OF HEAT TRANSFER COEFFICIENT USING DITTUS-BOELTER EQUATION

S/NO	1	2	3	4	5
Temperature(°c)	66.00	60.10	54.45	48.25	42.00
Velocity(m/s)	0.66	0.79	0.92	1.07	1.20
Density(kg/m^3)	1.0383	1.0516	1.0646	1.0749	1.0982
Specific Heat Capacity(m^{2}s^{-2}K^{-1})	1008.5	1008.3	1008.2	1008.0	1007.8
Dynamic Viscosity(kgm^{-1}s^{-1})	202.1E-7	200.18E-7	198.4E-7	196.9E-7	193.6E-7
Reynolds Number (-)	10070.64	12325.75	14661.86	17348.53	20216.86
Prandtl Number (-)	0.702	0.7022	0.7028	0.703	0.705
Heat Transfer Coefficient(Wm^{-2}K^{-1})	25.5297	30.0116	34.4787	39.4635	44.6411

3.1. MODELLED RESULT

The values of the variables of Table 3 are used to obtain the values of π_1, π_2, π_3, as shown in Table 4.

TABLE 4
π- GROUPS

S/No	1	2	3	4	5
π_1	0.001121	0.0009245	0.0007842	0.0006681	0.0005831
π_2	0.0000993	0.0000811	0.0000696	0.0000572	0.0000495
π_3	0.03696	0.03583	0.03492	0.03402	0.03361

Plotting the values of π_3 versus π_1 and π_3 versus π_2 we obtain Figures 2 and 3 below.

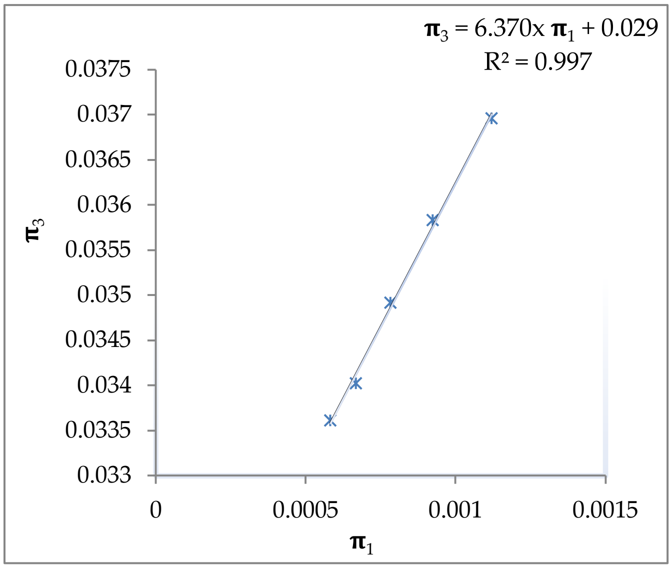

Fig 2. Graph of π_3 Versus π_1

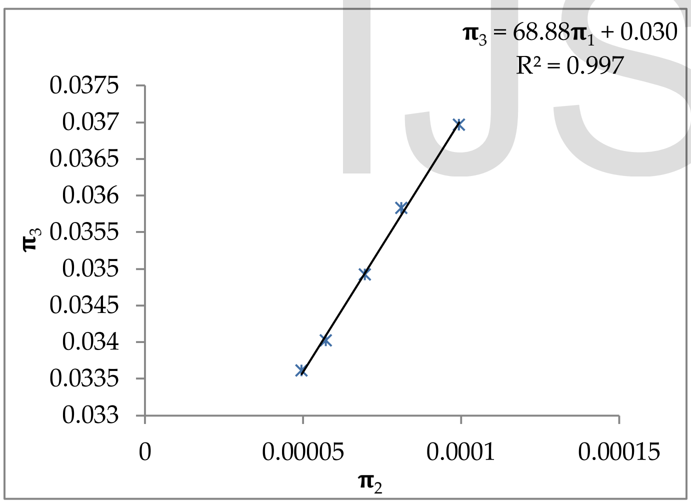

Fig 3. Graph of π_3 Versus π_2

Using various air flow velocities, experiments were peerformed in order to obtain the gradient for drying the unripe plantain. The experimental results are plotted as shown in Figures 4 to 6.

