Linear Programming Approach for the Determination of the Optimal Process Parameters in the Physically Refined Vegetable Oils

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Abstract: In this work, the optimal process conditions in the physically refined vegetable oils have been determined. The oils involved in this analysis were Palm Oil (PO), and Palm Kernel Oil (PKO). The oils, obtained from a local market in Enugu, North – East province of Nigeria, were characterized with a view to finding the ordinary values of the variables that were optimized, especially in the degumming, bleaching, and deodorization sections, as well as the cost of refining and the revenue accruable from the sale of the products. The AOCS method of AOCS, [1], was used in the analysis. Graphical method of linear programming and Sinplex Alogrithm methods were also used in order to optimize the variables. The objectives and the constraints were formulated into standard models with temperature, pressure, chemical concentration and clay dosage as the variables. It was found that optimal values of phosphotide in the refined oils, measured as Phosphorous, were 0.0105 and 0.0047Ppm; Colour pigment, measured in Red Units, 1.92 and 1.35, for PO and PKO, respectively, and odour, measured as FFA, was 0.1 and 0.05%, accordingly. The optimal cost of refining which was analysed using Simplex Alogrithm, gave N583.95 and 676.24 per tones, each of the PO and PKO. Optimal revenue was obtained as ₦ 106,400 and ₦128,700, per tone of the refined oils. Based on the optimized cost, while profit accruable per 100 tones of the refined raw oils, based on the sale of the products was ₦ 1,399,344 and ₦2,106825 for PO and PKO respectively. The results confirmed the standard **values** from the literatures.

Keywords; Linear Programming, Optimization, vegetable oils, characterization, cost of refining, Revenue, Profit.

INTRODUCTION

Vegetable oils are water insoluble substances of plant origin, which consists predominantly of glycerol esters of fatty acids or triglycerides, Moore [2]. Demand for refined vegetable oils has increased the world over. Its needs for consumption and industrial purposes will increase tremendously in the near future. In the energy sector, vegetable oil will provide the source of bio-energy and renewable energy requirements of the world, Pahl [3]. There is therefore, the need to stabilize the quality of the products for these purposes. The triglycerides contain approximately 95% of fatty acid and 5% glycerol, combined as part of the glyceride molecules and the reactive portion. Refining is carried out in order to purify the glycosides. Bailey [4], asserts that the chemical and physical properties of fats and oils are determined by the properties of their component fatty acids. Okiy [5], noted that, as the average molecular weight of the fatty acid increases, fats progressively have higher melting points and can more easily solidify. Vegetable oils are sometimes called "Fixed Oil" to distinguish them from volatile, ethereal or essential oils, Corelius,[6]. Athanassiadis, [7], observed that two refining methods are used, in the refining of vegetable oils,namely;

- Chemical and
- Physical processes.

The Chemical method involves the use of Caustic Soda (lye) to remove the Free Fatty Acid (FFA) present in the oil. This is a neutralization reaction between the (FFA) and the Caustic alkali.

The first chemical reaction applied to vegetable oils and fats was that of Saponification to produce soap, Mahatta, [8]. The industrialization of vegetable oils started with the erection of a cotton seed oil mill in South Carolina in 1826. The use of Caustic Soda to remove FFA was introduced in France in 1850, and this marked the beginning of Chemical refining process,[8]. The removal of FFA with Caustic lye was, however, improved on in the 1930's with the introduction of Centrifuges to remove the Soap Stock (Foot) resulting from the reaction between the FFA and the Caustic lye, Soon,[9].

In the late 20th century, following the efforts to reduce production cost physical refining process was developed, there by eliminating the use of Caustic Soda, lye[8]. The operation, which is essentially steam distillation of oil volatiles, involves heating under vacuum and stripping steam to remove odour and FFA from the oil. Fig. 1 shows the process routes for both physical and chemical refining processes.

Both qualitative and quantitative factors come into any comparative analysis of the two processes.

For Chemical Process

- There is a substantial colour reduction during neutralization stage.
- There is also fuel saving advantage during distillation, since the FFA has been greatly reduced during neutralization.
- Capacity and quality are said to be higher compared to physical process, based on the oil type.
- Anti-oxidants, Sterol and Tocopherol are preserved at the moderate temperature of this method. This enhances the quality and stability of the final product.

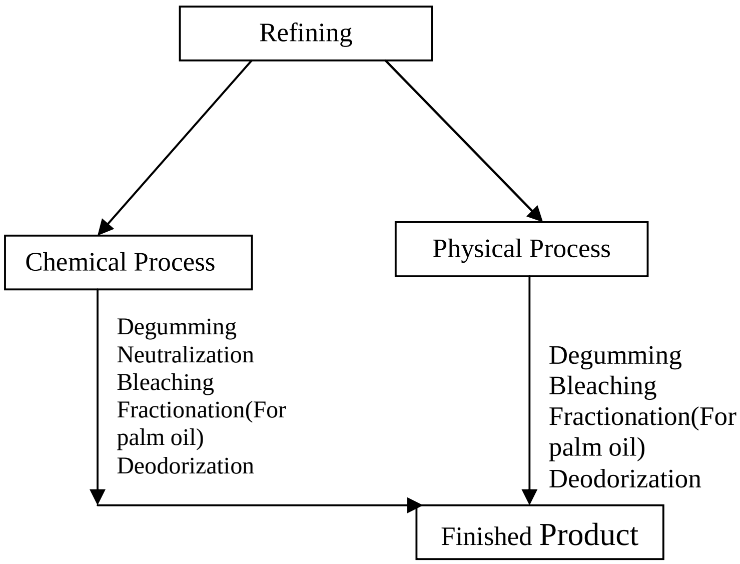

Fig 1, The Refining Process Routes

For Physical Process

- Initial colour of the oil is not affected due to the absence of neutralization stage.
- The low absolute pressure, which improves temperature in the distillation equipment, can enable a reduction of colour to compare with those of chemical process.
- This low absolute pressure also leads to low water consumption.
- There is also a reduced oil loss, and savings on chemical,
- There is reduction in manpower requirements,
- Pollution and corrosion problems are minimal, and these result in low production cost.

