FREE CONVECTION EFFECTS ON THE OSCILLATORY MAGNETOHYDRODYNAMIC FLOW OF A VISCO-ELASTIC FLUID PAST AN INFINITE VERTICAL POROUS PLATE WITH CONSTANT SUCTION AND HEAT DISSIPATION

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ABSTRACT

This paper deals with free convection effects on the oscillatory magnetohydrodynamic flow of a visco-elastic fluid past an infinite vertical porous plate with constant suction and heat dissipation. The problem is formulated developing equation of continuity, equation of momentum and equation of energy constant suction and dissipation have been taken into account. Above equations have been solved with the help of small parameter regular perturbation technique pertaining to the allowed boundary conditions. It is observed that the strength of the external magnetic field reduces the mean velocity of flow like the elastic parameter (R_c).

Keywords : MHD, Visco-elastic fluid, porous plate, constant suction, heat transfer.

1. INTRODUCTION

The unsteady flow of visous incompressible fluids past two-dimensional bodies was studied by Lighthill[1] in case of small amplitude oscillatory free stream. In solving the problem he assumed that the unsteady flow is superimposed on the mean steady flow. This method was employed by Stuart[2] to analyse the effects of oscillatory free stream on the flow past an infinite porous plate with constant suction. In the analysis of temperature field, he assumed that the plate is thermally instulated. These studies are concerned with horizontal bodies. Soundalgekar[3] studied the unsteady flow of Walters′ liquid B′ past a vertical porous plate with constant suction when the free convection currents are present in the boundary layer. Free convection effect on the flow of an elastico-viscous fluid past an exponentially accelerated vertical plate has been analysed by Dash and Biswal[4]. They have further investigated the problem of unsteady free convection flow of an visco-elastic fluid past an infinite plate with constant suction and heat sources[5]. Biswal and Pradhan[6] have studied the magnetohydrodynamic unsteady free convection flow past an infinite plate with constant suction and heat sinks including dissipative heat. Biswal and Mahalik[7] have analysed the unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous flat plate with heat sources/sinks. The objective of the present study is to analyse the free convection effects on the oscillatory MHD flow of a visco-elastic fluid past an infinite vertical porous plate with constant suction.

2. FORMULATION OF THE PROBLEM

Here the x'-axis is taken along the vertical, infinite plate in the direction of the flow and the y'-axis is taken normal to the plate. A two-dimensional unsteady magnetohydrodynamic flow, in the upward direction, of an incompressible, elastico-viscous liquid ($Walters'$ liquid B') is considered. All the fluid properties are assumed to be constant except that the influence of the density variation with temperature is considered only in the body force term. The influence of the density variations in other terms of the momentum and the energy equations and the variations of expansion co-efficient with temperature are considered negligible. This is the well-known Boussinesq approximation. Under these assumptions, the physical variables are functions of y' and t' only except the pressure which is a function of x' only. The constitutive equations of $Walters'$ liquid B' is given by

$$P'_{ij} = 2\eta e_{ij} - 2K_0 \tilde{e}_{ij}, \qquad (2.1)$$

where P'_{ij} is the stress-tensor, $\tilde{e}_{ij}$ is the rate of strain tensor and in contravariant form, it is given by

$$\tilde{e}^{ij} = \frac{\partial e^{ij}}{\partial t} + e^{ij},_k V^k - e^{ik} V^j,_k - e^{kj} V^i,_k + e^{ij} V^k,_k , \tag{2.2}$$

Taking into account the body force, the equations governing the flow of an incompressible visco-elastic ($Walters'\ B'$) fluid are

Momentum equation:

$$\rho'\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial P'}{\partial x'} - \rho' g + \eta_0 \frac{\partial^2 u'}{\partial y'^2}$$

$$- K_0\left[\frac{\partial^3 u'}{\partial y'^2 \partial t} + v'\frac{\partial^3 u'}{\partial y'^3}\right] - \sigma B_0^2 u' \tag{2.3}$$

Energy equation :

$$\rho' C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = K\frac{\partial^2 T'}{\partial y'^2} + \eta_0\left(\frac{\partial u'}{\partial y'}\right)^2$$

$$- K_0 \frac{\partial u'}{\partial y'}\left\{\frac{\partial^3 u'}{\partial t' \partial y'} + v'\frac{\partial^2 u'}{\partial y'^2}\right\} \tag{2.4}$$

Continuity equation :

$$\frac{\partial V'}{\partial y'} = 0 , \tag{2.5}$$

and $\eta_0 = \int_0^\infty N(\tau) d\tau$

the limiting viscosity at small rates of shear, $K_0 = \int_0^\infty \tau N(\tau) d\tau$

where N(τ) is the distribution function of relaxation times τ. Also, g is the acceleration due to gravity. The last term in the equation (2.4) represents the heat dissipated due to the elastic property of the fluid.

The boundary conditions are :

$$u'(0) = 0 , \qquad T'(0) = T'_\omega ,$$

$$u'(\infty) = U'(t'),\ T'(\infty) = T'_\infty \tag{2.6}$$

In the free stream, from equn. (2.3), we obtain

$$\rho' \frac{\partial U'}{\partial t'} = -\frac{\partial P'}{\partial x'} - \rho'_\infty g , \tag{2.7}$$

and from the equation of state, we have

$$g(\rho'_\infty - \rho') = g\beta'\rho'(T' - T'_\infty), \tag{2.8}$$

Where β' is the coefficient of volume expansion,

From equn. (2.3), (2.7) and (2.8), we obtain, on eliminating $-\frac{\partial p'}{\partial x'}$ and $(\rho'_\infty - \rho')$,

$$\rho'\left(\frac{\partial u'}{\partial t'} + V'\frac{\partial u'}{\partial y'}\right) = \rho'\frac{\partial u'}{\partial t'} + gb'\rho'(T' - T'_\infty)$$

$$+ \eta_0\frac{\partial^2 u'}{\partial y'^2} - K_0\left[\frac{\partial^3 u'}{\partial y'^2 \partial t'} + v'\frac{\partial^3 u'}{\partial y'^3}\right] - \sigma B_0^2 u' \tag{2.9}$$

Again, from equn. (2.5), for constant suction, we have

$$V' = -V_0, \tag{2.10}$$

Where the negative sign indicates that the suction is towards the plate.

On introducing the following non-dimensional quantities,

$$\left.\begin{array}{l} y = \dfrac{y'V_0}{v}, \quad t = \dfrac{t'V_0^2}{4v}, \quad u = \dfrac{u'}{U_0}, \\ U = \dfrac{U'}{U_0}, \quad \omega = \dfrac{4v\omega'}{V_0^2}, \quad \theta = \dfrac{T' - T'_\infty}{T_\omega - T'_\infty} \\ R_c = \dfrac{K_0 V_0^2}{\rho v^2}, \quad P = \dfrac{\eta_0 C_p}{K}, \quad G = \dfrac{g\beta' v(T'_\omega - T'_\infty)}{U_0 V_0^2}, \\ E = \dfrac{U_0^2}{C_p(T'_\omega - T'_\infty)}, \quad M = B_0\left(\dfrac{\sigma}{\rho' U_0}\right)^{1/2} \end{array}\right\} \tag{2.11}$$

in equns. (2.9) and (2.4), we obtain

$$\frac{1}{4}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{1}{4}\frac{\partial u}{\partial t} + G\theta + \frac{\partial^2 u}{\partial y^2} - M^2 u - R_c\left[\frac{1}{4}\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\partial^3 u}{\partial y^3}\right] \tag{2.12}$$

and

$$\frac{P}{4}\frac{\partial\theta}{\partial t} - P\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2 - R_c PE\frac{\partial u}{\partial y}\left\{\frac{1}{4}\frac{\partial^2 u}{\partial y \partial t} - \frac{\partial^2 u}{\partial y^2}\right\} \tag{2.13}$$

All the physical variables are defined in notation. Equations (2.12) and (2.13) show that the problem is governed by the coupled non-linear equations.

The boundary conditions (2.6) now become

$$\left.\begin{array}{ll} u(0) = 0, & \theta(0) = 1, \\ u(\infty) = U(t), & \theta(\infty) = 0 \end{array}\right\} \tag{2.14}$$

In order to solve these coupled non-lenear equations when the free-stream is varying periodically with time, we follow Lighthill's method. Then, in the neighbourhood of the plate, we assume

$$\left.\begin{aligned} u &= u_0 + \varepsilon e^{iwt} u_1 \\ \text{And} \quad \theta &= \theta_0 + \varepsilon e^{iwt} \theta_1 \end{aligned}\right\} \quad (2.15)$$

And for the free-stream,

$$U(t) = 1 + \varepsilon e^{iwt}, \quad (2.16)$$

Substituting (2.15) and (2.16) in (2.12), (2.13), equating harmonic terms, we have

$$R_c u_0''' + u_0'' + u_0' + M^2 u_0 = -G\theta_0, \quad (2.17)$$

$$R_c u_1''' + \left(1 - \frac{iwR_c}{4}\right) u_1'' + u_1' - \frac{iw}{4} u_1 + M^2 u_1 = -\frac{iw}{4} G\theta_1, \quad (2.18)$$

$$\theta_0'' + P\theta_0' = -PE\left(u_0'^2 + R_c u_0' u_0''\right) \quad (2.19)$$

$$\theta_1'' + P\theta_1' - \frac{iwp}{4}\theta_1 = -2PEu_0'u_1' + R_c PE\left(\frac{iw}{4} u_0' u_1' - u_0' u_1'' - u_1' u_0''\right) \quad (2.20)$$

Where primes now denote the derivative with respect to y. The corresponding boundary conditions are:

$$\left.\begin{aligned} &u_0 = 0,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0 \ \text{ at } y = 0 \\ &u_0 = 1,\ u_1 = 1,\ \theta_0 = 0,\ \theta_1 = 0 \ \text{ at } y \to \infty \end{aligned}\right\} \quad (2.21)$$

3. SOLUTIONS OF THE EQUATIONS

Now, equations (2.17) and (2.18) are of the third-order differential equations for $R_c \neq 0$ (non-Newtonian fluids) and they reduce to the case of Newtonian fluid when $R_c=0$. Obviously, the elastic property of the fluid increases the order of the momentum equation of the fluid from two to three. The given boundary conditions are not sufficient to have an unique solution of the equation of motion. To overcome this difficulty, we follow Beard and Walters[8] and expand u_0 and u_1 in powers of R_c, as $R_c<<1$. This is also possible physically for Walters[9] has derived these equations for liquids with short memories on the assumptions that R_c is very small. Thus, we have

$$\left.\begin{aligned} u_0 &= u_{01} + R_c u_{02} \\ u_1 &= u_{11} + R_c u_{12} \\ \theta_0 &= \theta_{01} + R_c \theta_{02} \\ \theta_1 &= \theta_{11} + R_c \theta_{12} \end{aligned}\right\} \quad (3.1)$$

