Churning Multiple Communication Sources in Cooperative Control of a Mobile Robot

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Abstract— Cooperative control of a mobile robot with multiple communication sources is investigated. In the leader-follower framework, the robot is assumed as a follower and it follows the instructions of the leader. However, the communication sources are not only the leader but also its other associate following the leader. In this setup, the computation at the mobile robot need to churn the data it receives from multiple sources and simultaneously perform the task of its leader. Suppose the leader is assigned to seek a target cooperatively along with its followers. In this paper, a cooperative control algorithm compatible to the robot receiving information from more than one source is developed. The algorithm is executed for a communication pattern in decentralized architecture. Several examples of churning are illustrated and it is shown that churning significantly influences the performance. Further, the initial condition and the direction of the velocity of the follower with respect to its leader are critical to develop a formation.

Index Terms — Communication, Churning, Cooperation, Computation, Follower, Leader, Multiple.

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Nomenclature:

a	: Open loop system matrix.
A	: Closed loop system matrix.
b	: Control influence matrix.
C	: Communication matrix elements.
c	: Interconnections
f	: Feed forward control.
k	: Feedback controller.
N	: Number of robots engaged in the formation.
p	: Position.
$\dot{p}$	: Velocity.
$\bar{r}$	: Radius of a circle for cooperation.
t	: Time instants.
T	: Formation matrix.
u	: Control input.
x	: State vector.
y	: Communication vector.
α	: Cooperative control index indicating velocity directions.

Subscripts:

a	: Leader.
i, j, 1, 2	: Followers.
i	: Follower who gives information to its associate j.
1	: Follower who gives information to its associate 2.
j	: Follower receiving information from a and i.
2	: Follower receiving information from a and 1.
r	: Leader or follower.

1 INTRODUCTION

Cooperative control of multi-vehicle systems has been an active research area in recent years [1,2]. Higher level control is generally mixed with decision making policies [3,4]. Various lower level control methods for multi-vehicle systems are discussed in the first paper [5] of the special issue [1]. Some of the control methods are demonstrated experimentally [6]. Such an exercise for an air vehicle or a mobile robot cooperating with multiple systems with multiple communication sources require further understanding of churning [7]. Churning is a computational option to invite data hierarchically from multiple sources and process them at the centralized or decentralized platform onboard a system cooperating with those systems providing the data. In this paper, a decentralized architecture for leader-follower framework is considered. All the followers are assumed to be cooperating with the leader which is assigned to seek a target. The leader is providing its information to all the followers. In addition, one of the follower systems further receives information from its associate. While seeking the target, this paper develops a cooperative control algorithm and investigates the cooperative behavior of the follower system that receives information from the multiple sources. It is shown that churning significantly affects performance. Further, the initial conditions and the direction of the velocity vectors are critical to develop a formation with cooperation among multiple communication sources.

In leader-follower framework, the information flow among the systems is shown to be processed at the feed forward path of the representative systems [8,9]. Thus, a feed forward gain proportional to a communication pattern is required to cooperate with the systems providing the information. In doing so, since the information sharing among the systems is from multiple sources, it is required to consider a specific communication pattern and determine gains of the feed forward controller. Thus, various options arise to represent the communication pattern and churn the data for a cooperative performance. In this paper, the information for cooperation is assumed to be available alternatively. Accordingly, a control algorithm for the follower that receives such information is developed to seek a target along with the leader. Unlike a swarm which seeks a moving target [10], in order to understand the effects of churning on cooperation, the

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target is assumed to be fixed. Although cooperative performance deteriorates at the expense of data churning, the system with multiple communication sources is capable of performing a certain task assigned to either one of the systems sharing the information.

The paper is organized using an example that enhances the tutorial value. Mainly, an interconnection in terms of the velocity vector direction for cooperation among the distributed systems is discussed. Then, various issues to process multiple communications at the local control structure of the system are presented. In decentralized platform, for a follower with multiple communication sources, an algorithm for churning the data is developed and examples are illustrated. Conclusions and future directions in this research area are summarized.

