Mixed Convective MHD Flow of Visco-elastic Fluid Past A Vertical Infinite Plate With Mass Transfer

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Abstract-A theoretical analysis of mixed convective unsteady flow of a visco-elastic incompressible fluid past an accelerated infinite vertical porous plate subjected to a uniform suction has been investigated under the influence of a uniform transverse magnetic field. Approximate solutions for fluid velocity, temperature, concentration field and skin friction have been obtained by using perturbation technique. The effects of the various parameters involved in the solution have been studied. The profiles of fluid velocity and the skin friction are presented graphically to observe effects of the visco-elastic parameter.

Keywords: Heat transfer, mass transfer, MHD Flow, mixed convection, suction, visco-elastic fluid.

1. INTRODUCTION

The subject of free convective and heat transfer flows through a porous medium under the influence of a magnetic field has attracted the attention of a number of researchers because of their possible applications in transportation cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vaporization in combustion chambers. In mass transfer process, heat transfer considerations arise due to chemical reaction and often due to the very nature of the process. Magnetohydrodynamics is currently undergoing a period of great enlargement and differentiation of subject matter. The interests in these new problems generates from their importance in liquid metals, electrolytes and ionized gases, fossil fuel, combustion, energy process, solar energy and space vehicle re-entry, control of pollutant spread in ground water, to name just a few applications.

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The viscous force imparted by a flowing fluid in a dense swarm of particles has been investigated by Brinkman [1]. Hasimoto [2] had studied the boundary layer growth on a flat plate with suction or injection. Berman [3] has discussed the two-dimensional steady-state flow in a channel having a rectangular cross section and two equally porous walls. Sellars [4] has extended the work of Berman for high section Reynolds number. The flow between two vertical plates under the assumption that the wall temperature varies linearly in the direction of flow in presence of a transverse magnetic field has been investigated by Mori [5]. The problem of fluid motion in renal tubules which is complicated by the existence of radial velocities generated by re-absorption process has been investigated by Macey [6]. England and Emery [7] have studied the effects of thermal radiation upon laminar free convection boundary layer of a vertical plate for absorbing and non-absorbing gases. Soundalgekar and Thakar [8] have examined the radiation effects on free convection flow of an optically thin gray gas past a semi-infinite vertical plate. Das et.al [9] have discussed the radiation effects on flow past an impulsively started vertical infinite plate. The steady flow of a non-Newtonian fluid past a porous plate with suction has been examined by Mansutti et.al [10]. Sattar [11] has investigated the free convection and mass transfer flow past an infinite vertical porous plate with time dependent temperature and concentration. Choudhury and Das [12] have investigated the MHD boundary layer flow of a non-Newtonian fluid past a flat plate. The mass transfer effects on unsteady flow past an accelerated vertical porous plate have been discussed by Das et.al [13]. Reddy et.al [14] have investigated the unsteady mixed convective flow with mass transfer past an accelerated infinite vertical porous flat plate with suction in presence of transverse magnetic field.

The objective of this study is to extend the work of Reddy et.al with visco-elastic flow characterized by second-order fluid whose constitutive equation is given by [Coleman and Noll (1960)]

$$\sigma = -pI + \mu_1 A_1 + \mu_2 A_2 + \mu_3 (A_1)^2 \qquad (1.1)$$

where σ is the stress tensor, p is the isotropic mean pressure, μ_1, μ_2, μ_3 are material constants describing viscosity, elasticity and cross-viscosity respectively and A_i are kinematic Rivlen-Ericson tensors defined as

$$A_{(1)ij} = v_{i,j} + v_{j,i}$$
$$A_{(2)ij} = a_{i,j} + v_{j,i} + 2v_{m,i}v^m_{,i}$$

where $a_i = \frac{\partial v_i}{\partial t} + v_j v_{i,j}$

Here v_i and a_i are respectively the components of velocity and acceleration in x^i direction. Also $\mu_2 < 0$ from thermodynamic consideration [Coleman and Markivitz (1964)].