It can therefore, be said that physical refining method is more cost effective than the chemical method, and hence its wide spread adoption in the present day refining of vegetable oils.

Refining of vegetable oils is, therefore, meant to compromise the two opposing groups of compounds:

Group I: All undesirable components alien to the triglycerides, which impart unpleasant odour and taste to the oil. This group comprises the main oil soluble compounds such as, phosphorous compound,(gum), coloring matters, heavy metals, unsaturated carbohydrates, and oxidative products, Aldehydes and Ketones. They are identified by gas chromatographic analysis of steam distillate produced under high vacuum and temperature of the unrefined vegetable oil. Their elimination is necessary in order to produce a vegetable oil that is as pure as possible, [9]

Group II: Components which are desirable because of their nutritive and anti-oxidant properties. They are the compounds to be preserved in the oil, and include, tocopherol and tocotriols, which have a strong anti-oxidant effect on the oil, especially, the palm oil. They are also valued for their nutritive value. Tocopherol is the precursor of vitamin E, [10]

In order to stabilize quality of the refined vegetable oil products, factors such as temperature, concentration of the refining chemicals, and of course, time of contact, among others, all of which have a great influence on this operation, must be optimized. Other variables which are very important and which play vital roles in quality and stability of refined vegetable oils are their Peroxide Value, Iodine Value, Colour, Acid Value, Anisidine Value, as well as, their Iron and Cupper contents, [11]. It is therefore, necessary to carry out an optimization study and analysis of these variables so as to establish the best value of each of these variables in the finished product, so that the quality standard is maintained and at a reasonable profit.

Dantzig, [12], had defined optimization as a process in optimal control theory which enables the determination of the process variables that will optimize the output, subject to some restrictions. In order to find the optimal value, the problem has to be defined so as to formulate an objective, and present same as an objective function, recognize the constraints and evaluate the alternatives, with a view to selecting the apparent best course of action, which will lead to optimal solution. This necessitated the formulation of mathematical models which have to bear with the actual physical situation, and make the problem to be purely mathematical. This mathematical model enables the forecasting of the effects of the factors which are crucial to the solution of the problem. The objective in vegetable oil refining could be to find the optimal temperature of the process, the contact time and the best concentration of the refining chemicals, or any other variables as noted earlier. Since the objective cannot always be achieved without considering external influences, a number of restrictions have to be imposed which will determine the level to which the objective will be achieved. The problem is thus reduced to finding the best set of values of a function of several variables, and the set of conditions required to achieve the best results from given situations.

Many optimization problems are not linear, but in this work, only the linear cases have been considered, since all or most of the functions that were generated were linear. In finding the optimal value of each of the variables (parameters), two linear optimization methods have been applied, namely;

- Graphical and
- Numerical methods, notably the Simplex Algorithm method.

2.0 MATERIAL AND METHODS

2.1 REFINING OF RAW OILS: ALFA – LAVAL method, (1978), was used.

In order to obtain the necessary data for the linear optimization calculations, the oils were subjected to refining operations. The refining methods that were applied on the crude oils include, degumming, bleaching, and deodorization.

2.2Degumming of the Raw Oils:

a) Aim: Degumming of the raw oil was done to reduce the phosphotide, so as to minimize the foaming tendency of the finished product observed during frying.

b) Materials/Equipment:

- Freshly prepared raw PO, and PKO
- Phosphoric acid,
- Thermometer,
- Laboratory glassware,
- Measuring cylinders,
- Separating funnel and
- stirrer.

c) Experimental procedure: about one per cent (1%), by weight phosphoric acid was added to l00g of the raw oil samples in conical flasks. The mixture was heated to aconstant temperature of 338K, and stirring done with the magnetic stirrer for 30 minutes. The whole mass was poured into a separating funnel and allowed to settle for 30 minutes. The lower layer (the lecithin), was run off through a valve. Temperature, concentration of the degumming chemical and time of contact were subsequently varied. [13]. Results are shown in tables II

2.3 Bleaching of raw palm oil

Aim: The bleaching experiment was aimed at reducing the carotene pigments, so as to minimize the formation of hydroperoxides during deodorization and storage. The experiment was done with the activated clay, [14]

Material/Equipment:

- Bleaching Clay,
- Oil samples,
- Beakers,
- Filter paper,
- Funnel,
- Heating mantle,
- Thermometer,
- Stirrer and
- Oven.

Experimental procedure: One per cent (1%), by weight, of clay was added to l00g of the oil sample. The mixture was heated to a constant temperature of 373K, with stirring for 30 minutes. The oil was then filtered at the same temperature, and the filtrate characterized. [15].

2.4 Deodorization Experiment

Aim: Deodorization, which essentially is steam distillation, was aimed at removing odour, colour, FFA and undesirable flavour in the oil. This was done at a temperature of 473 °K and for 60 minutes. At these conditions, the β - Carotene pigment bond are broken and the pigments, as well as Iron metal, which is a pro-oxidant, are removed with the odoriferous materials, thereby improving the colour and taste of the refined product.

Material/Equipment:

- Bleached oil samples,
- Distillation apparatus,
- Bunsen burner,
- Vacuum plant,
- Steam generation equipment.

Experimental procedure: 1 liter of degummed and bleached oil each, was taken into the distillation equipment and pre - heated to a temperature of 373K. Steam was generated by heating water in a around bottom flask and passed into the oil through a delivery tube. Temperature was then increased to 473 K, and vacuum applied by means of the vacuum pump and maintained at 20mmHg absolute. Vaporized moisture, odoriferous matter, FFA, and colour pigments were condensed in the reflux condenser in which water is used as a cooling medium. The condensate, which was essentially Fatty acid, was collected in a beaker. This is a batch process. The oil was then analyzed for FFA, Colour, PV, AV, P and Fe [1].