2 SYSTEM MODELING WITH MULTIPLE COMMUNICATION SOURCES

In leader-follower framework, the position (p_r) and velocity ($\dot{p}_r$) of a finite set of mobile robots in R^2 are shown in Fig.1. The dynamics of each robot is assumed as an integrator. The discrete-time model with sampling one time unit is,

$$\begin{bmatrix} p_r(t+1) \\ \dot{p}_r(t+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_r(t) \\ \dot{p}_r(t) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u_r(t) \qquad (1a)$$

or,

$$x_r(t+1) = a_r x_r(t) + b_r u_r(t),\; x_r(t) \in R^2 \qquad (1b)$$

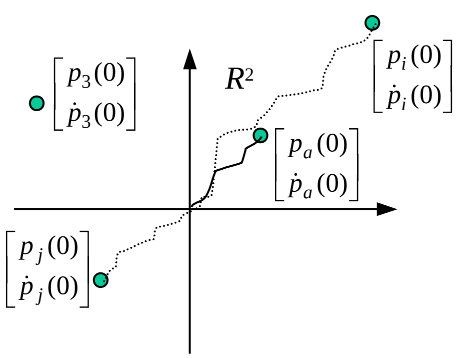

Fig. 1. Follower $x_j(t)$ cooperating wit $x_a(t)$ and $x_i(t)$.

Here t refers the time units for the discrete time model taking the values 0,1,…∞. In Figure 1, the leader is denoted by the sub script a. Consistent with the Fig. 1, the followers are denoted by the subscripts i, j and 3, respectively. The leader or follower system is denoted by $r \in [a,1,\ldots,j,\ldots N]$. Systems $x_a(t)$, $x_i(t)$ and $x_j(t)$ are assumed to be in a formation to seek a target at the origin. Leader $x_a(t)$ communicates with its followers $x_i(t)$ and $x_j(t)$ respectively. In addition, $x_i(t)$ also communicates with its associate $x_j(t)$ for further cooperation. A cooperation is considered by an algebraic constraint given by,

$$p_a^2(t) + p_j^2 = r_a^2 \qquad (2)$$

That is, the follower movements are such that at each time instant they satisfy a circle criterion with respect to the leader movements in Eq. (2). The circle criterion is further chosen to study the influence of the changing velocity directions of the follower system with respect to the leader. Consider a linear time-invariant feedback controller k_j and define a two-degree of freedom control law,

$$u_j(t) = k_j x_j(t) + f_j(t)\,. \qquad (3)$$

The closed loop dynamics becomes,

$$x_j(t+1) = A_j x_j(t) + b_j f_j$$

$$A_j = (a_j + b_j k_j)\,;$$

$$x_j(0) = \begin{bmatrix} p_j(0) \\ \dot{p}_j(0) \end{bmatrix}$$

Each follower system from its initial condition $x_r(0)$ is driven to origin in a certain duration of time along with their leader. The feed forward control input $f_j(t)$ is introduced to enhance cooperation between the leader and its followers. Given the leader information, the feed forward control proportional to the leader information is applied at the follower systems. However, at the follower system $x_j(t)$, the feed forward control is proportional to the leader as well as its associate inputs. The objective of this paper is determine such a feed forward control for the follower system $x_j(t)$ such that the cooperative constraint in Eq. (2) is met.

The cooperation given by Eq. (2) is augmented in the systems as follows. An interconnection is developed by differentiating Eq. (2) with respect to time.

$$p_a \dot{p}_a + p_j \dot{p}_j = 0 \Leftrightarrow \frac{p_a}{p_j} = -\frac{\dot{p}_j}{\dot{p}_a} = -\alpha_a \qquad (4)$$

Eq.(4) leads to $x_a(t) - x_j(t)$ interconnection as follows,

$$c_j(t) = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \alpha_a \end{bmatrix}}_{T_a} \begin{bmatrix} p_a(t) \\ \dot{p}_a(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \alpha_a & 0 \\ 0 & -1 \end{bmatrix}}_{T_b} \begin{bmatrix} p_j(t) \\ \dot{p}_j(t) \end{bmatrix} \qquad (5)$$