2. MATHEMATICAL FORMULATION

Let us consider the unsteady mixed convective mass transfer flow of a second order fluid past an accelerating vertical infinite porous plate in the presence of a transverse magnetic field B0. The x-axis is taken along the plate in the vertically upward direction and y-axis is taken normal to the plate. Here u and v are the components of the velocity in the x and y directions respectively. It is also assumed that the plate is accelerating with a velocity u=U0 in its own plane for t≥0. The equations governing the flow under Boussinesq's approximation are

$$\frac{\partial v}{\partial y} = 0 \Rightarrow v = -V_0 \qquad (2.1)$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu_1 \frac{\partial^2 u}{\partial y^2} + \nu_2\{\frac{\partial^2}{\partial y^2}(\frac{\partial u}{\partial t}) + v\frac{\partial^3 u}{\partial y^3}\}]$$
$$+ g\beta(T - T_\infty) + g\bar{\beta}(C - C_\infty) - \frac{\bar{\sigma}B_0^2}{\rho}u \qquad (2.2)$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \kappa\frac{\partial^2 T}{\partial y^2} \qquad (2.3)$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad (2.4)$$

where ν_1 and ν_2 are the kinematic viscosities, κ is the thermal diffusivity, D is the molecular diffusivity, β is the co-efficient of volumetric expansion for heat transfer, $\bar{\beta}$ is the co-efficient of volumetric expansion for mass transfer, ρ is the fluid density, $\bar{\sigma}$ is the electrical conductivity, T is the temperature of the fluid, C is the concentration and g is the acceleration due to gravity.

The necessary boundary conditions are

$$y = 0 : u = U_0, v = -V_0, T = T_w, C = C_w$$
$$y \to \infty : u \to 0, T \to T_\infty, C \to C_\infty \qquad (2.5)$$

where Tω and Cω are the temperature and the concentration of the fluid at the plate respectively; T_∞ and C_∞ are respectively the temperature and the concentration of the fluid far away from the plate.

We introduce the following non-dimensional quantities

$$y^* = \frac{V_0 y}{\nu_1}, u^* = \frac{u}{U_0}, t^* = \frac{tV_0^2}{\nu_1},$$
$$\theta^* = \frac{T - T_\infty}{T_w - T_\infty}, \phi^* = \frac{C - C_\infty}{C_w - C_\infty} \qquad (2.6)$$

Using (2.6), the equations (2.2)-(2.4) reduce to the forms (dropping the stars '*'):

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha[\frac{\partial^2}{\partial y^2}(\frac{\partial u}{\partial t}) - \frac{\partial^3 u}{\partial y^3}]$$
$$+ G_r\theta + G_m\phi - Mu \qquad (2.7)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \qquad (2.8)$$

$$\frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial y} = \frac{1}{S_c}\frac{\partial^2 \phi}{\partial y^2} \qquad (2.9)$$

where

$G_r = \frac{\nu_1 g\beta(T_w - T_\infty)}{U_0 V_0^2}$ is the Grashof number for heat transfer,

$G_m = \frac{\nu_1 g\bar{\beta}(C_w - C_\infty)}{U_0 V_0^2}$ is the Grashof number for mass transfer,

$M = \frac{\bar{\sigma}B_0^2\nu_1}{\rho V_0^2}$ is the Hartmann number, $P_r = \frac{\nu_1}{\kappa}$ is the Prandtl number,

$S_c = \frac{\nu_1}{D}$ is the Schmidt number

and $\alpha = \frac{\nu_2 V_0^2}{\nu_1}$ is the visco-elastic parameter.

The corresponding boundary conditions are

$$\begin{aligned} &y = 0 : u = 1, \theta = 1, \phi = 1 \\ &y \to \infty : u = 0, \theta = 0, \phi = 0 \end{aligned} \quad (2.10)$$

3. METHOD OF SOLUTION

To solve the equations (2.7)-(2.9), we assume

$$\begin{aligned} u(y,t) &= u_0(y)e^{i\omega t}, \\ \theta(y,t) &= \theta_0(y)e^{i\omega t}, \\ \phi(y,t) &= \phi_0(y)e^{i\omega t} \end{aligned} \quad (3.1)$$