Substituting (3.1) in equations (2.17) to (2.20), equating the co-efficients of R_c^0 and R_c^1 and neglecting the terms containing higher powers (≥2) of R_c, we have

$$R_c\left(u_{01}''' + R_c u_{02}'''\right) + u_{01}'' + R_c u_{02}'' + u_{01}' + R_c u_{02}' + M^2\left(u_{01} + R_c u_{02}\right)$$

$$= - G\,(\theta_{01} + R_c\,\theta_{02})$$

or $$u_{01}''' + u_{02}'' + u_{02}' + M^2 u_{02} = -G\theta_{02}, \quad (3.2)$$

and $$u_{01}'' + u_{01}' + M^2 u_{01} = -G\theta_{01}, \quad (3.3)$$

$$R_c\left(u_{11}''' + R_c u_{12}'''\right) + \left(1 - \frac{iwR_c}{4}\right)\left(u_{11}'' + R_c u_{12}''\right)$$

$$+ u_{11}' + R_c u_{12}' + \left(M^2 - \frac{iw}{4}\right)\left(u_{11} + R_c u_{12}\right) = -\frac{iwG}{4}\left(\theta_{11} + R_c\theta_{12}\right)$$

or $$u_{11}''' - \frac{iw}{4}u_{11}'' + u_{12}'' + u_{12}' + \left(M^2 - \frac{iw}{4}\right)u_{12} = -\frac{iwG}{4}\theta_{12}, \quad (3.4)$$

and $$u_{11}'' - u_{11}'\left(M^2 - \frac{iw}{4}\right)u_{11} = -\frac{iwG}{4}\theta_{11} \quad (3.5)$$

$$\theta_{01}'' + R_c\theta_{02}'' + P\left(\theta_{01}' + R_c\theta_{02}'\right)$$

$$= -PE\left[u_{01}'^2 + R_c^2 u_{02}'^2 + 2u_{01}' R_c u_{02}' + R_c\left(u_{01}' + R_c u_{02}'\right)\left(u_{01}'' + R_c u_{02}''\right)\right]$$

$$= -PE\left[u_{01}'^2 + R_c^2 u_{02}'^2 + 2R_c u_{01}' u_{02}' + R_c\left(u_{01}' u_{01}'' + R_c u_{02}' u_{01}'' + R_c u_{01}' u_{02}'' + R_c^2 u_{02}' u_{02}''\right)\right]$$

$$= -PE\left[u_{01}'^2 + R_c^2 u_{02}'^2 + 2R_c u_{01}' u_{02}' + R_c u_{01}' u_{01}'' + R_c u_{01}'' u_{02}' + R_c u_{01}' u_{02}'' + R_c^2 u_{02}' u_{02}''\right]$$

or $$\theta_{01}'' + P\theta_{01}' = -PE\left[u_{01}'^2\right] \quad (3.6)$$

and $$\theta_{02}'' + P\theta_{02}' = -PE\left[2u_{01}' u_{02}' + u_{01}' u_{01}'' + u_{01}'' u_{02}' + u_{01}' u_{02}''\right] \quad (3.7)$$

$$\theta_{11}'' + R_c\theta_{12}'' + P\left(\theta_{11}'' + R_c\theta_{12}''\right) - \frac{iwp}{4}\left(\theta_{11}'' + R_c\theta_{12}''\right)$$

$$= -\,2PE\,\left(u_{01}' + R_c u_{02}'\right)\left(u_{11}' + R_c u_{12}'\right)$$

$$+R_c PE\left[\frac{iw}{4}\left(u_{01}' + R_c u_{02}'\right)\left(u_{11}' + R_c u_{12}'\right)\right.$$

$$\left. -\left(u_{01}' + R_c u_{02}'\right)\left(u_{11}'' + R_c u_{12}''\right) - \left(u_{11}' + R_c u_{02}'\right)\left(u_{01}'' + R_c u_{02}''\right)\right]$$

$$= -\,2PE\,\left(u_{01}' u_{11}' + R_c u_{02}' u_{11}' + R_c u_{01}' u_{12}' + R_c^2 u_{02}' u_{12}'\right)$$

$$+R_c PE\left[\frac{iw}{4}\left(u'_{01}u'_{11}+R_c u'_{02}u'_{11}\right.\right.$$

$$+R_c u'_{01}u'_{12}+R_c^2 u'_{02}u'_{12}-u'_{01}u''_{11}-R_c u'_{02}u''_{11}-R_c u'_{01}u''_{12}$$

$$\left.\left.-R_c^2 u'_{02}u''_{12}-u'_{11}u''_{01}-R_c u'_{12}u''_{01}-R_c u'_{11}u''_{02}-R_c^2 u'_{12}u''_{02}\right)\right]$$

or
$$\theta''_{11}+P\theta'_{11}-\frac{iwp}{4}\theta_{11}$$

$$=-2PE\left(u'_{01}u'_{11}\right) \tag{3.8}$$

and

$$\theta''_{12}+P\theta'_{12}-\frac{iwp}{4}\theta_{12}$$

$$=-2PE\left(u'_{02}u'_{11}+u'_{01}u'_{12}\right)$$

$$+PE\left[\frac{iw}{4}\left(u'_{01}u'_{11}-u'_{01}u''_{11}-u'_{11}u''_{01}\right)\right] \tag{3.9}$$

With boundary conditions

$$\left.\begin{aligned} &u_{01}=u_{02}=0,\ \theta_{01}=1,\ \theta_{02}=0,\ u_{11}=u_{12}=0 \text{ at } y=0\\ &u_{01}=u_{11}=1,\ \theta_{11}=\theta_{12}=0,\ u_{02}=u_{12}=0,\ \theta_{01}=\theta_{02}=0 \text{ at } y\to\infty \end{aligned}\right\} \tag{3.10}$$

We still have coupled non-linear equations (3.2) to (3.9). These are again linearised by using

$$\left.\begin{aligned} u_{01}&=u_{011}+Eu_{012}\\ u_{02}&=u_{021}+Eu_{022}\\ u_{11}&=u_{111}+Eu_{112}\\ u_{12}&=u_{121}+Eu_{122}\\ \theta_{01}&=\theta_{011}+E\theta_{012}\\ \theta_{02}&=\theta_{021}+E\theta_{022}\\ \theta_{11}&=\theta_{111}+E\theta_{112}\\ \theta_{12}&=\theta_{121}+E\theta_{122} \end{aligned}\right\} \tag{3.11}$$

as E is very very small for incompressible fluids, i.e. E<<1.

The corresponding boundary conditions are

$$\left.\begin{aligned} &u_{01}=1,\ u_{02}=0,\\ &u_{11}=1,\ u_{12}=0,\\ &\theta_{01}=0,\ \theta_{02}=0, \end{aligned}\right\} \quad \text{as } y\to\infty$$

$\theta_{11} = 0,\ \theta_{12} = 0,$

and

$$\left.\begin{array}{l} u_{01} = u_{02} = 0; \\ u_{11} = u_{12} = 0; \\ \theta_{01} = 1,\ \theta_{02} = 0; \\ \theta_{11} = \theta_{12} = 0 \end{array}\right\} \quad \text{at } y = 0$$

$$u_{01}''' + u_{02}'' + u_{02}' + M^2 u_{02} = -G\theta_{02}$$

$$u_{011}''' + Eu_{012}''' + u_{021}'' + Eu_{022}'' + u_{021}' + Eu_{022}' + M^2(u_{021} + Eu_{022}) = -G(\theta_{021} + E\theta_{022})$$

or $$u_{011}''' + u_{021}'' + u_{021}' + M^2 u_{021} = -G\theta_{021}, \tag{3.12}$$

and $$u_{012}''' + u_{022}'' + u_{022}' + M^2 u_{022} = -G\theta_{022} \tag{3.13}$$

Again we have

$$u_{01}'' + u_{01}' + M^2 u_{01} = -G\theta_{01}$$

$$u_{011}'' + Eu_{012}'' + u_{011}' + Eu_{012}' + M^2(u_{011} + Eu_{012}) = -G(\theta_{011} + E\theta_{012})$$

or $$u_{011}'' + u_{011}' + M^2 u_{011} = -G\theta_{011}, \tag{3.14}$$

and $$u_{012}'' + u_{012}' + M^2 u_{012} = -G\theta_{012}, \tag{3.15}$$

Again we have

$$u_{11}''' - \frac{iw}{4}u_{11}'' + u_{12}'' + u_{12}' + \left(M^2 + \frac{iw}{4}\right)u_{12} = -\frac{iwG}{4}\theta_{12}$$

$$u_{111}''' + Eu_{112}''' - \frac{iw}{4}(u_{11}'' + Eu_{12}'') + u_{121}'' + Eu_{122}''$$

$$+ u_{121}' + Eu_{122}' + \left(M^2 - \frac{iw}{4}\right)(u_{121} + Eu_{122}) = -\frac{iwG}{4}(\theta_{121} + E\theta_{122})$$

or, $$u_{111}''' - \frac{iw}{4}u_{111}'' + u_{121}'' + u_{121}' + \left(M^2 - \frac{iw}{4}\right)u_{121} = -\frac{iwG}{4}\theta_{121}, \tag{3.16}$$

and $$u_{112}''' - \frac{iw}{4}u_{112}'' + u_{122}'' + u_{122}' + \left(M^2 - \frac{iw}{4}\right)u_{122} = -\frac{iwG}{4}\theta_{122}, \tag{3.17}$$

$$u_{11}'' + u_{11}' + \left(M^2 - \frac{iw}{4}\right)u_{11} = -\frac{iwG}{4}\theta_{11}$$

$$u_{111}'' + Eu_{112}'' + u_{111}' + Eu_{112}' + \left(M^2 - \frac{iw}{4}\right)(u_{111} + Eu_{112})$$

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$$= -\frac{iwG}{4}\left(\theta_{111} + E\theta_{112}\right)$$

or $$u''_{111} + u'_{111} + \left(M^2 - \frac{iw}{4}\right)u_{111} = -\frac{iwG}{4}\theta_{111} \quad (3.18)$$

and $$u''_{112} + u'_{111} + \left(M^2 - \frac{iw}{4}\right)u_{112} = -\frac{iwG}{4}\theta_{112}, \quad (3.19)$$

Again we have

$$\theta''_{01} + p\theta'_{01} = -PE[u'_{01}]^2$$

$$\theta''_{011} + E\theta''_{012} + P\left(\theta'_{011} + E\theta'_{012}\right)$$

$$= -PE[u'_{011} + Eu'_{012}]^2$$

$$= -PE\left[u'^2_{011} + E^2 u'^2_{012} + 2Eu'_{011}u'_{012}\right]$$

$$= -PEu'^2_{011} - PE^2 u'^2_{012} - 2PE^2 u'_{011}u'_{012}$$

or, $$\theta''_{011} + P\theta'_{011} = 0, \quad (3.20)$$

and $$\theta''_{012} + P\theta'_{012} = -P\left(u'_{011}\right)^2 \quad (3.21)$$

Further we have

$$\theta''_{02} + P\theta'_{02} = -PE\left[2u'_{01}u'_{02} + u'_{01}u''_{01} + u''_{01}u'_{02} + u'_{01}u''_{02}\right]$$

or $$\theta''_{021} + E\theta''_{022} + P\left(\theta'_{021} + E\theta'_{022}\right)$$

$$= -PE\left[\left(2u'_{011} + Eu'_{012}\right)\left(u'_{021} + Eu'_{022}\right)\right.$$

$$\left(u'_{011} + Eu'_{012}\right)\left(u''_{011} + Eu''_{012}\right) + \left(u''_{011} + Eu''_{012}\right)\left(u'_{011} + Eu'_{022}\right)$$

$$\left. + \left(u'_{011} + Eu'_{012}\right)\left(u''_{021} + Eu''_{022}\right)\right]$$

or $$\theta''_{021} + P\theta'_{021} = 0, \quad (3.22)$$

and $$\theta''_{022} + P\theta'_{022} = -P\left[2u'_{011}u'_{021} + u'_{011}u''_{011} + u''_{011}u'_{021} + u'_{011}u''_{021}\right] \quad (3.23)$$