Using the aggregate model discussed in Ref.11 and its interconnections in Eq. (5), feed forward control at the follower system $x_i(t)$ is applied for α_a=1 and α_a= –1, respectively. Further, Eq.(4) implies that when α_a=1, the follower seeks the origin by cooperating with its leader in such a way the velocity components are the same in both direction and magnitude. Fig.2a and Fig.2c for various initial conditions in first and third quadrants depict this situation, where the follower system attempts to align itself with the leader using a feed forward control. Thus the initial conditions (15,1) and (-15,-1) are concluded as cooperative. In the absence of feed forward control, it is observed that the follower system becomes uncooperative to both the initial conditions. The interconnections α_a= –1 in Fig.2b and Fig.2d draw the same conclusions. However, the formations are different with the velocity vectors of the leader and follower systems acting in opposite directions.

If the design constraints such as the state and control input saturation limits are imposed, majority of the interconnections for complex maneuvers become infeasible. Consider the design constraints,

$$\{-25 \le p_r(t) \le 25, -5 \le \dot{p}_r(t) \le 5\}\,.$$

Clearly, the initial condition (–15,–1) with α_a=1 is cooperative with feed forward control. But it becomes invalid because the maneuver is complex. In addition, if the system begins to cooperate with other associate following the leader, cooperative performance with initial conditions such as (15,1) may further deteriorate. This aspect of the study is formulated next.

Problem Definition: A more complex problem is posed as follows. In addition to the leader to follower communication, let the follower $x_j(t)$ receives information from its associate $x_i(t)$ such that,

a) $x_j(t)$ cooperates with $x_i(t)$ and
b) Both $x_i(t)$ and $x_j(t)$ cooperate with $x_a(t)$.

In this case, the cooperative constraints in terms of the two algebraic constraints are,

$p_a^2(t)+p_j^2=r_a^2$ and

$p_i^2(t)+p_j^2=r_i^2$.

Now, follower j satisfies the following two independent interconnections, one to cooperate with the leader and another to cooperate with its associate,

$$\bar{c}_j(t)=\underbrace{\begin{bmatrix}1 & 0\\ 0 & \alpha_a\end{bmatrix}}_{\bar{T}_a}\begin{bmatrix}p_a(t)\\ \dot{p}_a(t)\end{bmatrix}+\underbrace{\begin{bmatrix}\alpha_a & 0\\ 0 & -1\end{bmatrix}}_{\bar{T}_b}\begin{bmatrix}p_j(t)\\ \dot{p}_j(t)\end{bmatrix} \qquad (6)$$

$$\tilde{c}_j(t)=\underbrace{\begin{bmatrix}1 & 0\\ 0 & \alpha_i\end{bmatrix}}_{\tilde{T}_a}\begin{bmatrix}p_i(t)\\ \dot{p}_i(t)\end{bmatrix}+\underbrace{\begin{bmatrix}\alpha_i & 0\\ 0 & -1\end{bmatrix}}_{\tilde{T}_b}\begin{bmatrix}p_j(t)\\ \dot{p}_j(t)\end{bmatrix} \qquad (7)$$

Upon transformation, the aggregate dynamics for the follower $x_j(t)$ that need to cooperate with both $x_a(t)$ and $x_i(t)$ is as follows:

$$\bar{x}_j(t+1)=A_j\bar{x}_j(t)+\bar{\Gamma}x_a(t)+b_jf_j(t), \qquad (8a)$$

where, $\bar{\Gamma}=\bar{T}_b^{-1}\bar{T}_aA_a-A_j\bar{T}_b^{-1}\bar{T}_a$.