Using (3.1), the equations (2.7)-(2.9) reduce to

$$\alpha u_0''' - (1 + \alpha i\omega)u_0'' - u_0' + (M + i\omega)u_0 = G_r\theta_0 + G_m\phi_0 \quad (3.2)$$

$$\theta_0'' + P_r\theta_0' - i\omega P_r\theta_0 = 0 \quad (3.3)$$

$$\phi_0'' + S_c\phi_0' - i\omega S_c\phi_0 = 0 \quad (3.4)$$

with modified boundary conditions

$$\begin{aligned} &y = 0 : u_0 = e^{-i\omega t}, \theta_0 = e^{-i\omega t}, \phi_0 = e^{-i\omega t} \\ &y \to \infty : u_0 = 0, \theta_0 = 0, \phi_0 = 0 \end{aligned} \quad (3.5)$$

On solving equations (3.2)-(3.5) using (3.1) and (3.5), we get

$$\begin{aligned} u(y,t) = [&e^{-m_3 y} + \frac{G_r}{R_1}(e^{-m_1 y} - e^{-m_3 y}) \\ &+ \frac{G_m}{R_2}(e^{-m_2 y} - e^{-m_3 y}) \\ &+ \alpha\{(m_1 + i\omega)\frac{G_r m_1^2}{R_1^2}(e^{-m_3 y} - e^{-m_1 y}) \\ &+ (m_2 + i\omega)\frac{G_m m_2^2}{R_2^2}(e^{-m_3 y} - e^{-m_2 y})\}]e^{i\omega t} \end{aligned} \quad (3.6)$$

$$\theta(y,t) = e^{-m_1 y}e^{i\omega t} \quad (3.7)$$

$$\phi(y,t) = e^{-m_2 y}e^{i\omega t} \quad (3.8)$$

where

$$\begin{aligned} m_1 &= \frac{1}{2}[P_r + \sqrt{P_r^2 + 4i\omega P_r}], \\ m_2 &= \frac{1}{2}[S_C + \sqrt{S_c^2 + 4i\omega S_c}], \\ m_3 &= \frac{1}{2}[1 + \sqrt{1 + 4(M + i\omega)}], \\ R_1 &= m_1^2 - m_1 - (M + i\omega), \\ R_2 &= m_2^2 - m_2 - (M + i\omega). \end{aligned}$$

The skin friction on the plate is given by

$$\tau = [\tau_{xy}]_{y=0} = [\frac{\partial u}{\partial y} + \alpha\{\frac{\partial}{\partial y}(\frac{\partial u}{\partial t}) + \frac{\partial^2 u}{\partial y^2}\}]_{y=0} \quad (3.9)$$

The coefficient of the rate of heat transfer and the coefficient of the rate of mass transfer at the plate, which in the non-dimensional form in terms of Nusselt number Nu and Sherwood number Sh respectively are given by

$$N_u = (\frac{\partial\theta}{\partial y})_{y=0} \text{ and } S_h = (\frac{\partial\phi}{\partial y})_{y=0} \quad (3.10)$$

4. RESULTS AND DISCUSSION

In order to study the effects of the visco-elastic parameter α on the mixed convective unsteady flow with mass transfer, we have carried out numerical calculations for the dimensionless velocity component u and the skin friction τ at the plate for various values of the flow parameters involved in the solution. The corresponding results for Newtonian fluid can be deduced from the above results by setting α=0 and these results show conformity with earlier results.

In order to understand the physics of the problem, analytical results are discussed with the help of graphical illustrations. Figures 1 to 4 depict the variations of the velocity profile u versus y for various values of Prandtl number (Pr), Schmidt number (Sc), Grashof number for heat transfer (Gr), Grashof number for mass transfer (Gm), magnetic parameter (M), visco-elastic parameter (α) keeping the frequency of oscillation ω=0.1 and the time t=0. The figures reveal that the velocity diminishes in both Newtonian and non-Newtonian cases. It is also noticed that the nature of velocity distribution is unaltered when the magnetic intensity M increases (Figures 1 and 2), Grashof number for heat transfer Gr increases (Figures 1 and 3), Prandtl number Pr decreases (Figures 1 and 4) with increasing values of visco-elastic parameter $|\alpha|$ (α=0, -0.25, -0.4) and fixed values of other flow parameters. Variations of the skin friction τ versus the magnetic parameter M, the frequency of oscillation ω, Grashof number for heat transfer Gr and Grashof number for mass transfer Gm are illustrated in the figures 5 to 8 respectively. The figures reveal that the skin friction τ enhances due to increase of M, Gr and Gm (Figures 5, 7 and 8) respectively whereas increase of ω depresses the skin friction (Fig. 6). But in all the cases, the rising trend in τ is observed with the increase in the absolute value of α in combination with other flow parameters. It can be remarked from expressions (3.10) that the temperature and the concentration fields are not affected by the visco-elastic parameters.