2.5 Properties of the PO and PKO

The results of the physical and chemical analyses of the raw and refined oils used in the investigation are shown in Tables I and II respectively

Table I Physico-Chemical Properties of the raw Oils used in the experiments (Egbuna [16], [17])

Properties	Palm Oil (PO)	Palm Kernel Oil (PKO)
Colour (Physical Appearance)	Light Orange	Golden Yellow
Odour	Bland	Bland
Taste	Bland	Bland
Specific Gravity	0.9282	0.8732
Melting Point(°C)	35	32
Moisture (%)	0.05	0.02
Refractive Index	1.4562	1.4488
Free Fatty Acid (%)	0.2	0.1
Lovibond Red Unit (1" Cell)	3.2	1.8
Anisidine Value (M.eq/kg)	4.05	4.50
Peroxide Value (M.eq/kg)	3.00	2.45
Phosphorous (Ppm)	0.01	0.005
Iron (Ppb)	4.3	2.7
Saponification Value	210	250
Iodine Value	48	20

From the tables, it can be seen that there is a marked difference between the raw and refined oils, in terms of their properties. However, while the difference is high in properties like, colour, taste, odour, moisture content, FFA, AV, PV, P, and Fe, it is low in others such as, specific gravity, melting point, refractive index, saponification value and iodine value. Since these values are close to the standard values as shown in the tables, they can be accepted as true representative values and can be used in subsequent analysis. The difference, is occasioned by the removal of a reasonable amount of the contaminants during refining. The standard values for refined PO and PKO are shown in Table III

Table II, Physio-Chemical Properties of the Refined Oils used in the experiments (Egbuna,[18])

Properties	Palm Oil (PO)	Palm Kernel Oil (PKO)
Colour (Physical Appearance)	Deep Orange	Light Yellow
Odour	...of Palm Oil	Characteristics of PKO
Taste	...of Palm Fruit	PKO Taste
Specific Gravity	0.9182	0.8752
Melting Point(°C)	38	29
Moisture (%)	1.3	1.12
Refractive Index	1.4514	1.440
Free Fatty Acid (%)	3.80	4.2
Lovibond Red Unit (1" Cell)	23	4.2
Anisidine Value (M.eq/kg)	8.2	6.5
Peroxide Value (M.eq/kg)	5.8	2.45
Phosphorous (Ppm)	9.0	6.5
Iron (Ppb)	3.0	3.5
Saponification Value	200	240
Iodine Value	45	22

Table III. Laboratory Refining Experimental results compared with the international standard for Palm Oil and Palm Kernel Oil, (DegummingTemperature is 65^{0}C, and bleaching earth dosage is 1%

Parameters	Refined Oil						International Standard					
	Degummed Oil		Bleached Oil		Deodorized Oil		Degummed Oil		Bleached Oil		Deodor Oil	
	PO	PKO	PO	PKO	PO	PKO	PO	PKO	PO	PKO	PO	PKO
Colour (Red unit) in Inch Cell	19.5	3.5	11.5	2.0	3.2	2.5	20.0	3.0	10.5	2.0	2.5	2.5
FFA%	3.5	4.0	2.8	3.5	0.12	0.09	3.2	4.2	3.5	3.1	0.1	0.1
PV m.eg/kg	5.8	5.32	4.2	3.6	3.00	2.45	4.8	5.2	3.2	3.1	1.0	1.12
AV m.eg/kg	7.5	6.21	6.4	4.6	4.05	4.50	6.6	6.0	6.0	4.2	3.7	3.21
IV (Ppb)	50.6	23	45.2	22.5	45.0	21	50.6	23	46.0	21	45.0	20
Phosphorous	0.54	0.32	0.5	0.25	0.015	0.012	0.52	0.51	0.35	0.20	0.012	0.012
Iron	350	3.50	20.0	3.00	4.30	2.70	280	2.30	200	2.50	0.05	0.05

3.0 ANALYSIS OF RESULTS/DISCUSSION

3.1 Optimization of Process Conditions

Determination of the optimal process variables in this section is done by applying linear programming, making use of graphical and Simplex methods. Aneke, [19], gave the graphical method to be adopted when handling a problem of two variables. The refining stages that will be optimized with this method are:

a) Pretreatment (Degumming)
b) Bleaching
c) Deodorization

3.2 Optimal degumming conditions

In this stage, the main objective is to minimize the phosphotide content of the unrefined oils. Phosphotide is the major cause of foaming in vegetable oils,[9]. It is also required to optimize the use of operating labour and refining chemical -phosphoric acid.

Process specifications

From table II, the phosphorous, measured as phosphotide, content of degummed Palm Oil (PO) and Palm Kernel Oils (PKO) are 0.01 and 0.005kg respectively, for every 1000kg sample of the crude oil.

Temperature of degumming = 65 ± 5°C

H_3PO_4 requirements = 7 kg per 1000Kg of PO
= 5 kg per 1000Kg of PKO

Labour requirements 0.5 hours for refining PO
0.3 hours for refined PKO

The maximum amount of chemical and labour hours required for PO and PKO refining are 12Kg and 0.8 hrs respectively.

Objective function:

In modeling this process, the objective function is presented as;

$$\text{Minimize } Z = 0.01x_1 + 0.005x_2$$

Where $0.01x_1$, and $0.005x_2$ = amount of Phosphotide contained in PO and PKO, respectively, and x_1 and x_2 are the decision variables for PO and PKO respectively.