The transformation $\bar{x}_j=\bar{T}_b^{-1}\bar{c}_j$ is with respect to the formation matrix $\bar{T}_b$. The leader communication is denoted using a structured matrix $\bar{T}_a$. Similarly, the dynamics of $x_j(t)$ with respect to its associate's communication $\tilde{T}_i$, is derived. The follower-follower interconnection with respect to the transformation $\tilde{x}_j=\tilde{T}_b^{-1}\tilde{c}_j$ is given by,

$$\tilde{x}_j(t+1)=A_j\tilde{x}_j(t)+\tilde{\Gamma}x_i(t)+b_jf_j(t), \qquad (8b)$$

where, $\tilde{\Gamma}=\tilde{T}_b^{-1}\tilde{T}_iA_i-A_j\tilde{T}_b^{-1}\tilde{T}_i$.

The feed forward controller at the follower $x_j(t)$ with multiple communications becomes,

$f_j(t)=g_a(t)x_a(t)$ (or)

$$f_j(t)=g_i(t)x_i(t) \qquad (9)$$

Eq.(9) blends the information it receives from both $x_a(t)$ and $x_i(t)$. Accordingly, the feed forward gains $g_a,g_b\in R^{1\times 2}$ are reconfigured. Various possibilities do exist to churn the data from systems $x_a(t)$ and $x_i(t)$. In the presence of transmission delay, the problem becomes complex. In order to avoid congestion, a simple procedure that accepts data from either $x_a(t)$ or $x_i(t)$ but not from both is presented. In the following sections, a decentralized computational platform is assumed to study the problem.

3 COMPUTATIONAL PLATFORM

In the previous section, modeling of spatially distributed systems with communication from multiple sources is presented for a decentralized architecture. Depending upon the communication sources, a procedure to handle multiple dynamic models presented in Eq. (9) becomes necessary. This in turn poses some computational issues in the feed forward path. Consider the formation of a leader $x_a(t)$ with two associates, namely $x_1(t)$ and $x_2(t)$. That is, the three systems, i =1 and j = 2 are considered. As a result, follower 2 receives information from its leader $x_a(t)$ and its associate $x_1(t)$. In Fig.3, a typical computational platform with feed back controller k_2 and reconfigurable feed forward controllers {g_a,g_1} for the system $x_2(t)$ is presented. The communications are accordingly received and processed at the feed forward path.

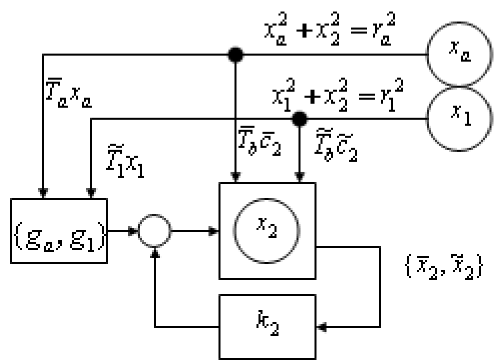

Fig. 3. A Computational platform for a follower system receiving information from multiple communication sources.

Suppose a centralized architecture is required. The integrated dynamics of the distributed systems for the centralized architecture is as follows:

$$\begin{bmatrix}x_a(t+1)\\ x_1(t+1)\\ x_2(t+1)\end{bmatrix}=\begin{bmatrix}a_a & 0 & 0\\ 0 & a_1 & 0\\ 0 & 0 & a_2\end{bmatrix}\begin{bmatrix}x_a(t)\\ x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}b_a & 0 & 0\\ 0 & b_1 & 0\\ 0 & 0 & b_2\end{bmatrix}\begin{bmatrix}u_a(t)\\ u_1(t)\\ u_2(t)\end{bmatrix} \qquad (10a)$$

It can accommodate unlimited information in a vector similar to the measurement vector in conventional control, which is referred as a communication vector,

$$y(t)=\begin{bmatrix}C_{aa} & 0 & 0\\ C_{1a} & C_{11} & 0\\ C_{2a} & C_{21} & C_{22}\end{bmatrix}\begin{bmatrix}x_a(t)\\ x_1(t)\\ x_2(t)\end{bmatrix}. \qquad (10b)$$