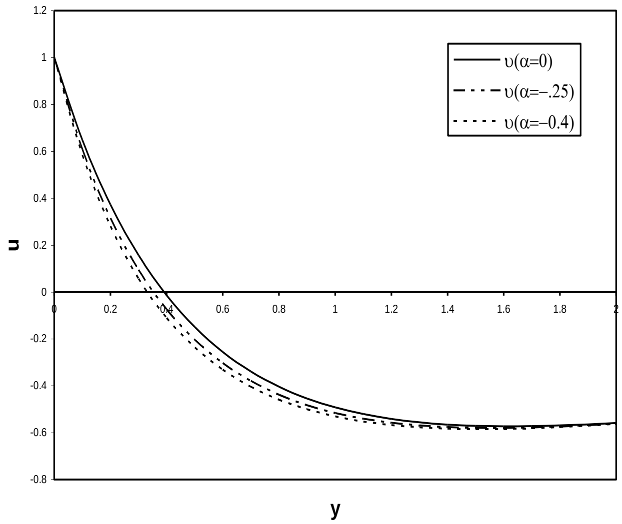

Fig. 1 Variation of velocity u versus y for $M=2, G_r=2, G_m=2, P_r=5, S_c=0.22, \omega=0.1, t=0$

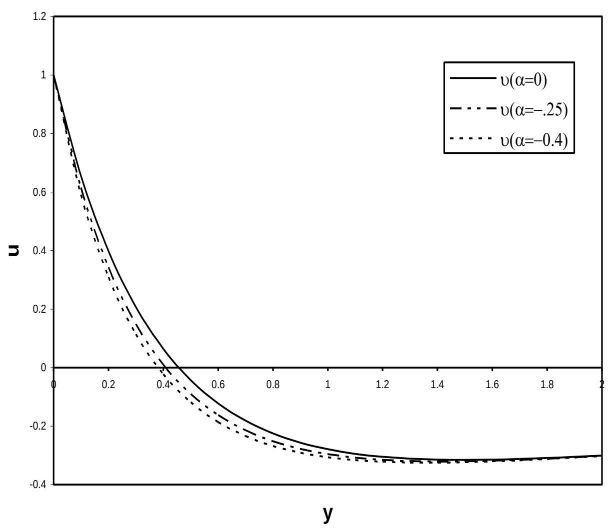

Fig. 2 Variation of velocity u versus y for $M=4, G_r=2, G_m=2, P_r=5, Sc=0.22, \omega=0.1, t=0$

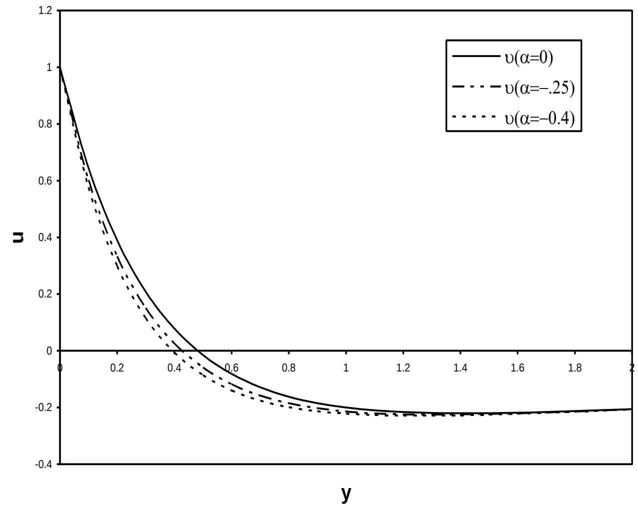

Fig. 3 Variation of velocity u versus y for $G_r=4, P_r=5, G_m=2, S_c=0.22, M=2, \omega=0.1, t=0$