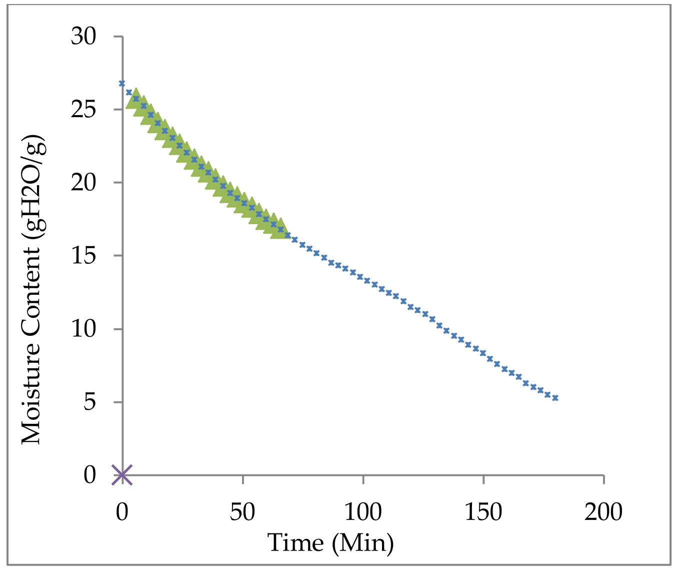

Fig. 4. Moisture Content Versus Time (at V_1)

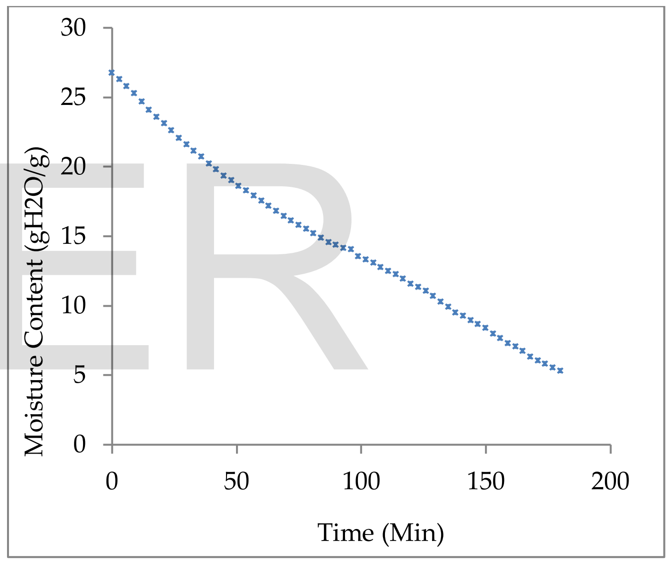

Fig. 5. Moisture Content Versus Time (at V_3)

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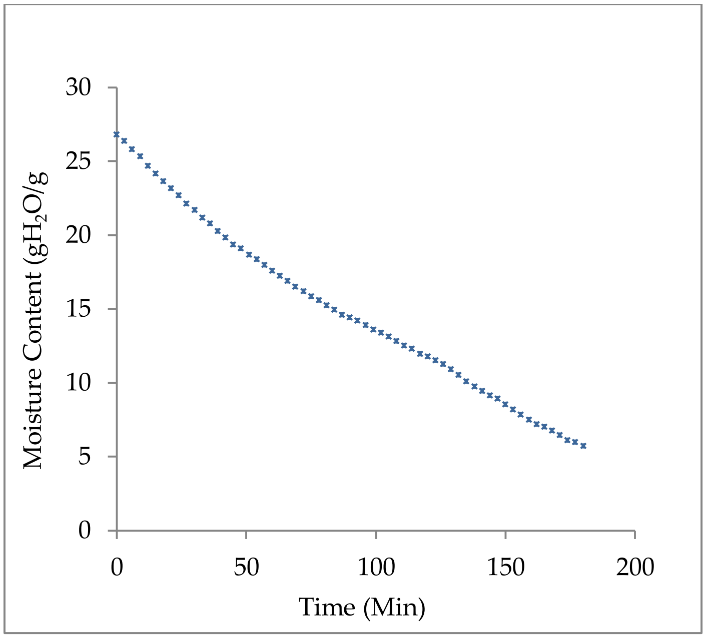

Fig. 6. Moisture Content Versus Time (at V5)

The drying rate (Evaporative Rate) which is a product of evaporative flux and latent heat of vaporization obtained from figure 4, 5 and 6, and the change in temperature base on V_1, V_3 and V_5 were used to obtain the experimental heat transfer coefficient as illustrated below: $T_{hf(1)} = 42.00^0C$, hence $\Delta T = 14.00^0C$

$T_{hf(3)} = 54.45^0C$, hence $\Delta T = 26.45^0C$

$T_{hf(5)} = 66.00^0C$, hence $\Delta T = 38.00^0C$

λ = Latent Heat of Vaporisation of water = 2501KJ/Kg (Pakorn et al, 2006)