The diagonal entries are the standard measured signals applicable for a conventional control system. The off-diagonal entries indicate the communication sources among the distributed systems. The triangular structure in Eq.(10b) has a special reference to the decentralized architecture. The control law for this specific communication vector will be of the form,

$$\begin{bmatrix} u_a(t) \\ u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} k_{aa} & 0 & 0 \\ k_{1a} & k_{11} & 0 \\ k_{2a} & k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x_a(t) \\ x_1(t) \\ x_2(t) \end{bmatrix}. \quad (10c)$$

The output vector is given by,

$$\begin{bmatrix} c_1(t) \\ \bar{c}_2(t) \\ \tilde{c}_2(t) \end{bmatrix} = \begin{bmatrix} T_a & T_b & 0 \\ \bar{T}_a & 0 & \bar{T}_b \\ 0 & \tilde{T}_i & \tilde{T}_b \end{bmatrix} \begin{bmatrix} x_a(t) \\ x_1(t) \\ x_2(t) \end{bmatrix}. \quad (10d)$$

Although the centralized platform is suitable to accommodate communication sources in the state-space model, it is difficult to synthesize a structured controller as in Eq.(10c). Further, the output vector is not dimensionally stable and its structure is expected to vary. Thus the study confines to the decentralized computing platform.

4 COOPERATIVE CONTROL ALGORITHM

When a communication delay is not present, a communication pattern adopted to develop the cooperative control algorithm is shown in Fig. 4. Here the system receiving information from

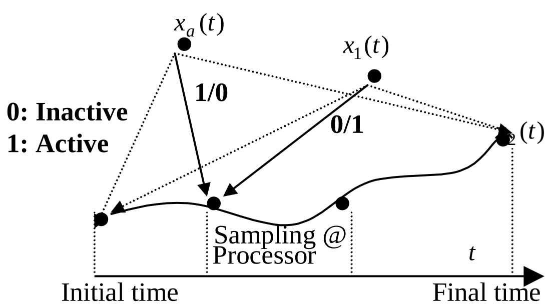

Fig. 4. A communication pattern for cooperative control algorithm

more than one system is identified as $x_2(t)$. When information from $x_a(t)$ is received, it is assumed that no data from $x_1(t)$ is received. Thus no data congestion occurs at system $x_2(t)$. The transmission time for this type of data transfer is further assumed twice the sampling time of the processor at each homogeneous system engaged in the formation.

Also, system $x_2(t)$ receives both position and velocity information from $x_a(t)$ and $x_1(t)$, respectively. The control structure at the feed forward path is fixed as a proportional plus derivative controller. An algorithm is developed to minimize the error between the leader trajectory and the follower trajectory. The feed forward gains for the communication pattern in Fig. 4 are accordingly determined. If $x_2(t)$ makes an attempt to cooperate with $x_a(t)$, then interconnection $\bar{c}_2$ becomes effective with gain $g_a(t)$. Similarly, if $x_2(t)$ makes an attempt to cooperate with $x_1(t)$, then the interconnection $\tilde{c}_2$ becomes effective with gain $g_1(t)$. The cooperative control algorithm is compared in the following two cases:

Case (*a*): $x_1(t)$ and $x_2(t)$ receive information from $x_a(t)$ and cooperate with $x_a(t)$. This is the standard leader-follower control problem where each follower receives information from single source, that is, the leader x_a.

Case (*b*): As in Case a. In addition, $x_2(t)$ begins to communicate with $x_1(t)$ to develop cooperation (may be to perform a different task when $x_1(t)$ takes a leadership).