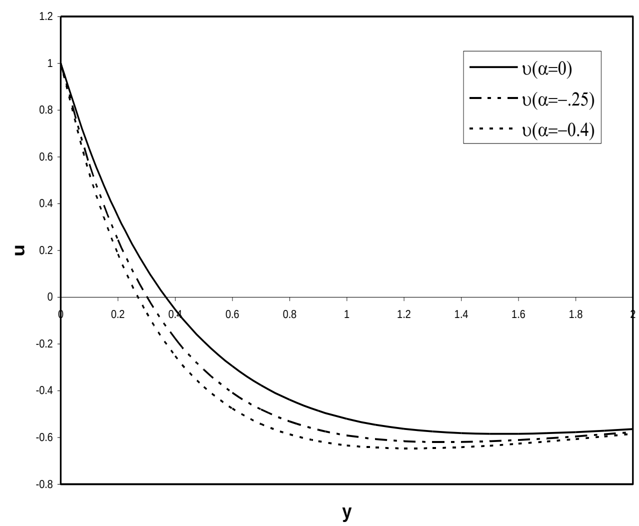

Fig. 4 Variation of velocity u versus y for $P_r=3, G_r=2, G_m=2, S_c=0.22, M=2, \omega=0.1, t=0$

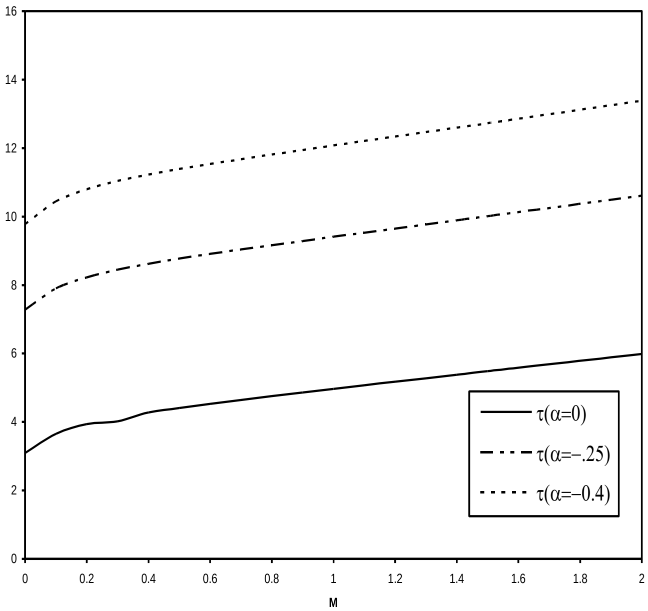

Fig. 5 Variation of skin friction τ versus M for G_r=2, G_m=2, P_r=5, S_c=0.22, ω=0.1, t=0

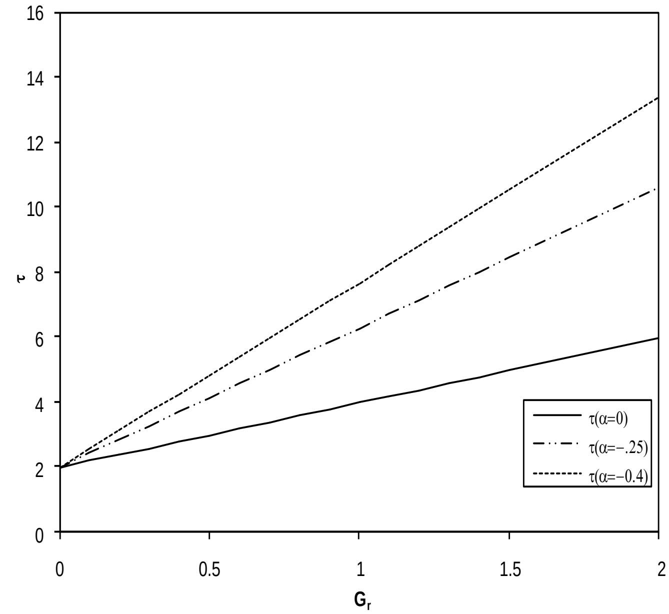

Fig. 7 Variation of skin friction τ versus G_r for P_r=5, G_m=2, S_c=0.22, M=2, ω=0.1, t=0