The constraint, inequalities ;

$$7x_1 + 5x_2 \geq 12$$
$$0.5x_1 + 0.3x_2 \geq 0.8$$
$$x_i \geq 0$$

Where; $7x_1$ = amount of phosphoric acid required for PO
$5x_2$ = amount of phosphoric acid required for PKO
$0.5x_1$ = hours of labour required for PO
$0.3x_2$ = hours of labour required for PKO

These constraints are also subject to a non negativity constraint of $x_i \geq 0$.

The modeled expression now becomes

$$\text{Minimize } Z = 0.01x_1 + 0.005x_2$$
$$\text{Subject to} \quad 7x_1 + 5x_2 \geq 12$$
$$0.5x_1 + 0.3x_2 \geq 0.8$$
$$x_i \geq 0$$

Graphical method was used to find the optimal values.

The standard form of the model is;

$$\text{Minimize } Z = 0.01x_1 + 0.005x_2$$
$$\text{Subject to} \quad 7x_1 + 5x_2 = 12 \quad (a)$$
$$0.5x_1 + 0.3x_2 = 0.8 \quad (b)$$
$$x_i \geq 0$$

From equation (a), given by; $7x_1 + 5x_2 = 12$

When $x_1 = 0$, $x_2 = 12/5 = 2.4 \approx (0, 2.4)$
$x_2 = 0$, $x_1 = 12/7 = 1.72 \approx (1.72, 0)$

Similarly, from equation (b),
$x_2 = 0.8/0.3 = 2.7 \approx (0 , 2.7)$
$x_1 = 0.8/0.5 = 1.6 \approx (1.6 , 0)$

The following coordinates were generated for the graph, of Fig. 2;

(0, 2.4),(1.72, 0); (0 , 2.7), (1.6 , 0)

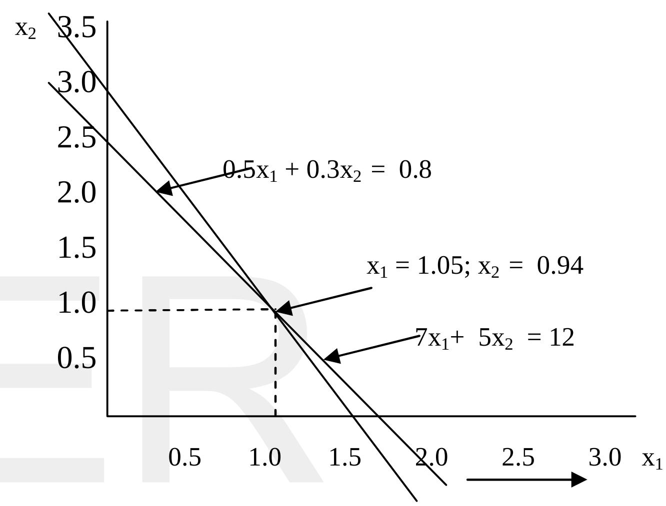

Fig.2, Optimal Degumming conditions

From the point of intersection, $x_1 = 1.05$, $x_2 = 0.94$

When the values are substituted in the objective function equation, we have;

For Palm oil, Minimize $Z = 0.01(1.05) + (0)x_2 = 0.0105$

For Palm kernel oil, Minimize $Z = 0(x_1) + 0.005(0.94) = 0.0047$

From the results, the optimized value of phosphotide in PO increased from a value of 0.01 to 0.0105ppm, while that of PKO was reduced from 0.005 to 0.0047%

It may therefore, be concluded that the optimal values of the decision variables x_1 and x_2 for PO and PKO are 0.105 and 0.0047%, respectively.

3.3 Optimal Bleaching Conditions

In this stage, the main objective is to minimize the colour pigment of the crude oils. Carotenoids are the major causes of red colour in vegetable oils. It is also required to optimize the use of operating temperature and bleaching clay.

Process specifications

From table II, the maximum red colour requirements of refined Palm Oil (PO) and Palm Kernel Oils (PKO), determined using Lovibond Tintometer in 1 inch cell; were, 3.2 and 1.8 red units respectively, for every 1000kg sample of unrefined oil.

Operating temperatures = 130^oC for PO
= $90\,^oC$ for PKO

Bleaching clay = 10Kg for every 1000Kg of PO refined
= 8Kg for every 1000Kg of PKO refined

The maximum temperature requirement was 150^oC, while the maximum amount of clay needed was 12Kg

Objective function:

Objective function is given by;

Minimize Colour, $Z = 3.2x_1 + 1.8x_2$

Where $3.2x_1$ = Minimum colour in PO
$1.8x_2$ = Minimum colour in PKO

and x_1 and x_2, the decision variables, are colour of PO and PKO respectively.

The constraints;

$$10x_1 + 8x_2 \geq 12$$
$$130x_1 + 90x_2 \geq 150$$
$$x_i \geq 0$$

Where; $10x_1$ = amount of Bleaching clay required for raw PO

$8x_2$ = amount of Bleaching clay required for raw PKO

$130x_1$ = operating temperature for PO bleaching

$90x_2$ = operating temperature for PKO bleaching

These constraints are also subject to a non - negativity constraint of $x_i \geq 0$.

The modeled expression now becomes,

Minimize Colour, $Z = 3.2x_1 + 1.8x_2$

Subject to $10x_1 + 8x_2 \geq 12$
$130x_1 + 90x_2 \geq 150$
$x_i \geq 0$

Graphical method was also used to find the optimal value.