Further we have

$$\theta''_{11} + P\theta'_{11} - \frac{iwp}{4}\theta_{11} = -2PE\left(u'_{01}u'_{11}\right)$$

or $$\theta''_{111} + E\theta''_{112} + P\left(\theta'_{111} + E\theta'_{112}\right) - \frac{iwp}{4}\left(\theta_{111} + E\theta_{112}\right)$$

$$= -2PE\left[\left(u'_{011} + Eu'_{012}\right)\left(u'_{111} + Eu'_{112}\right)\right]$$

or $$\theta''_{111} + P\theta'_{111} - \frac{iwp}{4}\theta_{111} = 0, \quad (3.24)$$

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and $$\theta''_{112} + P\theta'_{112} - \frac{iwp}{4}\theta_{112} = -2P[u'_{011}u'_{111}] \tag{3.25}$$

Further we have

$$\theta''_{12} + P\theta'_{12} - \frac{iwp}{4}\theta_{12} = -2PE(u'_{02}u'_{11} + u'_{01}u'_{12}) + PE \left[\frac{iw}{4}(u'_{01}u'_{11} - u'_{01}u''_{11} - u'_{11}u''_{01})\right]$$

or $$\theta''_{121} + E\theta''_{122} + P(\theta'_{121} + E\theta'_{122}) - \frac{iwp}{4}(\theta_{121} + E\theta_{122})$$

$$= -2PE[(u'_{021} + Eu'_{022})(u'_{111} + Eu'_{112} + Eu'_{012})(u'_{121} + Eu'_{122})]$$

$$+ PE\left(\frac{iw}{4}\right)[(u'_{011} + Eu'_{012})(u'_{111} + Eu'_{112}) - (u'_{011} + Eu'_{012})(u''_{011} + Eu''_{012})$$

$$- (u'_{111} + Eu'_{112})(u''_{011} + Eu''_{012})]$$

or $$\theta''_{121} + P\theta'_{121} - \frac{iwp}{4}\theta_{121} = 0 \tag{3.26}$$

and

$$\theta''_{122} + P\theta'_{122} - \frac{iwp}{4}\theta_{122} = -2P(u'_{021}u'_{111} + u'_{011}u'_{121}) + \frac{iwp}{4}(u'_{011}u'_{111} - u'_{011}u''_{111} - u'_{111}u''_{011}) \tag{3.27}$$

With the boundary conditions

$$\left.\begin{aligned} u_{011} &= u_{012} = 0, u_{021} = u_{022} = 0; u_{111} = 0 = u_{112}, u_{121} = 0 = u_{122}, \theta_{011} \\ &= 1, \theta_{012} = 0, \theta_{111} = \theta_{112} = \theta_{121} = 0 \text{ at } y = 0 \\ u_{011} &= 1, u_{012} = 0, u_{021} = u_{022} = 0, u_{111} = 1, u_{112} = 0, u_{121} = u_{122} = 0; \theta_{111} \\ &= \theta_{112} = \theta_{121} = \theta_{122} = 0, \theta_{011} = \theta_{012} = \theta_{021} = \theta_{022} = 0 \text{ at } y \to \infty \end{aligned}\right\} \tag{3.28}$$

Solving the above equations, we obtain expressions for mean velocity, transient velocity, mean temperature, transient temperature, skin-friction and the rate of heat transfer as follows.

Expressions for mean velocity:

$$u_{01} = u_{011} + Eu_{012}$$

$$= [1 + Ae^{-\alpha y} - e^{-(\alpha-\beta)y} - Ae^{-(\alpha+\rho-\beta)y}]$$

$$+ E\,[B_9 e^{-\alpha y} + B_2 e^{-\rho y} + B_3\, e^{-2\alpha y} + B_4\, e^{-2A_2 y}$$

$$+ B_5\, e^{-2A_3 y} + B_6\, e^{-2A_4 y} + b_7\, e^{-2A_5 y} + B_8\, e^{-2A_6 y}\,] \qquad (3.29)$$

$$u_{02} = u_{021} + E u_{022}$$

$$= [A_{14}\, e^{-\alpha y} - B_{11}\, e^{-(\alpha-\beta)y} + A_{12}\, e^{-(\alpha+\rho-\beta)y}]$$

$$+ E\,[B_{43} e^{-\alpha y} + B_{28} e^{-\rho y} + B_{29}\, e^{-2\alpha y} + B_{30}\, e^{-(2\alpha-\beta)y}$$

$$+ B_{31}\, e^{-(2\alpha+p-\beta)y} + B_{32}\, e^{-(2\alpha+p-2\beta)y} + B_{33}\, e^{-2(\alpha-\beta)y}$$

$$+ B_{34}\, e^{-2(\alpha+p-\beta)y} - B_{35}\, e^{-\beta y} - B_{36}\, e^{-py} - B_{37}\, e^{-2\alpha y}$$

$$+ B_{38}\, e^{-2A_2 y} - B_{39}\, e^{-2A_3 y} - B_{40}\, e^{-2A_4 y} - B_{41}\, e^{-A_5 y} - B_{42} e^{-A_6 y}\,] \qquad (3.30)$$

Mean Velocity:

$$U_0 = u_{01} + R_c\, u_{02},$$

(3.31)

Transient Velocity:

$$U_{11} = p_1 + up_2 + E\,[(a_{31} + ia_{32})\,(p_5 - ip_6) + (a_{17} + ia_{18})\,p_3 - ip_4)$$

$$+ (a_{21} - ia_{22})\,(e^{-a_1 y}.e^{-ia_2 y}) + (a_{25} - ia_{26})\,(e^{-a_1 y}.e^{+\beta y})$$

$$+ (a_{29} - ia_{30})\,(e^{-a_1 y}.e^{+ia_2 y}.e^{-py}.e^{+\beta y})]$$

$$= P_1 + iP_2 + E\,[(a_{31}P_5 + a_{32}P_6) + i\,(a_{32}P_5 - a_{31}P_6) + (a_{17}P_3 + a_{18}P_4)$$

$$+ i\,(a_{18}P_3^{-6} + (A_{21}\, e^{-a_1 y}\, Cos\, a_2 y) + i\,(a_{21}\, e^{-a_1 y}\, Sin\, a_2 y) - i\,(a_{22}\, e^{-a_1 y}\, Cos$$

$$+ (A_{22}\, e^{-a_1 y}\, Sin\, a_2 y) + e^{+(\beta-a_1)y}\,[(a_{25} - Cos\, a_2 y + a_{26}\, Sin\, a_2\, y$$

$$- i\,(a_{26}\, Cos\, a_2 y + a_{25}\, Sin\, a_2 y)] + e^{(\beta-a_1-p)y}\,[(a_{29}\, Cos\, a_2 y + a_{30}$$

$$+ i\,(a_{29}\, Sin\, a_2 y - a_{30}\, Cos\, a_2 y$$

$$U_{11} = p_1 + E\,[a_{31}\,P_5 + a_{32}\,P_6 + a_{17}\,P_3 + a_{18}\,P_4 + a_{21}\, e^{-a_1 y}\, Cos\, a_2 y$$

$$+ a_{22}\, e^{-a_1 y}\, Sin\, a_2 y + e^{(\beta-a_1)y}\,(a_{25}\, Cos\, a_2 y + a_{26}\, Sin\, a_2 y)$$

$$+ e^{(\beta-a_1-p)y}\,(a_{29}\, Cos\, a_2 y + a_{30}\, Sin\, a_2 y)]$$

$$+ i\,[P_2 + E\,\{a_{32}P_5 - a_{31}P_6 + a_{18}P_3 - a_{17}P_4 + a_{21}\, e^{-a_1 y}\, Sin\, a_2 y$$

$$- (a_{22}\, e^{-a_1 y}\, Cos\, a_2 y) - (a_{26}\, Cos\, a_2 y + a_{25}\, Sin\, a_2 y)$$

$$+ (a_{29}\, Sin\, a_2 y - a_{30}\, Cos\, a_2 y)\}], \qquad (3.32)$$

$$Q_r = P_1 + E\,[P_5 a_{31} + P_6\, a_{32} + P_3 a_{17} + P_4 a_{18} + (a_{21}\, Cos\, a_2 y + a_{22}\, Sin\, x\, e^{-a_1 y}$$

$$+ (a_{25} \, Cos \, a_2y + a_{26} \, Sin \, a_2y) \; e^{-(a_1\beta)y}$$
$$+ (a_{29} \, Cos \, a_2y + a_{30} \, Sin \, a_2y) \; e^{-(a_1y+p-\beta)y}] \quad (3.33)$$

$$Q_i = P_2 + E \, [P_5 \, a_{32} - P_6a_{31} + P_3a_{18} - P_4a_{17} + (a_{21} \sin a_2y - a_{22} \, Cos \, x \, e^{-a_1y}$$
$$+ (a_{25} \, Sin \, a_2y - a_{26} \, Sin \, a_2y) \; e^{-(a_1+p-\beta)y}] \quad (3.34)$$

$$u_{11} = Q_r + i \, Q_i, \quad (3.35)$$

$$P_r = a_{35} \, (P_1\text{-}P_5) - a_{36} \, (P_2 + P_6) + E \, [A_{151}P_3 + a_{152}P_4 + a_{101}P_3$$
$$- a_{102}P_4 - e^{-\alpha y} \, (a_{105} \, P_1 - a_{106}P_2) - e^{-(\alpha-\beta)y} \, (a_{109}P_1 - a_{110}P_2)$$
$$- e^{-(\alpha+p-\beta)y} \, (a_{119} \, P_1 - a_{120} \, P_2)] \quad (3.36)$$

$$P_i = a_{36} \, (P_1\text{-}P_5) + a_{35} \, (P_2+P_6) + E \, [A_{152} \, P_3 - a_{151}P_4 - (a_{102}P_3P_4)$$
$$- e^{-\alpha y} \, (a_{106} \, P_1 + a_{105}P_2) - e^{-(\alpha-\beta)y} \, (a_{110}P_1 + a_{109}P_2)$$
$$- e^{-(\alpha+p-\beta)y} \, (a_{114}P_1 + a_{113}P_2) - e^{-\alpha y} \, (a_{118}P_5 - a_{117}P_6)$$
$$- e^{-(\alpha+p-\beta)y} \, (a_{120}P_1 + a_{119}P_2)] \quad (3.37)$$

$$U_{12} = P_r + iPi, \quad (3.38)$$
$$U_1 = u_{11} + R_c \, u_{12}$$
$$= Q_r + iQ_i + R_c \, (P_r + iPi) = (Q_r + R_cP_r) + i \, (Q_i + R_cP_i), \quad (3.39)$$
$$u_1 = N_r + i \, Ni, \quad (3.40)$$
$$|u_1| = \sqrt{N_r^2 + N_i^2}, \quad (3.41)$$

Phase of transient velocity = $tan \; \theta = \frac{Q_i + R_cP_i}{Q_r + RP_r} = \left(\frac{Ni}{Nr}\right)$

$$\theta = \tan^{-1}\left(\frac{Ni}{Nr}\right) \quad (3.42)$$

Velocity = $u_0 + \varepsilon \, e^{iwt} \, u_1$

$$= u_0 + \varepsilon \, (Cos \, wt + i \, Sin \, wt) \, (N_r + i \, Ni)$$
$$= u_0 + \varepsilon \, (N_r \, Cos \, wt - N_i \, Sin \, wt) + i\varepsilon \, (N_r \, Sin \, wt + N_i \, Cos \, wt) \quad (3.43)$$