Fig.5 shows Case a, wherein, the followers $x_2(t)$ and $x_1(t)$ cooperate with $x_a(t)$. The initial condition for each system is considered in the first quadrant as (10,2). The effect of initial conditions when followers are in third quadrant is illustrated in Fig. 2. The trajectories of the followers cooperating with $x_a(t)$ suggest that the cooperative control constraint is satisfied after a finite number of time steps. The feed forward control input histories ($f_1(t)$ at $x_1(t)$ and $f_2(t)$ at $x_2(t)$) are also compared. In each of these simulations, the interconnection is defined using $\alpha_a = \alpha_1 = 1$. That is, the speed of each system does not exceed the speed of the other system. In Fig.6, Case b is illustrated. Because $x_2(t)$ receives communication from both $x_1(t)$ and $x_a(t)$, it is trying to cooperate with both of them. Further, same initial condition for each follower is assumed. Thus, for the assumed communication pattern in Fig. 4, the type of cooperation exists between the systems $x_2(t)$ and $x_1(t)$, and between the systems $x_2(t)$ and $x_a(t)$ does not change. When the initial conditions are modified, the cooperation is significantly modified as shown in Fig.7. As before, the interconnections are chosen using $\alpha_a = \alpha_1 = 1$. When multiple communication sources are introduced, the initial conditions significantly affect the formations. The cooperative constraints of system $x_2(t)$ with its leader $x_a(t)$ and its associate $x_1(t)$ can be inferred. With the existing communication pattern, it is observed that $x_2(t)$ in Fig.7 is uncooperative to the initial condition (–10,2). Although $x_2(t)$ is cooperative to (10,2), the speed constraint with $\alpha_a = \alpha_1 = 1$ becomes effective and churning occurs as it approaches the target.

5 CONCLUSIONS AND FUTURE WORK

Control of spatially distributed systems with multiple communication sources presents major challenges. This paper focuses on multiple models resulting from multiple communication sources. Unlike distributed systems with single communication source as in a standard leader-follower framework, when more than one communication source is introduced, churning inevitably occurs and cooperative performance deteriorates. Using an a communication pattern, multiple models among the distributed systems are used to develop a cooperative control algorithm. Various examples with initial condition uncertainty are illustrated. It is concluded that formation is sensitive to initial conditions and is difficult to achieve especially when multiple communication sources are present. However, such systems can cooperate with either one of the system providing the information and perform different tasks. Currently, churning in swarms is under investigation.

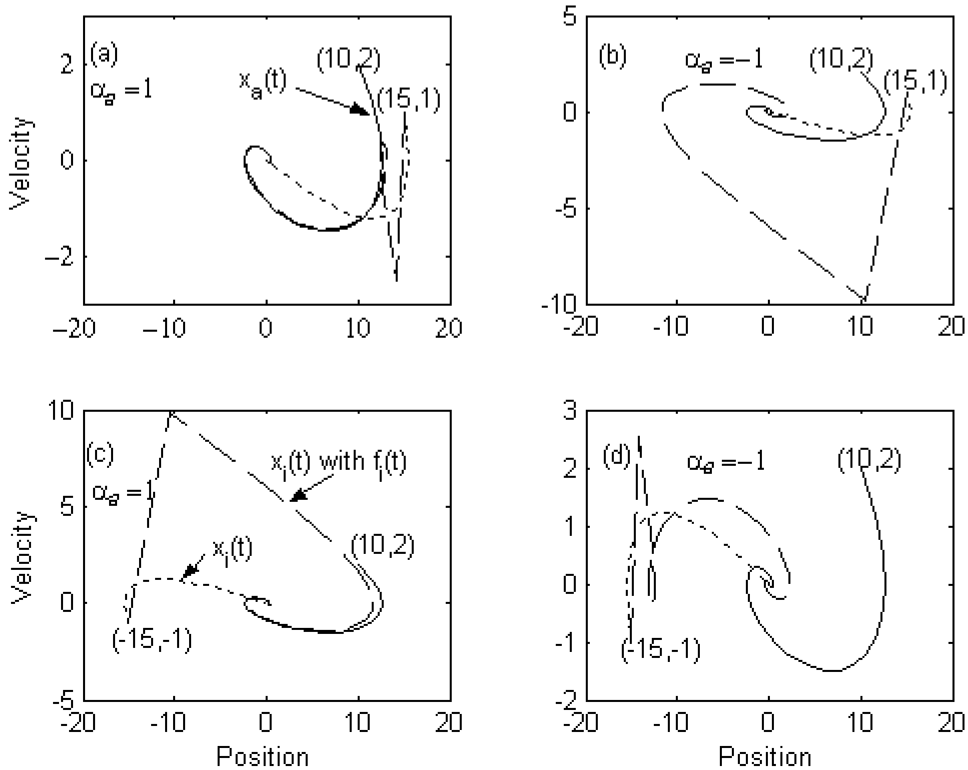

Fig. 2. Follower $\boldsymbol{x}_i(t)$ cooperating with the leader $\boldsymbol{x}_a(t)$