The standard form of the model is;

Minimize Colour, $Z = 3.2x_1 + 1.8x_2$

Subject to $10x_1 + 8x_2 = 12$ (a)
$130x_1 + 90x_2 = 150$ (b)
$x_i \geq 0$

From equation (a), given by; $10x_1 + 8x_2 = 12$

When $x_1 = 0$, $x_2 = 12/8 = 1.5 \approx (0, 1.5)$
$x_2 = 0$, $x_1 = 12/10 = 1.2 \approx (1.2, 0)$

From equation (b), it was found that

$x_2 = 150/90 = 1.7 \approx (0, 1.7)$

$x_1 = 150/130 = 1.25 \approx (1.15, 0)$

The coordinates are;

(0, 1.5),(1.2, 0);(0, 1.7), (1.15 , 0)

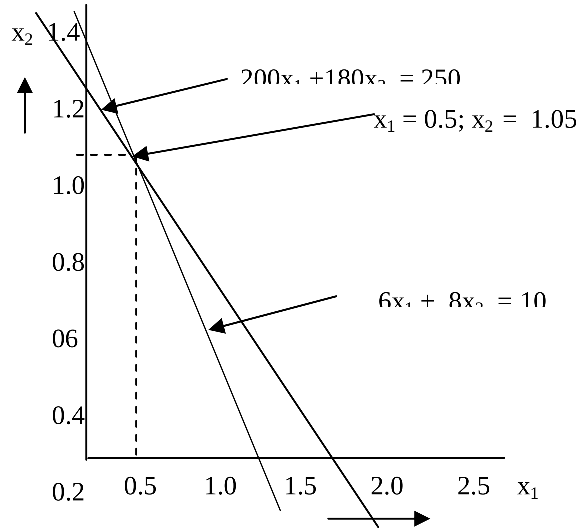

Fig.3, Optimal Bleaching conditions

From the point of intersection, $x_1 = 0.6$, $x_2 = 0.75$

When the values are substituted in the objective function equation, we have:

For Palm oil; Minimize $Z = 3.2(0.6) + (0)x_2 = 1.92$ Red units

For Palm kernel oil;

Minimize $Z = 0(x_1) + 1.8(0.75)=1.35$ Red units

From the results, the colour of PO has been reduced from a value of 3.2 to 1.92 red units, while that of CPKO has been reduced from 2 to 1.35 red units.

These are the optimal colour values for PO and PKO refining.

3.4 Optimal Deodorization conditions:

Here, the objective is to minimize the odoriferous materials in the refined oils. When an unsaturated fatty acid chain reacts with air at room temperature, (a process known as auto-oxidation, [21]), hydro-peroxides are formed. At high temperature, these peroxides break down to Hydrocarbons, Aldehydes and Ketones. These cleavage products impart odour and flavour to oil which is measured in terms of FFA.

Process specifications:

The process conditions for the optimization of odoriferous matters in the refined vegetable oils include;

Operating temperature = 200^oC for PO
= $180\,^oC$ for PKO

Operating pressure = 8 bar for BPO deodorized
= 6 bar for BPKO deodorized

The maximum temperature requirement is 250^oC, while the maximum absolute pressure needed is 10 bar.

From table 2, the maximum odoriferous matter calculated as FFA, of deodorized (PO) and (PKO) are 0.2 and 0.1 %, respectively.

Objective function:

In modeling this process, the objective function is;

Minimize Odour (measured as FFA) , $Z = 0.2x_1 + 0.1x_2$

Where $0.2x_1$ = Minimum FFA in refined PO

$0.1x_2$ = Minimum FFA in refined PKO

The constraints;

$$200x_1 + 180x_2 \geq 250$$
$$8x_1 + 6x_2 \geq 10$$
$$x_i \geq 0$$

Where; $200x_1$ = Operating temperature for deodorizing PO

$180x_2$ = Operating temperature for deodorizing PKO

$8x_1$ = Operating pressure for PO

$6x_2$ = Operating pressure for PKO

These constraint are also subject to a non negativity constraint of $x_i \geq 0$.

The modeled expression now becomes

Minimize Odour (as FFA), $Z = 0.2x_1 + 0.1x_2$

Subject to $200x_1 + 180x_2 \geq 250$

$8x_1 + 6x_2 \geq 10$

$x_i \geq 0$

Using graphical method to find the optimal values, we have;

Minimize Odour (as FFA), $Z = 0.2x_1 + 0.1x_2$

Subject to $200x_1 + 180x_2 = 250$ (a)

$8x_1 + 6x_2 = 10$ (b)

$x_i \geq 0$

From equation (a), given by; $200x_1 + 180x_2 = 250$

$x_2 = 250/180 = 1.4 \approx (0, 1.4)$

$x_1 = 250/200 = 1.25 \approx (1.25, 0)$

Similarly, from equation (b),

When $x_2 = 10/6 = 1.67 \approx (0, 1.67)$

$x_1 = 10/8 = 1.67 \approx (1.25, 0)$

The coordinates are;

(0, 1.4),(1.25, 0);(1.25, 0), (0, 1.67)

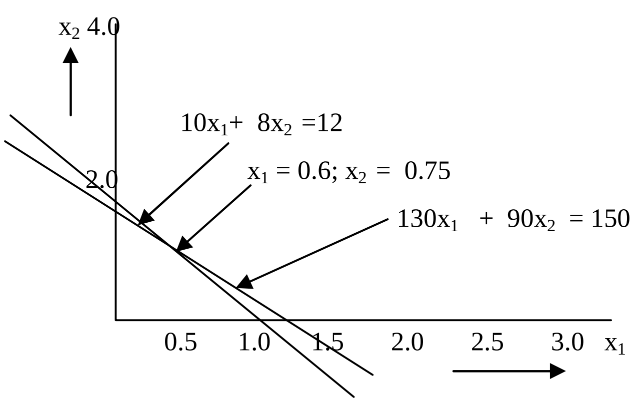

Fig.4, Optimal Deodorization conditions

From the point of intersection, $x_1 = 0.5$, $x_2 = 1.05$

When the values are substituted in the objective function equation, we have,

For Palm oil;

Minimize odour $Z = 0.2(0.5) + 0.1(0) = 0.1\%$

For Palm kernel oil,

Minimize $Z = 0.2(0) + 0.1(1.05) = 0.0.105\%$

From the results, the odour of deodorized PO, measured as FFA, will reduce from a value of 0.2 to 0.1%, while that of PKO will be increased from 0.1 to 0.105%.