Taking real part we have

$$u = u_0 + \varepsilon \, (N_r \, Cos \, wt - Ni \, Sin \, wt), \quad (3.44)$$

Temperature:

$$\theta_{01} = e^{-py} + E\,[B_1 e^{-py} + A_7 e^{-2\alpha y} + A_8\, e^{-2A_2 y} + A_9\, e^{-2A_3 y} + A_{10}\, e^{-2A_4 y} - A_{11}\, e^{-2A_5 y} - A_{12}\, e^{-2A_6 y}\,] \quad (3.45)$$

$$\theta_{02} = E\,[B_{27} e^{-py} + B_{21}\, e^{-2\alpha y} + B_{22}\, e^{-(2\alpha-\beta)y} + B_{23}\, e^{-(2\alpha+p-\beta)y} - B_{24}\, e^{-(2\alpha+p-\beta)y} - B_{25}\, e^{-2(\alpha-\beta)y} + B_{26}\, e^{-2(\alpha+p-\beta)y}\,] \quad (3.46)$$

Mean Temperature:

$$\theta_0 = \theta_{01} + R_c\,\theta_{02}, \quad (3.47)$$

Transient temperature:

$$Q_r' = E\,[-(a_9 + a_{11} + a_{13})\,P_3 + (-a_{10} - a_{12} + a_{14})\,P_4 + e^{-\alpha y}(a_9 P_1 - a_{10} P_2) + e^{-(\alpha-\beta)y}(a_{11} P_1 - a_{12} P_2) + e^{-(\alpha+P-\beta)y}(a_{13} P_1 + a_{14} P_2)], \quad (3.48)$$

$$Q_i' = E\,[-(a_{10} - a_{12} + a_{14})\,P_3 + (a_9 + a_{11} + a_{13})\,P_4 + (a_{10} P_1 + a_9 P_2)\,e^{-\alpha y} + e^{-(\alpha-\beta)y}(a_{12} P_1 - a_{11} P_2) + e^{-(\alpha+P-\beta)y}(a_{13} P_1 - a_{14} P_1)], \quad (3.49)$$

$$P_r' = a_{35}\,(P_1 - P_5) - a_{36}\,(P_2 + P_6) + E\,[(a_{97} P_3 + a_{98} P_4) + e^{-\alpha y}(a_{75} P_1 - a_{76} P_2) + e^{-(\alpha-\beta)y}(a_{79} P_1 - a_{80} P_2) + e^{-(\alpha+P-\beta)y}(a_{83} P_1 - a_{84}\,P_2) + e^{-\alpha y}(a_{87} P_5 + a_{88} P_6) + e^{-(\alpha-\beta)y}(a_{91} P_5 + a_{92} P_6) + e^{-(\alpha+P-\beta)y}(a_{95} P_5 + a_{96} P_6)] \quad (3.50)$$

$$P_r' = a_{36}\,(P_1 - P_5) - a_{35}\,(P_2 + P_6) + E\,[(a_{98} P_3 - a_{97} P_4) + e^{-\alpha y}(a_{76} P_1 - a_{75} P_2) + e^{-(\alpha-\beta)y}(a_{80} P_1 - a_{79} P_2) + e^{-(\alpha+P-\beta)y}(a_{84} P_1 - a_{83}\,P_2) + e^{-\alpha y}(a_{88} P_5 - a_{87} P_6) + e^{-(\alpha-\beta)y}(a_{92} P_5 + a_{91} P_6) + e^{-(\alpha+P-\beta)y}(a_{96} P_5 - a_{95} P_6)] \quad (3.51)$$

$$\theta_1 = (Q_r' + iQ_i') + R_c\,(P_r' + iP_i') = (Q_r' + R_c P_r') + i(Q_i' + R_c P_i') = N_r' + iN_i' \quad (3.52)$$

Where $N_r' = Q_r' + R_c P_r'$

$N_i' = Q_i' + R_c P_i'$

Taking only real part, we have,

$$\theta = \theta_0 + \varepsilon\,(N_r' Coswt - N_i' Sinwt), \quad (3.53)$$

Mean Skin – friction (τ_{xy}^m):

$$\left(\tau_{xy}^{m}\right) = [-\alpha A + (\alpha\text{-}\beta) + A\,(\alpha+P-\beta)] + E\,[-\beta B_9 - PB_2 - 2\alpha B_3 - 2A_2B_4$$

$$- 2\,A_3B_5 - 2A_4\,B_6 - A_5B_7 - A_6B_8] + R_c\,[-\alpha B_{14} - (\alpha-\beta)B_{11} - (\alpha+P-\beta)$$

$$+ E\,\{-\alpha B_{43} - PB_{28} - 2\alpha B_{29} - (2\alpha-\beta)B_{30} - (2\alpha+P-\beta)B_{31} - (2\alpha+P-2\beta)$$

$$-2\,(\alpha+\beta)B_{33} - 2\,(\alpha+P-\beta)B_{54} + \beta B_{35} + PB_{36} + 2\alpha B_{37} + 2A_2B_{38} + 2A_3B_{39}$$

$$+2A_4B_{40} + A_5B_{41} + A_6B_{42}\} + \{-\alpha A + (\alpha-\beta) + A\,(\alpha+P-\beta) + E$$

$$\{-\beta B_9 - PB_2 - 2\alpha B_3 - 2A_2B_4 - 2A_3B_5 - 2A_4B_6 - A_5B_7 - A_6B_8\}] \qquad (3.54)$$

$$\gamma_1 = \quad \tau_{xy}^{m}$$

$$M_r' = -2M^2 + E\,[(2M^2-1)a_{31} + \frac{w}{4}a_{32} - \frac{1}{2}(P^2+P)a_{17} + \frac{wp}{4}\,a_{18}$$

$$-(\alpha+2M^2)\,a_{21} + \frac{w}{4}a_{22} - (\alpha+2M^2-\beta)a_{25} + \frac{1}{4}\,wa_{26}$$

$$-(\alpha+p-\beta+2M^2)\,a_{29} + \frac{w}{4}a_{30}] + R_c\,[(1\text{-}4M^2)\,a_{35}$$

$$-\frac{w}{2}a_{36}] + ER_c\,[(2M^2-1)\,a_{151} - \frac{w}{4}a_{152} - \frac{1}{2}(P^2+P)a_{101}$$

$$-\frac{1}{4}\,wpa_{102} + (\alpha+2M^2)\,a_{105} + \frac{1}{4}\,wa_{106} + (\alpha+2M^2+P-\beta)\,a_2$$

$$-\frac{w}{4}a_{30} + (\alpha+2M^2+p-\beta)a_{113} + \frac{1}{4}w\,a_{114} + (\alpha+1\text{-}2M^2)\,a_{117}$$

$$-\frac{1}{4}\,w\,a_{118} + (\alpha+2M^2+p-\beta)\,a_{119} + \frac{1}{4}\,wa_{120}$$

$$+ (1-2M^2)\,a_{123} - \frac{1}{4}w\,a_{124} + \frac{1}{2}\,(P^2+P)\,a_{127}$$

$$-\frac{1}{2}w\,p\,a_{128} + (\alpha+2M^2)\,a_{131} - \frac{1}{4}w\,a_{132}$$

$$-(\alpha+2M^2-\beta)\,a_{135} + \frac{w}{4}\,a_{136} - (\alpha+p+2M^2-\beta)\,a_{139}$$

$$+\frac{1}{4}\,w\,a_{140} - (1-2M^2)\,a_{141}\ \frac{1}{4}\,wa_{142} - \frac{1}{2}(P^2+P)\,a_{143}$$

$$+\frac{1}{4}\,w\,pa_{144} - (\alpha+2M^2)\,a_{145} - \frac{1}{4}\,w\,a_{146} - (\alpha+2M^2-\beta)\,a_{147}$$

$$-\frac{1}{4}\,w\,a_{148} - (\alpha+P+2M^2-\beta)\,a_{149} - \frac{1}{4}\,w\,a_{150}], \qquad (3.55)$$

$$M_i' = \frac{w}{4} + E[(2M^2-1)\,a_{22} - \frac{1}{4}w\,a_{31} - \frac{1}{4}w\,pa_{17} + \frac{1}{2}\,(P^2+P)a_{18} + (\alpha+2M^2)\,a_{22}$$

$$-\frac{1}{4} w\, a_{21} + \frac{1}{4}\; w\, a_{25} + (\alpha+2M^2-\beta)a_{26} + \frac{1}{4}\; wa_{29} + (\alpha+p+2M^2-\beta)a_{30}]$$

$$+R_c\, [\frac{1}{2} w\, a_{35} + 1-4M^2)a_{36}]+ER_c\, [\frac{1}{4} w\, a_{151} + (2M^2-1)1_{152} + \frac{1}{4} w\, pc$$

$$-\frac{1}{2}\;(P^2+P)\, a_{102} + (\alpha+2M^2)\, a_{106} \;-\frac{1}{4} w\, a_{105} - \frac{w}{4} a_{29}\, (\alpha+p+2M^2-\beta)\, a_3$$

$$-(\alpha+p+2M^2-\beta)\, a_{114} - \frac{1}{4} w\, a_{113} + \frac{1}{4}\; w\, a_{117} + (\alpha+1-2M^2)\, a_{118}$$

$$+ (\alpha+2M^2+P-\beta)\, a_{120} - \frac{1}{4}\; w\, a_{119} + \frac{1}{4} w\, a_{123} + (1-2M^2)\, a_{124}$$

$$-\frac{1}{2} w\, pa_{127} - \frac{1}{2}(P^2+P)a_{128} - \frac{1}{4} w\, a_{131} - (\alpha+2M^2)\, a_{132} - \frac{1}{4} w\, a_{135}$$

$$-(\alpha+2M^2-\beta)a_{136} + \frac{1}{4} w\, a_{139} + (\alpha+P+2M^2-\beta)1_{140} - \frac{1}{4} wa_{141}$$

$$+ (1-2M^2)a_{142} - \frac{1}{4}\; wp\, a_{143} - \frac{1}{2}(P^2+P)\, a_{144} - (\alpha+2M^2)\, a_{146} + \frac{1}{4}\; wa_{145}$$

$$-(\alpha+2M^2-\beta)a_{148} + \frac{1}{4} w\, a_{147} - (\alpha+p+2M^2-\beta)\, a_{150} + \frac{1}{4} w\, a_{149}] \qquad (3.56)$$