From the plots of Figures 4, 5 and 6 the drying rates at constant rate periods are

$\frac{dw}{dt}_{(1)} = -0.176 g/min = 0.00000285 Kg/s$

$\frac{dw}{dt}_{(3)} = -0.173 g/min = 0.00000288 Kg/s$

$\frac{dw}{dt}_{(5)} = -0.171 g/min = 0.00000293\ Kg/s$

Hence the experimental heat transfer coefficient

$$h_E = \frac{2501000}{3 \times 0.030795}\left[\frac{0.00000285}{14.00} + \frac{0.00000288}{26.45} + \frac{0.00000293}{38.00}\right] \quad (39)$$

$h_E = 31.6379\ Wm^{-2}K^{-1}$

Model Heat Transfer Coefficient,
$(h_B) = 34.8524 Wm^{-2}K^{-1}$
Experimental Heat Transfer Coefficient,
$(h_E) = 31.6379 Wm^{-2}K^{-1}$
% error = $\frac{34.8524 - 31.6379}{34.8524} \times 100\% = 9.22\%$

Therefore a good agreement of about 91% was achieved.
As demonstrated in this document, the numbering for sections upper case Arabic numerals, then upper case Arabic numerals, separated by periods. Initial paragraphs after the section title are not indented. Only the initial, introductory paragraph has a drop cap.

3.2 DISCUSSION OF RESULTS

The values of the velocities and temperatures from the calibration and values from the thermo-physical table obtained from literature were substituted into the Dittus-Boelter equation to obtain the relationship between heat transfer coefficients (dependent variables) and velocities (independent variables), which form the basis for the use of BuckingHam's Pi-Theorem.

The BuckingHam's Pi-Theorem, which uses dimensional analysis, was then used to obtain the Pi groups (π_1, π_2, π_3). Based on the Pi groups obtained, plots π_3 versus π_1 and π_3 versus π_2 were obtained and regressed, using Microsoft Excel to obtain the modeled equations

The experiment when carried out at velocities of 0.66, 0.92 and 1.20 ms^{-1}, it was observed that the moisture in unripe plantain evaporated faster to approach dryness in the order; 0.66< 0.92< 1.2ms^{-1}. This is due to the fact that residence time of the hot air increases at a lower velocity than at a higher velocity.

From the plots of Figure 2 and 3, the equations at the constant drying periods at various velocities were also obtained.

The relative humidity of the drying environment was relatively constant throughout the experiment, since the drying is done in an enclosed system (Batch dryer) and at an average relative humidity of 75±5% of the laboratory.

Finally, a comparison of heat transfer coefficients obtained from the theoretical Buckingham Pi-Theorem (model) and that obtained from the experimental result, in the range of velocities illustrated showed minimal variation of less than 10%.

4 CONCLUSION

The heat and mass transfer analogy from Newton law of cooling has been shown to be a reliable correlation for obtaining heat transfer coefficient experimentally; also proven is the fact that the BuckingHam Pi-Theorem is a simplified and good method of obtaining correlation from experimental results. The comparison of both experimental and modeled heat transfer coefficients shows a percentage error of 9.22% which is within acceptable level.

NOMENCLATURE

D	Diameter (m)
h	Heat Transfer Coefficient ($kgs^{-3}K^{-3}$)
n	Constant (-)
Nu	Nusselt Number (-)
x	Distance across the plate (m)
Pr	Prandtl Number (-)
Re	Reynolds Number (-)
ΔT	Change in Temperature (^oc)
w	Moisture Content (gg^{-1})
A_s	Total Surface Area of Plantain Chips (m^2)

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C_p Specific Heat Capacity ($m^2s^{-2}K^{-1}$)
d_1, d_2 Diameters of Dryer (m)
λ Heat of Vaporization ($KJkg^{-1}$)
ρ Density (kgm^{-3})
μ Dynamic Viscosity ($kgm^{-1}s^{-1}$)
D_L Rate of Diffusion
G_r Grashof Number
h The heat transfer coefficient ($Wm^{-2}K^{-1}$)
h_B Modeled Heat Transfer Coefficient ($Wm^{-2}K^{-1}$)
h_E Experimental Heat Transfer Coefficient ($Wm^{-2}K^{-1}$
k_v Thermal Conductivity of Plantain ($kgm^{-3}K^{-1}$)
T_{hf} Temperature of Hot Fluid (^oc)
T_s Surface Temperature (^oc)
V_A , V_B Velocities of Air (ms^{-1})
π_1, π_2, π_3 Pi-Groups (-)

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