3.5 Optimizing Revenue in the Refining of Vegetable Oils

Two variables involved in optimizing revenue are refined palm oil (PO)and palm kernel oil,(PKO).

Labour for refining PO per tone of oil refined = 0.96 hours

Labour for refining PKO per tone of oil refined = 0.4hours

Maximum labour hours required = 1.2 hours

For 1000Kg of oil, clay requirement becomes

Clay requirements for PO = 10 kg

Clay requirement for PKO = 8 kg

Maximum amount of clay required = 16kg

Revenue accruable (₦/Tone), of refined;

PO (Cooking oil) = N 152,000.00

PKO (Pure vegetable oil) = N 143000.00

Decision valuables;

x_1 - PO (Cooking oil)

x_2 - PKO (Pure vegetable oil)

Objective Function;

Maximize Z, = ₦ $152{,}000x_1$ + ₦ $143{,}000x_2$

Constraints;

$$0.94x_1 + 0.4x_2 \leq 1.2$$
$$10x_1 + 8x_2 \leq 16$$
$$x_i \geq 0$$

Standard form becomes

Maximize Z, = ₦ $152{,}000x_1$ + ₦ $143{,}000x_2$

$0.96x_1 + 0.4x_2 = 1.2$ a

$10x_1 + 8x_2 = 16$ b

$x_i \geq 0$

From equation (a),

$x_2 = 1.2/0.4 = 3 \approx (0, 3)$

$x_1 = 1.2/0.96 = 1.25 \approx (1.25, 0)$

From equation (b),

$x_2 = 16/.8 = 2.0 \approx (0 , 2.0)$

$x_1 = 16/10 = 1.6 \approx (1.6 , 0)$

The coordinates are;

(0, 3.0),(1.25 , 0); (0, 2.0), (1.6, 0)

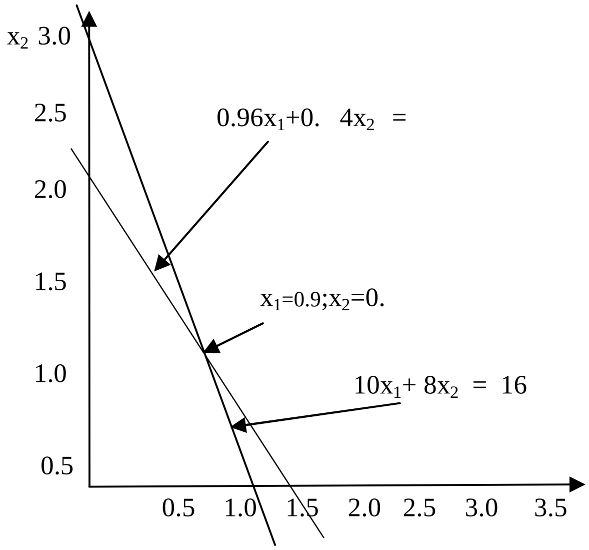

Fig.5, Optimal Revenue in the refining of vegetable oil

The intersection point gave, $x_1 = 0.9$, $x_2 = 0.9$

When the values are substituted in the objective function equation, we have,

The Minimize revenue,

$$Z = 152{,}000(0.9) + 143{,}000(0.9) = ₦265500$$

From the results, the maximum optimized revenue that can be generated per tone of each of the two products is ₦265500.00. Difference is ₦29,500.00 from the market value of ₦ 29,5000.00.

3.6 Optimizing the cost of refining PKO and PO.

Table IV gives the information required for refining one tone of raw oils. This information was obtained from Oil mill of ENVOY oil industries ltd Onitsha, South – East Province of Nigeria. The Simplex method of [12], was used in solving a problem of three variables as observed when there was need to optimize cost of refining when the amount of clay, phosphoric acid and calcium carbonate, are the refining variables. Backrishnan, [20], also adopted similar method when dealing with problems of several variables.

Table IV Cost of refining raw PKO AND PO

	Cost of material(₦) per kilogram		Quantity used (Kg) Per tone of		Cost (₦) per tone of	
Materials used	PKO	PO	PKO	PO	PKO	PO
Bleaching clay	29.56	29.56	8	10	236.48	295.60
Phosphoric acid	70.5	70.5	5	7	352.50	493.50
Calcium carbonate	11.83	11.83	0.53	0.8	6.27	9.46

The total cost of refining one tone of raw

PKO = ₦ 595.25

PO = ₦ 798.56

The market prices of refined PKO and PO are also shown in table IV

Table V Market prices of refined PO and PKO

Product	**Qty of oil produced per (tones) from**		**Cost (₦) per tone of**		**Cost (₦) per 100 tone of**	
	PKO	**PO**	**PKO**	**PO**	**PKO**	**PO**
Distilled fatty acid	4.05	5.05	110,000	96,000	445,500	484,800
Finished product	95.95	94.95	143,000	152000	13,720850	14432400
Grand Total realized from refined the products					14,166350	14,917200

The Simplex method of linear programming was used to determine the optimal cost of refining the oil whether PKO or PO.

The requirements for each tone of Raw PKO, are;

For PKO

- Not less than 10kg of bleaching clay at ₦ 236.48
- Not less than 5kg of Phosphoric acid at ₦ 352.50
- Not less than 0.53kg of Calcium carbonate at ₦ 6.27
- Let x_1 = amount (Kg) of bleaching clay used per tone of PKO

x_2 = amount (Kg) of phosphoric acid used per tone of PKO

x_3 = amount (Kg) calcium carbonate used per tone of PKO

It is required to optimize the use of these chemicals or minimize the cost of refining a tone of RPKO.