Where $\gamma_2 = M_r' + iM_i'$, (3.56a)

$$\gamma_3 = [\alpha^2 A + (\alpha-\beta)^2 + (\alpha+p-\beta)^2 A] + E\, [\beta^2 B_9 + p^2 B_2 + 4\alpha^2 B_3 + 4A_2{}^2 B$$

$$+4\, A_3{}^2 B_5 + 4A_4{}^2 B_6 + A_5{}^2 B_7 + A_6{}^2 B_8] + R_c\, [\alpha^2 B_{14} + (\alpha-\beta)^2\, B_{11}$$

$$+ (\alpha+p-\beta)^2 B_{12}] + R_c E\, [\alpha^2 B_{43} + p^2 B_{28} + 4\alpha^2 B_{29} + (2\alpha-\beta)^2 B_{36}$$

$$+ (2\alpha+p-\beta)^2\, B_{31} + (2\alpha+p-2\beta)^2 B_{32} + 4\, (\alpha-\beta)^2\, B_{33} + 4\, (\alpha+p-\beta)^2 B_{34}$$

$$+ \beta^2 B_{35} + p^2 B_{36} + 4\alpha^2 B_{37} + 4A_2{}^2 B_{38} + 4A_3{}^2 B_{39} + 4A_4{}^2 B_{40}$$

$$+A_5{}^2 B_{41} + A_6{}^2 B_{42}] \qquad (3.57)$$

$$M_r = \gamma_1 + \varepsilon\, M_r'\, Cos\, wt - \frac{1}{4}\, \varepsilon W\, R_c\;\; M_r'\;\, Sin\, wt + R_c \gamma_3$$

$$-\; M_i'\;\, Sin\, wt + \frac{1}{4}\;\, \varepsilon W\, R_c\;\; M_i'\;\, Cos\, wt$$

$$= \gamma_1 + R_c\, (\gamma_3 - \frac{1}{4} \gamma_3) + \varepsilon M_r'\; (Cos\, wt + \frac{1}{4}\;\, wR_c\, Sin\, wt)$$

$$-\; M_i'\; (Sin\, wt - \frac{1}{4}\;\, \varepsilon\, wR_c\, Cos\, wt), \qquad (3.58)$$

$$M_i = \varepsilon M_i'\; (1+\frac{1}{4} w\, R_c\, Sin\, wt) + M_r'\; (sin\, wt - \frac{1}{4}\;\, \varepsilon w\, R_c\, Cos\, wt), \qquad (3.59)$$

$$\tau_{xy} = \sqrt{M_r^2 + M_i^2}\,, \tag{3.60}$$

$$B_r' = E\,[\tfrac{1}{2}(P^2+P)(a_9 + a_{11} + a_{13}) - \tfrac{1}{4}wp\,(a_{10} + a_{12} - a_{14}) - (\alpha+2M^2)a_9 + \tfrac{1}{4}wa_{10} - (\alpha+2M^2-\beta)\,a_{11} + \tfrac{1}{4}wa_{12} - (\alpha+p+2M^2-\beta)a_{13} - \tfrac{1}{4}wa_{14}] + R_c\,[-2M^2a_{35} + \tfrac{1}{4}wa_{36} - (1-2M^2)\,a_{35} + \tfrac{1}{4}wa_{36}] + R_cE\,[-\tfrac{1}{2}(P^2+P)a_{97} + \tfrac{1}{4}wp\,a_{98} - (\alpha+2M^2)\,a_{75} + \tfrac{1}{4}wa_{76} - (\alpha+2M^2-\beta)\,a_{79} + \tfrac{1}{4}wa_{80} - (\alpha+p+2M^2-\beta)\,a_{81} + \tfrac{1}{4}wa_{84} - (\alpha+1-2M^2)\,a_{87} + \tfrac{1}{4}wa_{88} - (\alpha+1-2M^2-\beta)\,a_{91} + \tfrac{1}{4}wa_{92} - (\alpha+p-\beta+1-2M^2)\,a_{95} + \tfrac{1}{4}wa_{96}], \tag{7.3.61}$$

$$B_i' = E\,[\tfrac{1}{4}wp\,(a_9 + a_{11} + a_{13}) + \tfrac{1}{2}(P^2+P)\,(a_{10}+a_{14}) - \tfrac{1}{4}wa_9 - (\alpha+2M^2)\,a_{10} - \tfrac{1}{4}w\,a_{11} - (\alpha+2M^2-\beta)\,a_{12} - \tfrac{1}{4}w\,a_{13} - (\alpha+p-\beta+2M^2)\,a_{14}] + R_c\,[(-\tfrac{1}{4}wa_{35} - 2M^2a_{36} - \tfrac{1}{4}wa_{35} - (1+2M^2)\,a_{36}] + R_cE[-\tfrac{1}{4}wpa_{97} - \tfrac{1}{2}(P^2+P)a_{98} - \tfrac{1}{4}wa_{75} - (\alpha+2M^2)\,a_{76} - \tfrac{1}{4}wa_{79} - (\alpha+2M^2-\beta)a_{80} - \tfrac{1}{4}wa_{83} - (\alpha+2M^2-\beta+p)\,a_{84} + \tfrac{1}{4}wa_{87} - (\alpha+1-2M^2)a_{88} + \tfrac{1}{4}wa_{91} - (\alpha+1-2M^2-\beta)\,a_{92} - \tfrac{1}{4}wa_{95} - (\alpha+p-\beta+1-2M^2)a_{96}] \tag{3.62}$$

$$\gamma_4 = -p + E\,[-pB_1 - 2\alpha A_7 - 2A_2A_8 - 2A_3A_9 - 2A_4A_{10} + A_5A_{11} + A_6A_{12} + R_cE[-pB_{27} - 2\alpha B_{21} - (2\alpha-\beta)\,B_{22} - (2\alpha+p-\beta)B_{23} - (2\alpha+p-\beta)B_2 - 2\,(\alpha-\beta)\,B_{25} - 2\,(\alpha+p-\beta)\,B_{26}, \tag{3.63}$$

$$B_r = \gamma_4 + \varepsilon\left(B_r'Coswt - B_i'Sinwt\right), \tag{3.64}$$

$$B_i = \varepsilon\left[B_i'Coswt + B_r'Sinwt\right], \tag{3.65}$$

$$Nu = \sqrt{B_r^2 + B_i^2}\,, \tag{3.66}$$

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$$tan\ \delta = \frac{B_i}{B_r}, \tag{3.67}$$

When $wt = \pi/2$, $B_r = \gamma_4 - B_i'\varepsilon$, $B_i = \varepsilon B_r'$

$$\begin{aligned} Nu_0 = & -p + E\,[-pB_1 - 2\alpha A_7 - 2A_2 A_8 - 2A_3 A_9 - 2A_4 A_{10} + a_5 A_{11} + A_6 A_1 \\ & + R_c E\,[-pB_{27} - 2\alpha B_{21} - 2\,(\alpha-\beta)B_{22} - (2\alpha+p-\beta)\,B_{23} \\ & - (2\alpha+p-\beta)B_{24} - 2\,(\alpha-\beta(B_{25} - 2\,(\alpha+p-\beta)\,B_{26}] \end{aligned} \tag{3.68}$$

Curve	R_c	G	M	E
I	0.00	5.0	1.0	0.01
II	0.05	5.0	1.0	0.01
III	0.10	5.0	1.0	0.01
IV	0.10	10.0	1.0	0.01
V	0.10	10.0	1.0	0.02
VI	0.10	10.0	2.0	0.02

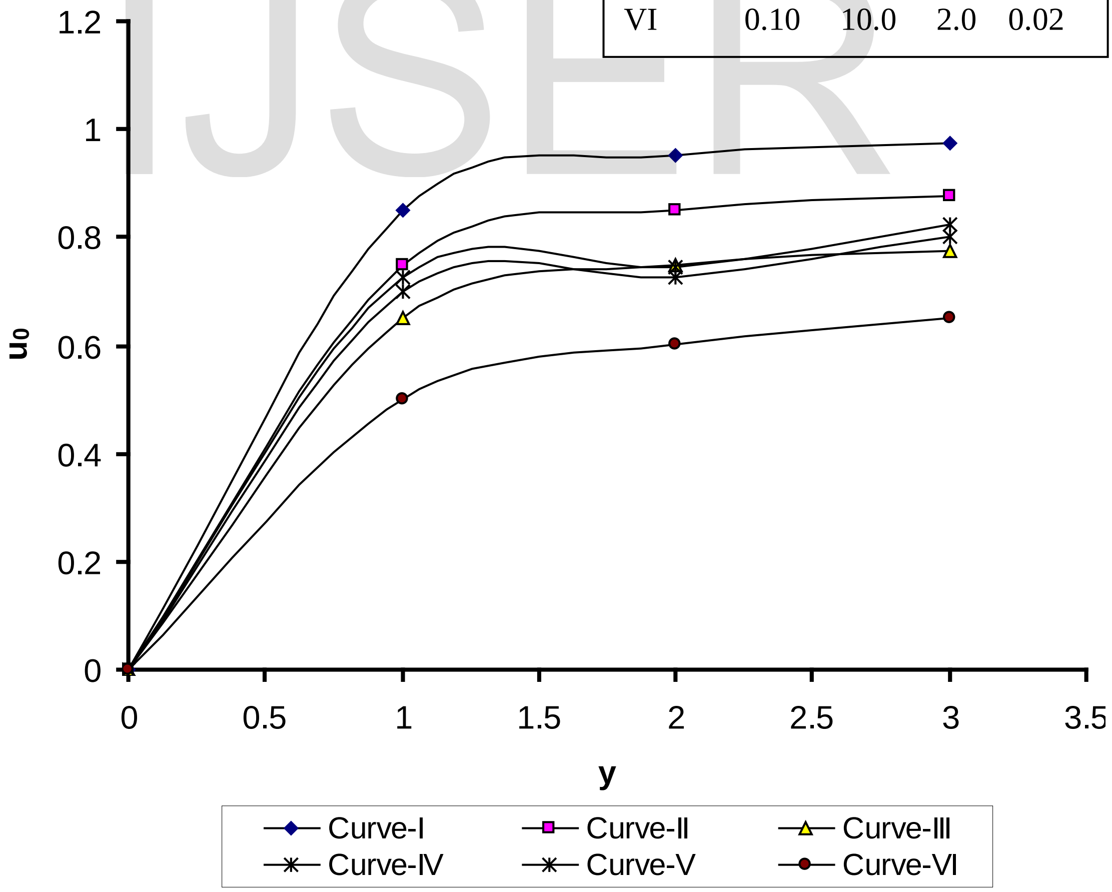

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Fig. 1 : Effects of R_c, M, G and E on the mean velocity u_0 for cooling of the plate.