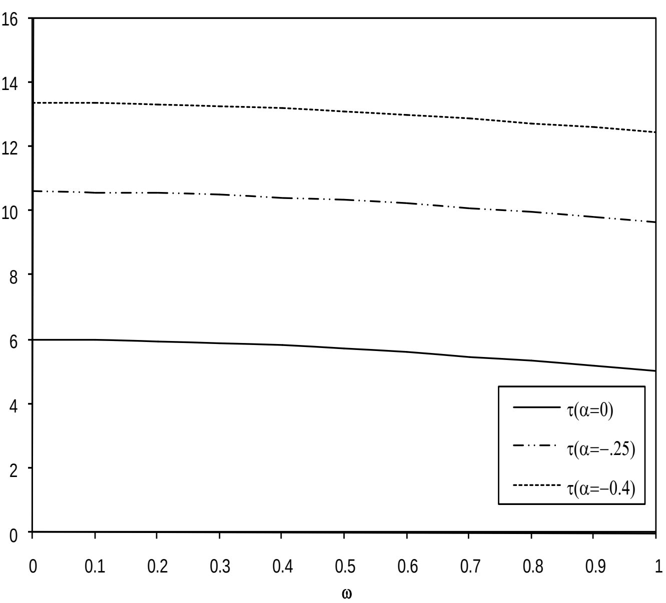

Fig. 6 Variation of skin friction τ versus ω for G_r=2, G_m=2, P_r=5, S_c=0.22, M=2, t=0

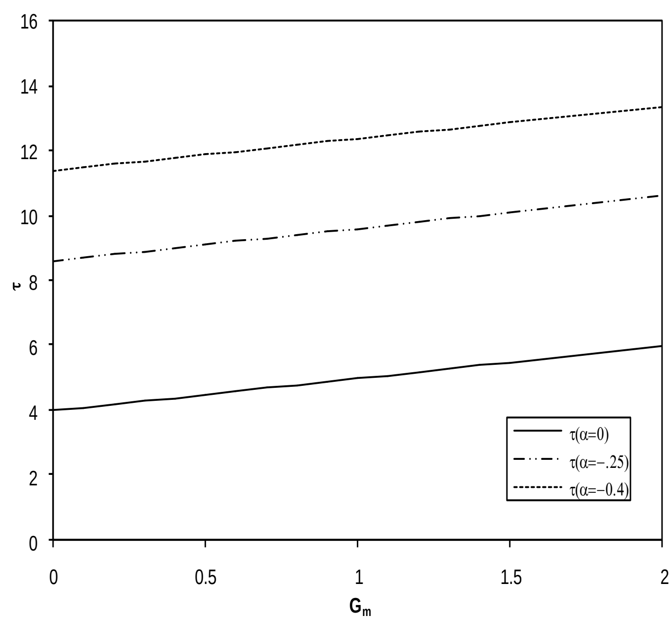

Fig. 8 Variation of skin friction τ versus G_m for P_r=5, G_r=2, S_c=0.22, M=2, ω=0.1, t=0

CONCLUSIONS

The problem of mixed convective MHD visco-elastic flow and mass transfer past an accelerated infinite vertical porous plate is studied analytically. The results of investigation may be summarized in the following conclusions:

- The velocity distribution is retarded in both in both Newtonian and non-Newtonian cases.
- The skin friction τ rises due to increase of magnetic parameter/ Grashof number for heat transfer / Grashof number for mass transfer while the increase of oscillation of frequency produces the opposite effect.
- The skin friction enhances under the effect of the visco-elastic parameter.
- The temperature and the concentration fields are unaffected due to the variation of visco-elastic parameter.

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