The total cost of refining a tone of RPKO

$$= 236.48x_1 + 352.50x_2 + 6.27x_3$$

$$\approx 37.79x_1 + 56.22x_2 + x_3$$

The Objective function model now becomes;

Minimize $Z = 37.79x_1 + 56.22x_2 + x_3$

Where Z = Cost of refining a tone of RPKO

The constraints model can also be given as;

$$x_1 \geq 8\text{Kg}$$

$$x_2 \geq 5\text{Kg}$$

$$x_3 \geq 0.53 \quad x_1 + x_2 + x_3 = 1000\text{Kg}$$

and amount of oil refined = 1000Kg

Presenting both objective function and the constraint inequalities in their standard form, we have;

Minimize $Z = 37.79x_1 + 56.22x_2 + x_3$

Subject to $x_1 \geq 8Kg$

$x_2 \geq 5Kg$

$x_3 \geq 0.53Kg$

$x_1 + x_2 + x_3 = 1000Kg$

$x_i \geq 0$,

Introducing artificial variables, we have

$a_1 + x_1 - x_4 = 8Kg$

$a_2 + x_2 - x_5 = 5Kg$

$a_3 + x_3 - x_6 = 0.53Kg$

$x_1 + x_2 + x_3 + x_7 = 1000Kg$

But $x_i \geq 0$, $(i = 1,2,\ldots\ldots.7)$

$a_j \geq 0$, $(j = 1,2, 3)$

When the penalty functions are introduced, we have;

$a_1 = 8 - x_1 + x_4$

$a_2 = 5 - x_2 + x_5$

$a_3 = 0.53 - x_3 + x_6$

$P = Za_j = 13.53 - x_1 - x_2 - x_3 + x_4 + x_5 + x_6$

The model problem now becomes

$(-p) = x_1 + x_2 + x_3 - x_4 - x_5 - x_6 = -13.53$

$-Z + 37.79x_1 + 56.22x_2 + x_3 = 0$

Subject to

$a_1 + x_1 - x_4 = 8Kg$

$a_2 + x_2 - x_5 = 5Kg$

$a_3 + x_3 - x_6 = 0.53Kg$

$x_1 + x_2 + x_3 + x_7 = 1000Kg$

$x_i \geq 0$, $(i = 1,2,\ldots\ldots.7)$

$a_j \geq 0$, $(j = 1,2,3)$

Simplex Tableaux 1

	C	x_1	a_2	x_3	x_4	x_5	x_6
a_1	8	1	0	0	-1	0	0
x_2	5	0	1	0	0	-1	0
a_3	0.53	0	0	1	0	0	-1
x_7	1000	1	1	1	0	0	0
-Z	0	37.79	0	1	0	56.22	0
-P	-13.53	-1	-1	-1	1	1	1

2

	C	x_1	x_2	x_3	x_4	x_5	x_6
a_1	8	1	0	0	-1	0	0
x_2	5	0	1	0	0	-1	0
a_3	0.53	0	0	1	0	0	-1
x_7	995	0	1	1	1	0	0
-Z	-281.1	37.79	0	1	0	56.22	0
-P	-853	-1	0	-1	1	0	1

3

	C	a_1	a_2	x_3	x_4	x_5	x_6
x_1	8	1	0	0	-1	0	0
x_2	5	0	1	0	0	-1	0
a_3	0.53	0	0	1	0	0	-1
x_7	987	0	0	1	1	1	0
-Z	-583.42	0	0	1	37.79	56.22	0
-P	-853	0	0	-1	0	0	1

4

	C	a_1	a_2	a_3	x_4	x_5	x_6
x_1	8	1	0	0	-1	0	0
x_2	5	0	1	0	0	-1	0
x_3	0.53	0	0	1	0	0	-1
x_7	986.47	0	0	0	1	1	1
-Z	-583.95	0	0	0	37.79	56.22	0
-P	0	0	0	0	0	0	0

Since all the variables in (-P) row are all zero , the penalty function and artificial variables are removed to obtain,

	C	x_4	x_5	x_6
x_1	8	-1	0	0
x_2	5	0	-1	0
x_3	0.53	0	0	-1
x_7	986.47	1	1	1
-Z	-583.95	37.79	56.22	1

There is also no negative coefficient in the (-Z) row, therefore, the following results become obvious.

$x_1 = 8$, $x_2 = 5$, $x_3 = 0.53$ $x_7 = 986.47$

And -Z = -583.95, which implies that Z = ₦ 583.95

The optimized cost of refining one tone of raw PKO now is, ₦ 583.95

As a check, the objective function is

Max. Z = ₦ $37.79x_1 + 56.22x_2 + x_3 = 0$

Then, we have, ₦ 37.79(8) + 56.22(5) + 0.53 = ₦ 583.95

The cost difference between the optimized cost and the company cost of refining a tone of raw PKO is therefore,

₦ 595.25 - ₦ 583.95 = ₦ 11.30

For PO

The requirements for each tone of raw RPO refined are:

- Not less than 10kg of bleaching clay at ₦ 295.60
- Not less than 7kg of Phosphoric acid at ₦ 493.50
- Not less than 0.8kg of Calcium carbonate at ₦ 9.46

Let x_1 = amount (Kg) of bleaching clay used per tone of RPO

x_2 = amount (Kg) of phosphoric acid used per tone of RPO

x_3 = amount (Kg) calcium carbonate used per tone of RPO

It is required to optimize the use of these chemicals or minimize the cost of refining a tone of PO.

The total cost of refining a tone of RPO

$= 295.60x_1 + 493.50x_2 + 9.46x_3$

$= 31.25x_1 + 52.17x_2 + x_3$

The Objective function model now becomes;

Minimize $Z = 31.25x_1 + 52.17x_2 + x_3$

Where Z = Cost of refining a tone of PO

The constraints model can also be given as;

$31.25x_1 + 52.17x_2 + x_3$, and amount of oil refined = 1000Kg, each.