Curve	R_c	G	M	E
I	0.05	–5.0	1.0	–0.01
II	0.05	–10.0	1.0	–0.01
III	0.05	–10.0	1.0	–0.02
IV	0.05	–10.0	2.0	–0.02
V	0.10	–10.0	2.0	–0.02

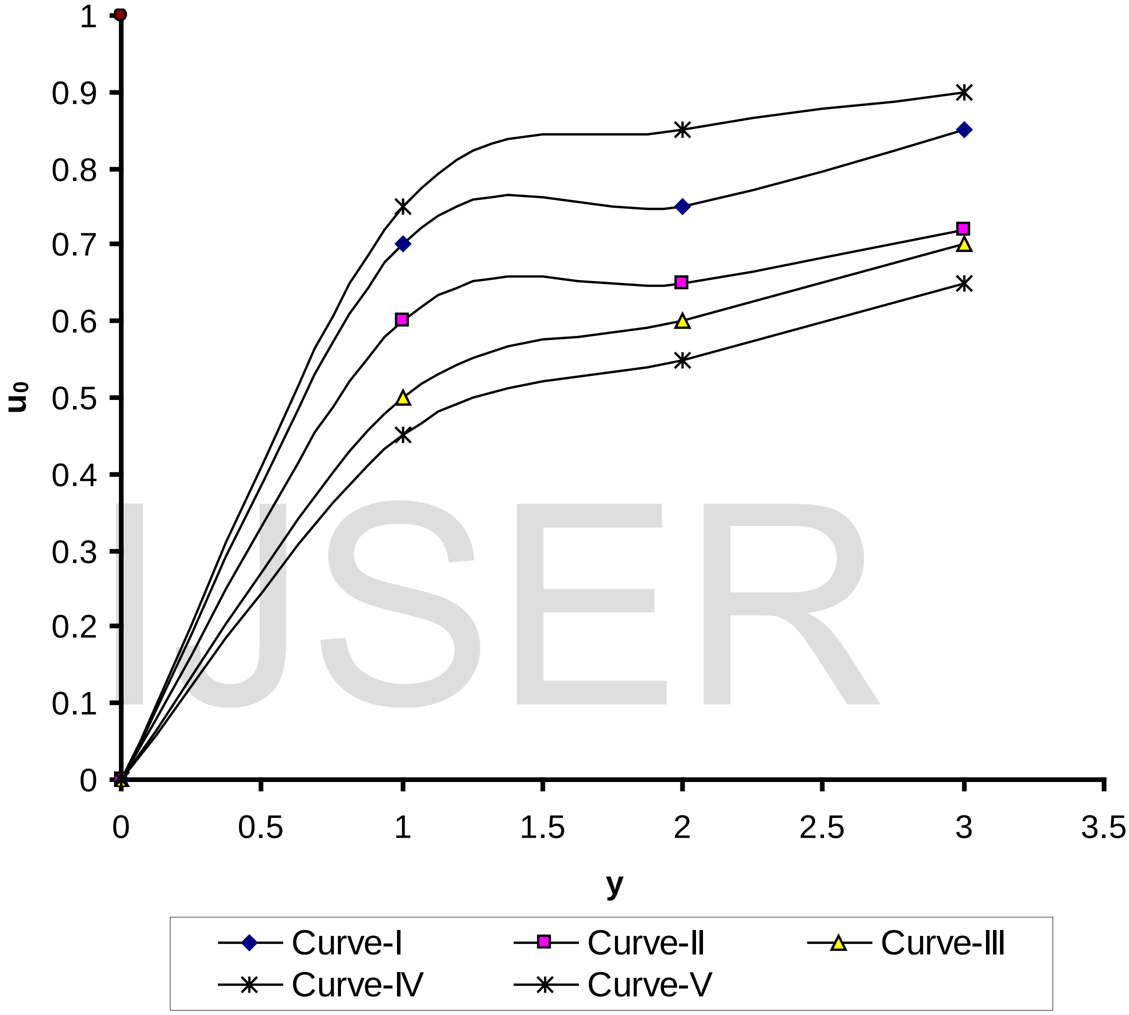

Fig. 2 : Effects of Rc, M, G, E on the mean velocity u_0 for heating of the plate.

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Curve	R_c	G	M	E
I	0.00	1.0	–0.02	–15.0
II	0.05	1.0	–0.02	–15.0
III	0.10	1.0	–0.02	–15.0
IV	0.10	2.0	–0.02	–15.0
V	0.10	3.0	–0.02	–15.0
VI	0.10	3.0	–0.01	–15.0
VII	0.10	3.0	–0.01	–10.0

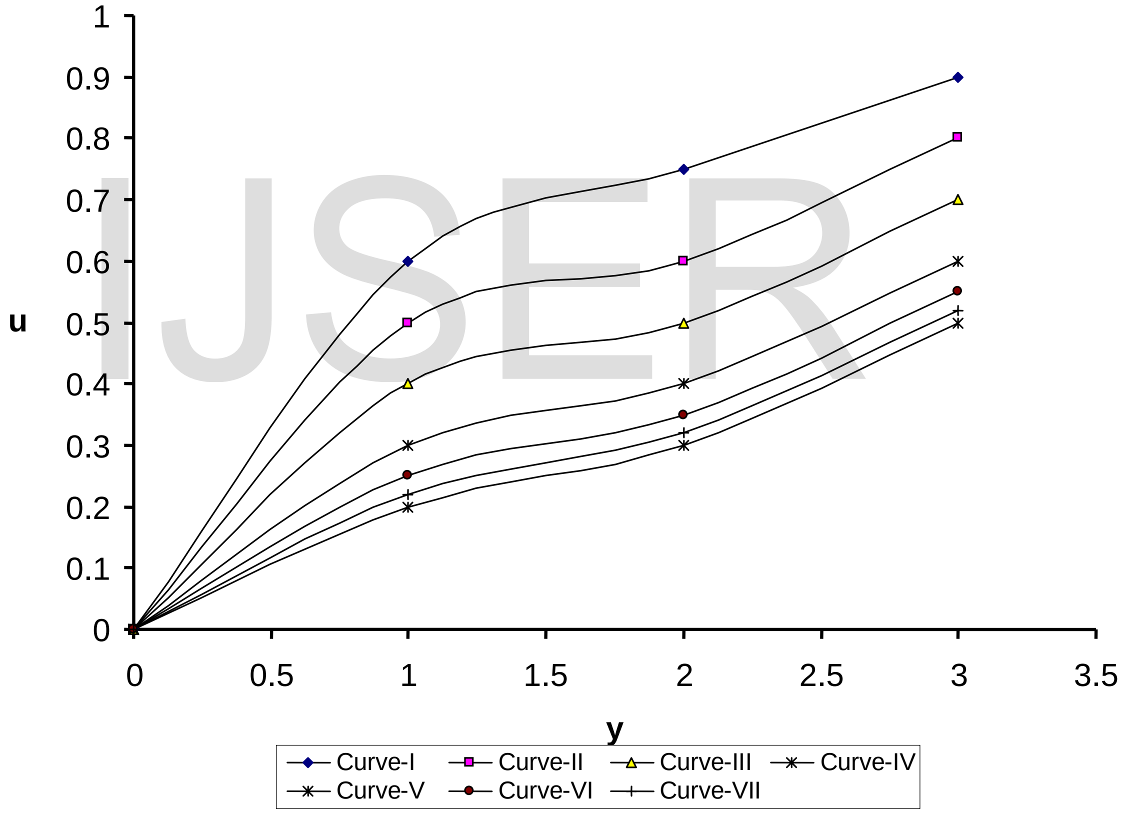

Fig. 3 : Effects of R_c, M, E, G on the transient velocity u for heating the plate where p=5.0, w=4.0, ε=0.002, E<0

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Curve	R_c	G	M	E
I	0.00	1.0	0.01	5.0
II	0.05	1.0	0.01	5.0
III	0.10	1.0	0.01	5.0
IV	0.10	2.0	0.01	5.0
V	0.10	3.0	0.01	5.0
VI	0.10	3.0	0.02	5.0
VII		3.0	0.02	10.0

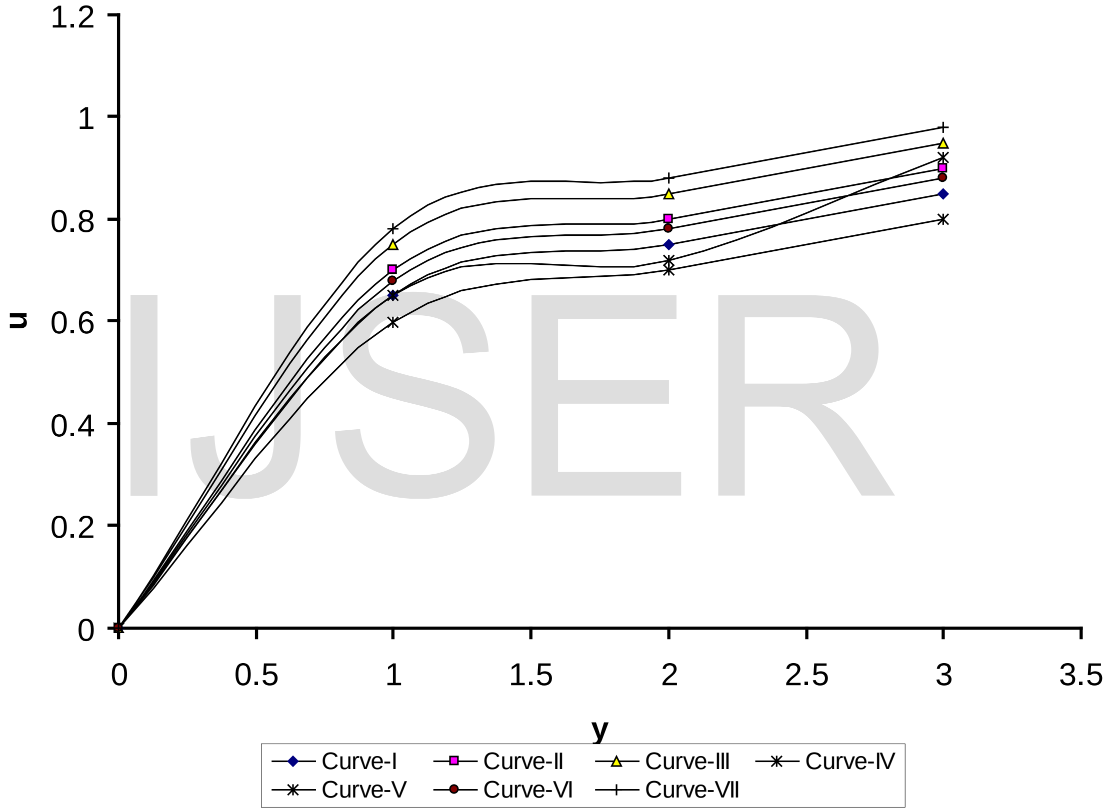

Fig. 4: Effects of R_c, M, E and G on transient velocity u for cooling of the plate where p=5.0, w=4.0, ε=0.002

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Curve	R_c	G	E	P
I	0.00	5	0.01	9
II	0.05	5	0.01	9
III	0.10	5	0.01	9
IV	0.10	5	0.01	16
V	0.10	5	0.02	16