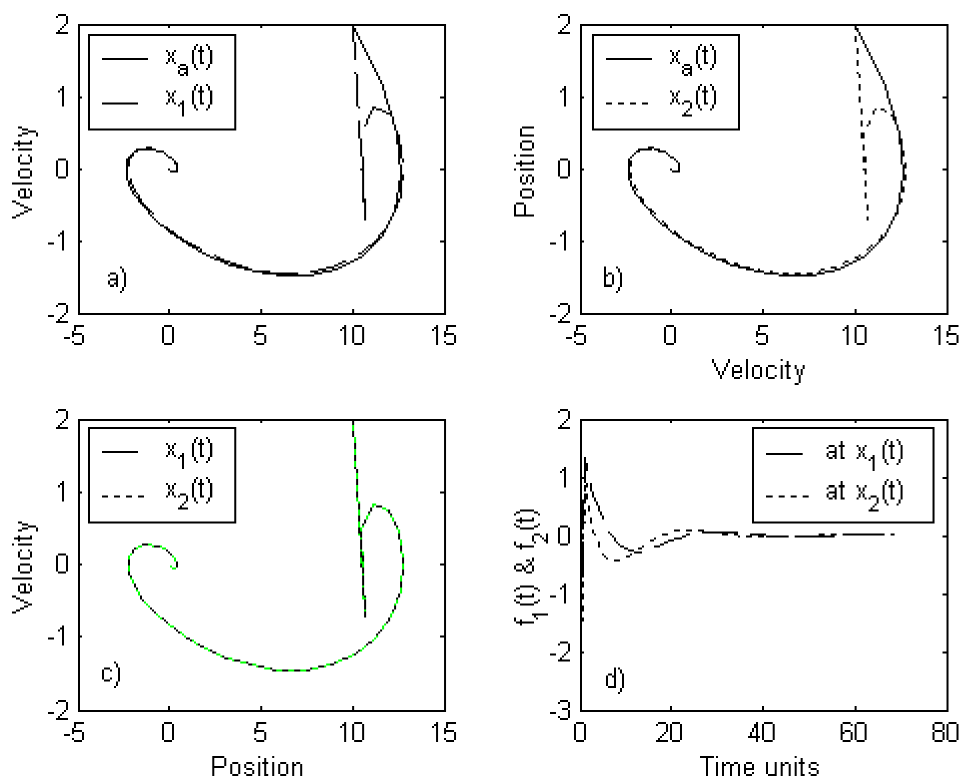

Fig. 5. Cooperative systems with single communication source.