Presenting both objective function and the constraint inequalities in their standard form, we have;

Minimize $Z = 31.25x_1 + 52.17x_2 + x_3$

Subject to $x_1 \geq 10$Kg

$x_2 \geq 7$Kg

$x_3 \geq 0.8$Kg

$x_1 + x_2 + x_3 = 1000$Kg

$x_i \geq 0$,

Introducing artificial variables, we have

$a_1 + x_1 - x_4 = 10$Kg

$a_2 + x_2 - x_5 = 7$Kg

$a_3 + x_3 - x_6 = 0.8$Kg

$x_1 + x_2 + x_3 + x_7 = 1000$Kg

But $x_i \geq 0$, (i = 1,2,…….7)

$a_j \geq 0$, (j = 1,2, 3)

When the penalty functions are introduced, we have;

$a_1 = 10 - x_1 + x_4$

$a_2 = 7 - x_2 + x_5$

$a_3 = 0.8 - x_3 + x_6$

$P = \underline{Za_j} = 17.8 - x_1 - x_2 - x_3 + x_4 + x_5 + x_6$

The model problem now becomes

$(-p) = x_1 + x_2 + x_3 - x_4 - x_5 - x_6 = -17.8$

$(-Z) + 31.25x_1 + 52.17x_2 + x_3 = 0$

Subject to

$a_1 + x_1 - x_4 = 10$Kg

$a_2 + x_2 - x_5 = 7$Kg

$a_3 + x_3 - x_6 = 0.8$Kg

$x_1 + x_2 + x_3 + x_7 = 1000$

$x_i \geq 0$, (i = 1,2,…….7

$a_j \geq 0$, (j = 1,2, 3)

Simplex Tableaux 1

	C	x_1	x_2	x_3	x_4	x_5	x_6
a_1	10	1	0	0	-1	0	0
a_2	7	0	1	0	0	-1	0
a_3	0.8	0	0	1	0	0	-1
x_7	1000	1	1	1	0	0	0
-Z	0	31.25	52.17	1	0	0	0
-P	-17.8	-1	-1	-1	1	1	1

2

	C	a_1	x_2	x_3	x_4	x_5	x_6
x_1	10	1	0	0	-1	0	0
a_2	7	0	1	0	0	-1	0
a_3	0.8	0	0	1	0	0	-1
x_7	993	0	1	1	1	0	0
-Z	-365.19	0	52.17	1	31.25	0	0
-P	-10.8	0	-1	-1	0	1	1

3

	C	a_1	a_2	x_3	x_4	x_5	x_6
x_1	10	1	0	0	-1	0	0
x_2	7	0	1	0	0	-1	0
a_3	0.8	0	0	1	0	0	-1
x_7	983	0	1	1	1	0	0
-Z	-675.44	0	0	1	31.25	52.17	0
-P	-0.8	0	0	-1	0	0	1

4

	C	a_1	a_2	a_3	x_4	x_5	x_6
x_1	10	1	0	0	-1	0	0
x_2	7	0	1	0	0	-1	0
x_3	0.8	0	0	1	0	0	-1
x_7	982.2	0	0	0	1	1	0
-Z	-676.24	0	0	0	31.25	52.17	1
-P	0	0	0	0	0	0	0

Since all the variables in (-P) row are all zero, , the penalty function and artificial variables are removed to obtain,

	C	x_4	x_5	x_6
x_1	10	-1	0	0
x_2	7	0	-1	0
x_3	0.8	0	0	-1
x_7	982.2	1	1	0
-Z	-676.24	31.25	52.17	1

There is also no negative coefficient in the (-Z) row, therefore, the following results apply.

$x_1 = 10, \quad x_2 = 7, \quad x_3 = 0.8 \quad x_7 = 982.2$

And $-Z = -676.24$, which implies that Z = ₦676.24

The optimized cost of refining one tone of RPO refined now becomes, ₦676.24

As a check, the objective function is

Max. Z = ₦ $31.25x_1 + 52.17x_2 + x_3 = 0$

Then, we have,

₦ 31.25(10) + 52.17(7) + 0.8 = , ₦ 676.24

The cost difference between the optimized cost and the company cost of refining a tone of raw PKO is therefore,

₦ 798.56 - ₦ 676.24 = ₦ 122.32 PO

3.7 Revenue Based On Optimized Cost

Now, cost of 100 tones of unrefined PO = ₦ 13,440,000
Refining cost of 100 tones of PO = ₦ 79,856
Total cost of refining (Finished product) = ₦ 13,519,856
Selling price of finished product = ₦14,917,200
Less refining cost = ₦ 13,519,856
Profit from 100tones of PO refined = ₦ 1,397,344
This amounts to ₦ 13,973.44 per 100 tones of refined PO

Similarly;

Now cost of 100 tones of unrefined PKO = ₦ 12,000,000
Refining cost of 100 tones of PKO = <u>₦ 59525</u>
Total cost of refining (Finished product) = ₦ 12,059,525
Selling price of finished product = ₦14,166,350
Less refining cost = ₦ 12,059,525
Profit from 100tones of PKO refined = ₦ 2,106,825. 00
This amounts to ₦ 21,068.25 per 100 tones of refined PKO

4.0 CONCLUSION

This work was aimed at finding the optimal values of the parameters involved in the refining og vegetable oils by physical refining method. The objective functions and the constraint that may limit the attainment of the objectives were formulated into a models, which were solved using linear programming methods of graphs and Simplex algorithm methods. The refining parameters of phosphorous, in the degumming section, colour, in the bleaching section and FFA in the deodorization section, were optimized, so also the optimal cost of refining g, the revenue generated, and of course, the profit accruable. The results confirm the values in the literature.

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