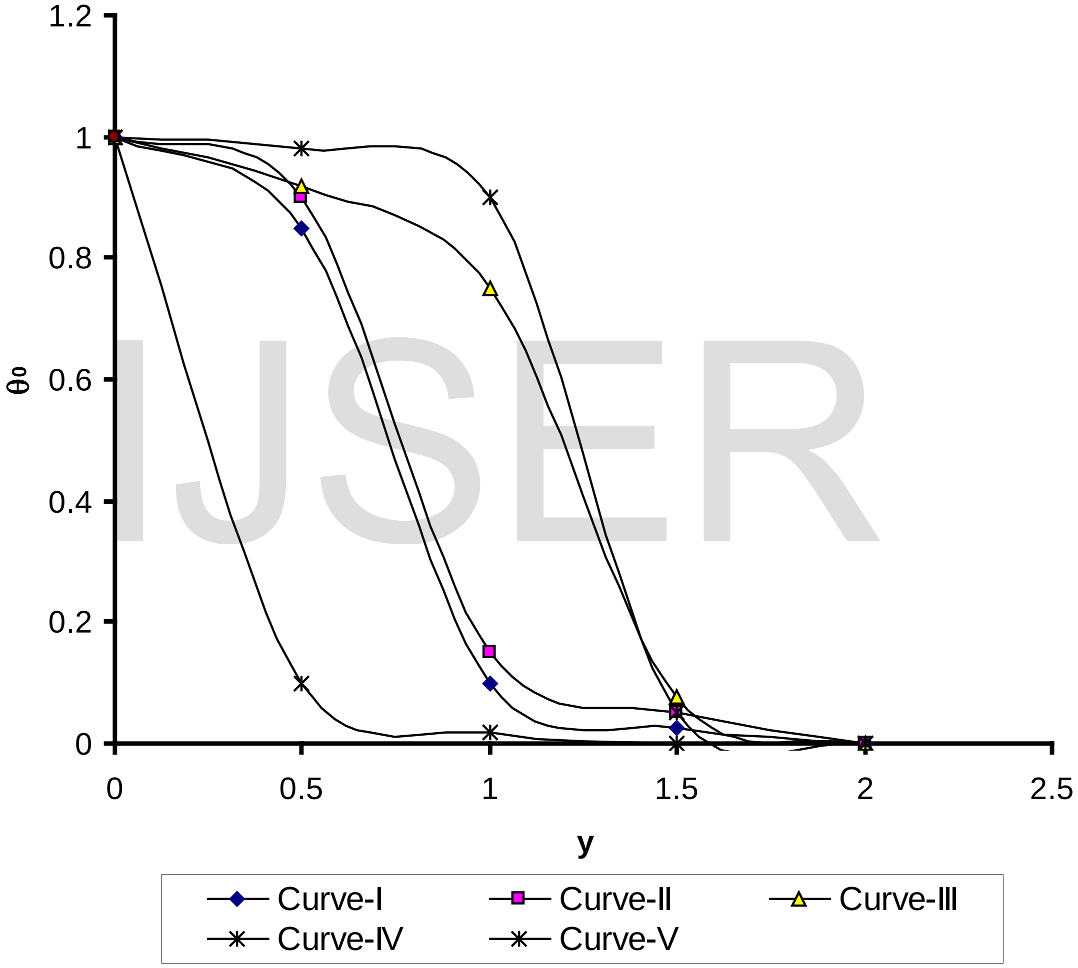

Fig.5 : Effect of R_c, G, E and P on mean temperature θ_0 when M=1.0, w=4.0, ε=0.002

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Curve	R_c	G	E	P
I	0.00	–5	–0.01	9
II	0.05	–5	–0.01	9
III	0.10	–5	–0.01	9
IV	0.10	–5	–0.01	16
V	0.10	–5	–0.02	16

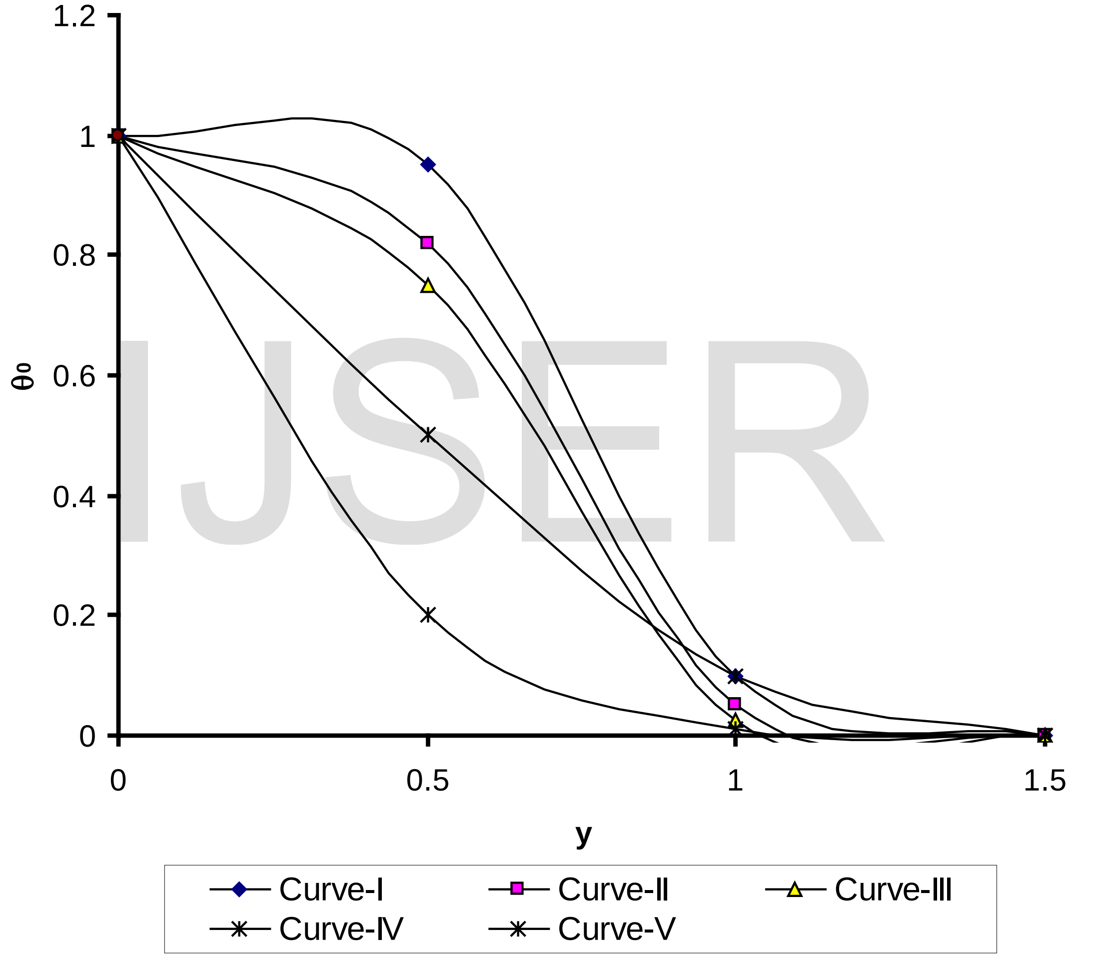

Fig. 6: Effect of R_c, G, P and E on the mean temperature θ_0 when M=1.0, w=4.0, ε=0.002

Curve	R_c	G	E	P	w
I	0.00	5	0.01	9	5
II	0.05	5	0.01	9	5
III	0.10	5	0.01	9	5
IV	0.10	10	0.01	9	5
V	0.10	10	0.02	9	5
VI	0.10	10	0.02	16	5
VII	0.10	10	0.02	16	10

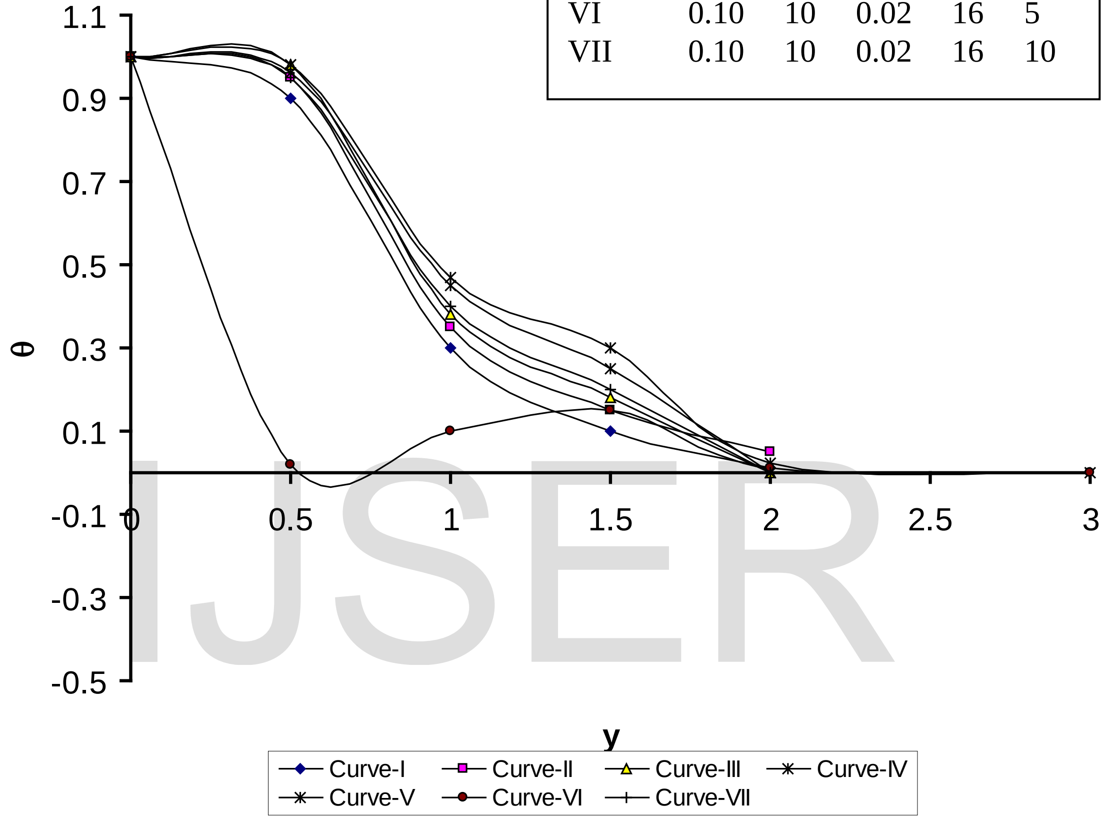

Fig.7 : Effects of R_c, G, E, P & w on the transient temperature θ when $wt = \pi/2$, ε=0.2

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Curve	R_c	G	E	P	w
I	0.00	–5	–0.01	9	5
II	0.05	–5	–0.01	9	5
III	0.10	–5	–0.01	9	5
IV	0.10	–10	–0.01	9	5
V	0.10	–10	–0.02	9	5
VI	0.10	–10	–0.02	16	5
VII	0.10	–10	–0.02	16	10

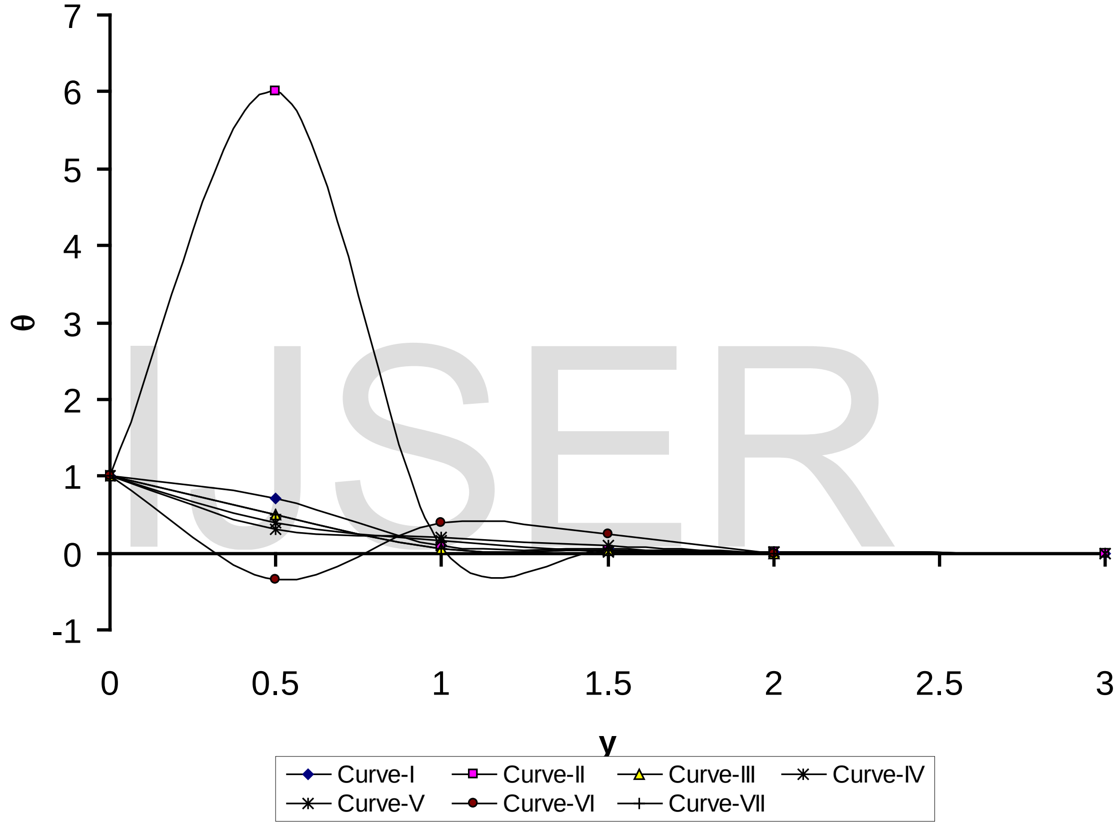

Fig.8 : Effects of R_c, G, E, P and w on the transient temperature θ when $wt = \pi/2$, ε=0.2

4. RESULTS AND DISCUSSIONS:

Free convection effects on the oscillatory magnetohydrodynamic flow of a visco-elastic fluid past an infinite vertical porous plate with constant suction and heat dissipation have been studied with the help of graphs and tables involving various fluid parameters like non-Newtonian parameter R_c, magnetic parameter M, Grashof number G, Eckert number E and Prandtl number P.