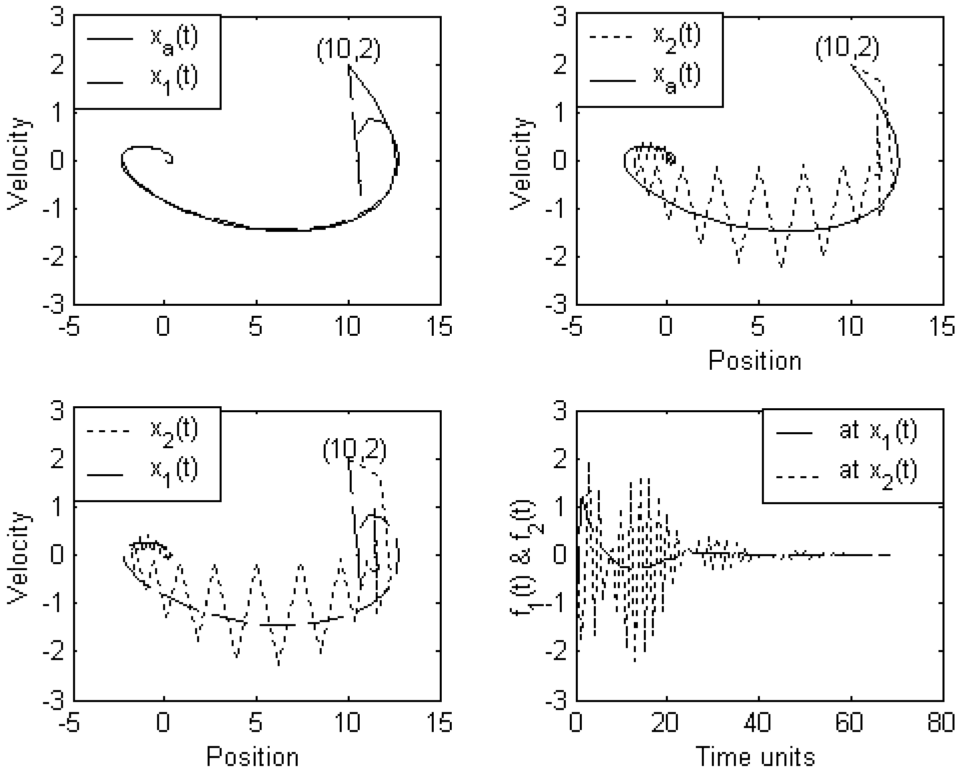

Fig. 6. Cooperative Systems With Multiple Communication Sources.

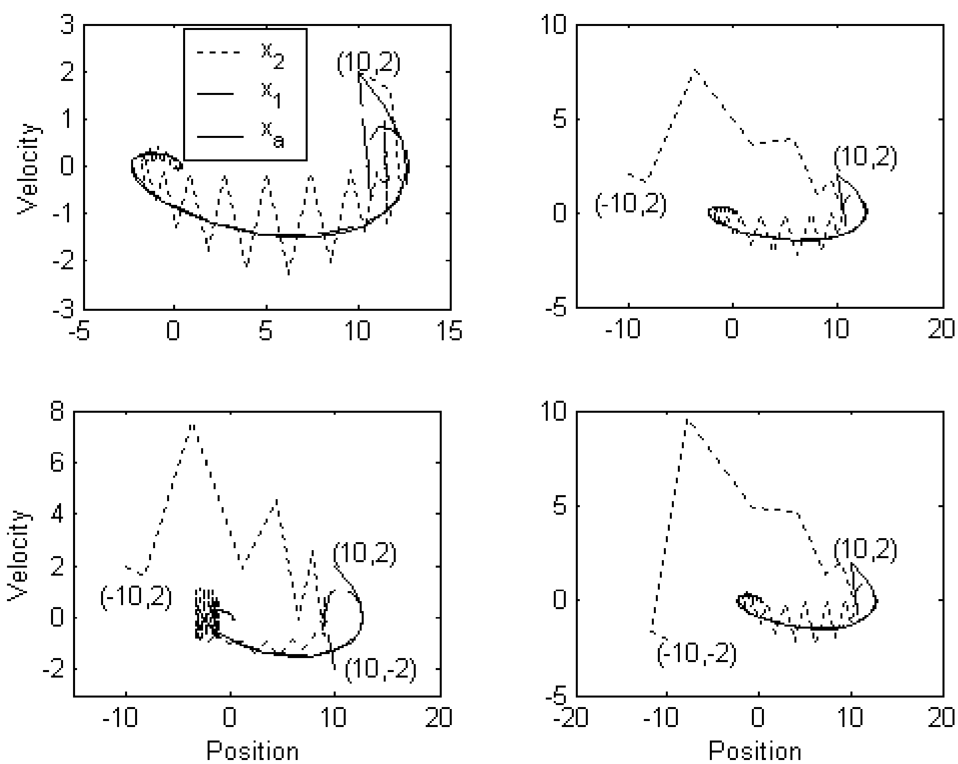

Fig. 7. Initial Condition Effects on Cooperative Control.

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