Fig.1 depicts the effects of R_c, M, G and E on the mean velocity u_0 for cooling of the plate (G>0). It is observes that the mean velocity decreases with the increase of R_c. Mean velocity rises with the increase of Grashof number G as well as Eckert number E. But, the magnetic field strength reduces the mean velocity u_0.

The effects of R_c, G, M and E on the mean. Velocity u_0 for heating of the plate (G<0) have been shown in the fig.2. It is marked that the fall of G reduces the mean velocity u_0. The same case is observed in case of Eckert number (curve II & III). Further, the increase in magnetic parameter reduces the mean velocity. But, the increase in the value of non-Newtonian parameter R_c enhances the mean velocity u_0.

Fig.3 illustrates the effects of R_c, M, E and G on the transient, velocity u for heating the plate, (G<0) where P=5.0, W=4.0, ε=0.002 and E<0. It is observed that the increase of R_c reduces the transient velocity. Also increase of G decreases the transient velocity. Further, rise in the magnetic field strength decelerates, the transient velocity. The pause effect is marked in case of Eckert number E.

Effects of R_c, M, E and G on transient velocity u for cooling of the plate (G>0) have been shown in the fig.4. From the curve, I, II and III of this figure, it is gleaned that the rise in R_c accelerates the unsteady flow. The increase in Grashof number reduces the flow. As the magnetic field strength rise the transient velocity decreases. But, the increase in the value of Eckert number enhances the unsteady flow.

Fig.5 explains the effects of R_c, G, E and P on mean temperature θ_0 when M=1.0, w=4.0 and ε=0.002. It is marked that the increase in R_c increases the mean temperature. The rise in the Prandtl number P reduces the mean temperature. Further, the increase in Eckert number raises the mean temperature (curve V).

Fig.6 depicts the effects of R_c, G, P and E on the mean temperature θ_0 for heating of the plate (G<0). It is observed that the increases in R_c reduces the mean temperature. Rise in P further decreases the mean temperature. But, the increase in Eckert number enhances the mean temperature.

Effects of R_c, G, E, P and w on the transient temperature θ for cooling of the plate (G>0) have been shows in the fig. 7. As R_c rises, the transient temperature increases. It is observed that the increase in G increases the value of θ. Also, the rise in Eckert number rises the transient temperature θ. Further, increase in the value of Prandtl number reduces the transient frequency enhances the value of θ.

Fig.8 shows the effects of R_c, G, E, P and w on the transient temperature θ for heating of the plate (G<0). The curves I and II of the figure present that the rise in R_c raises the transient temperature upto y=1.0 and then the temperature falls. As the value of G decreases the transient temperature falls. The decrease in the Eckert number decreases the temperature further.

The transient temperature first decreases and then rises with the rise of P. However, the temperature becomes zero at y=2.0. It is noticed that the increase in w increases the value of θ.

The value of mean skin-friction, amplitude of the skin-friction, phase of the skin-friction, mean rate of heat transfer, rate of heat transfer and phase of the rate of heat transfer are entered in the tables I to VI respectively.

Table – I

Value of mean skin-friction

G	E	P/R_c	0	0.05	0.1
–10	–0.01	5	–0.7582	–1.567	–2.8235
–5	–0.01	5	0.0125	–0.3829	–0.8975
–5	–0.01	10	0.5020	0.0526	–0.2958
+5	+0.01	5	1.9567	2.021	3.156
+5	+0.01	10	1.510	2.105	2.546
+5	0.02	10	1.435	1.998	2.365
+10	0.01	5	5.856	3.569	4.755

Table I shows the effects of G, E, P and R_c on the mean skin-friction. It is observed that the rise in R_c reduces the mean skin-friction for heating of the plate. But, a reverse affect is marked for cooling of the plate (G>0). As P rises, the mean skin-friction increases for G<0 and a reverse effect is observed for G>0. The same effect is also marked in case of Eckert number for cooling of the plate. Also, rise in G raises the mean skin-friction.

Table – II

Value of the amplitude of the skin-friction τ_{xy}

G	E	P	w/R_c	0	0.05	0.10
–10	–0.01	5	4	1.356	1.285	0.989
–10	–0.01	5	10	1.735	1.628	1.453
–5	–0.01	5	10	1.878	1.753	1.567
–5	–0.02	5	10	1.885	1.736	1.523
–5	–0.01	10	10	1.896	1.739	1.544
+5	+0.01	5	4	1.362	1.295	0.928
+5	+0.01	5	10	1.898	1.756	1.536
+5	+0.01	10	10	1.886	1.693	1.458
+10	+0.01	5	10	1.998	1.781	1.457
+5	+0.02	5	10	1.889	1.684	1.436

Values of the amplitude of the skin-friction τ_{xy} are entered in the Table II. It is observed that the amplitude falls with the rise of R_c for both G<0 and G>0. The rise in w raises the value of amplitude of the skin-friction. The decrease in the values of the Eckert number increases the amplitude of τ_{xy}. As P increases, the amplitude increases for G<0 and increases for G>0.

Table – III

Values of the phase of the skin friction tan θ

G	E	P	w/R_c	0	0.05	0.1
–10	–0.01	5	4	0.408	0.395	0.344
–10	–0.01	5	10	0.532	0.499	0.442
–5	–0.01	5	10	0.598	0.543	0.496
–5	–0.01	10	10	0.524	0.469	0.397

–5	–0.02	5	10	0.474	0.421	0.398
–10	+0.01	5	10	0.536	0.493	0.445
5	+0.01	5	4	0.470	0.436	0.412
5	+0.01	5	10	0.588	0.552	0.497
5	+0.01	10	10	0.610	0.575	0.462
5	+0.02	5	10	0.620	0.574	0.482
10	+0.01	5	10	0.640	0.582	0.523

Table III contains the values of the phase of the skin-friction (tan θ) for both heating and cooling of the plate. It is noticed that the rise in R_c reduces the value of the phase for both G<0 an dG>0. The increase in w increases the value of the phase. The decrease in Eckert number decreases the value of the phase. It is marked that the rise in the value of G raises the phase. The increase in Prandtl number reduces the phase for G<0, but a reverse effect is marked in case of G>0.

Table – IV

Values of the mean rate of heat transfer Nu_0

G	E	P/R_c	0	0.05	0.1
–5	–0.01	5	4.052	4.054	4.067
–5	–0.01	10	9.02	9.06	9.08
–5	–0.02	10	9.08	9.16	9.18
–10	0.01	5	4.95	4.68	4.23
+5	0.01	5	4.624	4.620	4.614
+5	0.01	10	8.954	8.942	8.925
+5	0.02	10	9.443	9.426	9.410
+10	0.01	5	4.856	4.850	4.843

The values of the mean rate of heat transfer Nu_0 are entered in Table-IV. It is observed that the increase in the non-Newtonian parameter R_c increases the mean rate of heat transfer for heating of the plate and a reverse effect is marked for cooling of the plate (G>0). As P rises, the value of Nusselt number increases for both G<0 and G>0. The fall in Eckert number enhances the mean rate of heat transfer. It is also observed that the fall in G reduces the mean rate of heat transfer. Increase in G increases the meat rate of heat transfer for cooling of the plate.

Table – V

Values of the amplitude of the rate of heat transfer Nu/E

G	P	w/R_c	0	0.05	0.1
–5	5	4	5.306	5.229	5.016
–5	5	10	7.450	7.567	7.892
–5	10	10	13.44	13.84	13.660
–10	5	10	7.881	8.256	9.763
+5	5	4	6.934	6.736	6.256
+5	5	10	7.998	7.653	7.242
+5	10	10	14.872	15.624	15.939
10	5	10	9.672	9.480	9.236

Values of the amplitude of the rate of heat transfer are entered in Table-V for both heating and cooling of the plate. It is noticed that the increase in R_c reduces the amplitude of both G<0 and G>0. As w rises, the amplitude also rises for both heating and cooling of the plate. Similar effect is marked in case of P. The decrease in the value of G increases the amplitude. The same effect is marked in case of rise in G for cooling of the plate (G>0).

Table – VI

Values of the phase of the rate of heat transfer tan δ

G	P	w/R_c	0	0.05	0.10
–5	5	1	0.864	0.878	0.902
–5	5	10	1.922	2.040	2.398
–5	10	10	0.854	0.996	1.022
–10	5	10	3.812	4.226	5.678
5	5	4	0.426	0.418	0.395
5	5	10	0.716	0.688	0.592
5	10	10	0.778	0.743	0.692
10	5	10	0.448	0.421	0.315

The values of the phase of the rate of heat transfer (tan δ) are entered in Table-VI. It is noticed that the increase in R_c increases the phase for G<0 and decreases the phase for G>0. The increase in w increases the phase for both cooling and heating of the plate. As P rises, the value of tan δ decreases for G<0 and increases for G>0.

CONCLUSIONS:

The following conclusions are drawn from the results obtained by analyzing the problem of free convection effects on the oscillator magnetohydrodynamic flow of a visco-elastic fluid past an infinite vertical porous plate with constant suction and heat dissipation.

i) The strength of the external magnetic field reduces the mean velocity.

ii) The mean velocity decreases as R_c increases.

iii) The fall in the value of Grashof number reduces the mean velocity.

iv) Increase in G decreases the transient velocity.

v) Rise in R_c accelerates the unsteady flow.

vi) Rise in the Prandtl number P reduces the mean temperature

vii) The increase in Eckert number raises the mean temperature.

viii) As the value of G decreases, the transient temperature falls.

ix) The transient temperature first decreases and then rises with the rise of P.

x) Rise in G raises the mean skin-friction.

xi) With the rise of the Prandtl number, the mean skin-friction, rises for G<0 and falls for G>0.

xii) The amplitude of the skin-friction falls with the rise of R_c for both heating and cooling of the plate.

xiii) Mean rate of heat transfer increases with R_c, the non-Newtonian parameter.

xiv) The phase of the skin-friction increases with the angular frequency ‘w’.

xv) The phase of the rate of heat transfer also rises with the rise of w for both G<0 and G>0.

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