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Assignment of Personnels when Job Completion time follows Gamma distribution using Stochastic Programming Technique

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ABSTRACT
Recruitment of persons to various jobs according to required talents in an organization plays an important role in the growth of the organization. The formation of a number of groups of the persons from the population based on their efficiency in completion of jobs is being done by using the theory of cluster analysis. In this paper we formulate the problem of assignment of persons from various groups to different jobs who may complete them in minimum time as stochastic programming problem. The job completion times are assumed to follows Gamma distribution. By using the chance constrained programming technique we transform the stochastic programming problem to an equivalent deterministic problem with linear objective function and some non-linear (convex) constraints. First we assume that the completion time variables are identically distributed Gamma variables. The model is then extended to the case of non-identically distributed time variables. The illustrative examples are also given for both the models.

Key Words: Cluster analysis, Gamma distribution, Linear stochastic programming, Non-linear programming,
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1. INTRODUCTION

Growth and Development of any organization depends mainly on the talent of the persons recruited for its various assignments. A talented person will perform his/her work more accurately and will take less time in completion. The time of completion of a job is a very important factor and obviously it may be taken as a random variable because the same person under similar conditions will not necessarily complete a same piece of work in another occasion in same time. In this paper we consider the problem of assignment of personnel in which time to complete the job by a person follows Gamma Distribution, and formulate it as a stochastic linear programming problem(*SLPP*). The deterministic equivalents of the *SLPP's* are obtained in different situations and the solution is obtained using a suitable software. The use of stochastic programming in different situations has been made by various authors, see for example , Biswal *et al*. (1998), Yadavalli, *et al*.(2007) and many others. A similar problem has been discussed by Jeeva *et al*. (2004) with Weibull distribution.

Consider an organization with a finite number of jobs say, n . The organization wants to recruit persons to these jobs, for which they have received a large number of applications. The persons are categorized in various clusters using cluster analysis technique. We are interested in obtaining the optimal number of persons to be selected from different clusters to perform various jobs so as to minimize the completion time of all the jobs .The time taken by each person to complete a given job is assumed to be a random variable and follows Gamma distribution. Gamma Distribution is commonly used to model the time to complete a job.(See Leonhard and Davis,1995).

Suppose the applications received for different assignments were classified into m homogeneous groups according to the similarity of their qualifications. The classification may be performed using K-means method of cluster analysis technique (Gorden,A.D,1981). Let $C_1, C_2, \ldots, C_m$ be the m homogeneous clusters into which the population of applications has been divided. We will use the stochastic linear programming technique for finding the optimal assignment of applicants to various jobs.

2. GAMMA DISTRIBUTION

A continuous random variable T assuming only non-negative values is said to follow a gamma probability distribution if its probability density function is given by

$$f(t) = \frac{\alpha}{\Gamma(\beta)}(\alpha t)^{\beta-1} e^{-\alpha t}, \; t > 0$$

$= 0$, elsewhere.

The parameters α and β are positive real numbers *i.e.* $\alpha > 0$ *and* $\beta > 0$ .We use the notation $T \sim G(\alpha, \beta)$.

Its moment generating function is given as

$$M_T(u) = E(e^{ut}) = (1 - \frac{u}{\alpha})^{-\beta}.$$

The $r\,th$ moment about origin of T is given by

$$E(T^r) = \frac{1}{\alpha^r} \cdot \frac{\Gamma(\alpha+\beta)}{\Gamma(\beta)}$$

Hence the mean and variance of T are obtained as

$$E(T) = \frac{\beta}{\alpha} \text{ and } V(T) = \frac{\beta}{\alpha^2}.$$

Note that if we take $\beta = 1$, we get exponential probability distribution and for $\alpha = \frac{1}{2}$ and $\beta = \frac{n}{2}$ we get χ^2 distribution with n degrees of freedom.

3. FORMULATION OF THE PROBLEM

Let t_{ij} be the time taken for completing $i\,th$ job by a person from $j\,th$ cluster $(i = 1, \ldots, n, \; j = 1, \ldots, m)$.We assume that t_{ij} are random variables following Gamma distributions. Let x_{ij} be the number of individuals selected for $i\,th$ job from $j\,th$ cluster. Our aim is to find x_{ij} , $i = 1, \ldots, n, \; j = 1, \ldots, m$ so as to complete the project in minimum time. *i.e.* we want to minimize $\sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij} x_{ij}$. Suppose that the organization would be happy if the completion time of $i\,th$ job does not exceed a_i time units. However, it would tolerate a violation of this condition with a small probability $p_i^{'}$.This means that the following constraints should be satisfied

$$P \left\{ \sum_{j=1}^{m} t_{ij} x_{ij} \le a_i \right\} \ge 1 - p_i^{'} , \quad i = 1, \ldots, n. \tag{3.1}$$

Another requirement of the organization is that the expected total man hours from $j\,th$ cluster should preferably not exceed b_j .The corresponding constraints for this requirement with tolerance probabilities of violation $p_j^{''}$ are

$$P \left\{ \sum_{i=1}^{n} t_{ij} x_{ij} \le b_j \right\} \ge 1 - p_j^{''} \quad j = 1, \ldots, n. \tag{3.2}$$

Also the minimum number of individuals required to complete the $i\,th$ job is given as $x_i^{'}, i = 1, \ldots, n.$ So

$$\sum_{j=1}^{m} x_{ij} \ge x_j^{'}, i = 1, \ldots, n.$$

Further the minimum number of individuals to be selected from each cluster are also fixed by the organization. Let the number for $j\,th$ cluster be $x_j^{''}$. Then we should have

$$\sum_{i=1}^{n} x_{ij} \ge x_j^{''}, \; j = 1, \ldots, m.$$

Since the total number of persons selected from various clusters should be equal to the number of persons assigned to various jobs, we obviously have

$$\sum_{j=1}^{m}\sum_{i=1}^{n} x_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}$$

Further we should have the restrictions

$x_{ij} \geq 0$ and integers $i = 1, \ldots, n,\ j = 1, \ldots, m$.

The problem of finding the optimal number of individuals from various clusters to complete the assigned jobs in minimum time is formulated as the following chance constrained linear programming problem, (Charnes &Cooper, 1959).

SLPP

$$\left.\begin{aligned}
&E(Z) = E\left(\sum_{i=}^{n}\sum_{j=1}^{m} t_{ij}x_{ij}\right) && (i)\\
&\text{Subject to}\\
&P\left[\sum_{j=1}^{m} t_{ij}x_{ij} \leq a_i\right] \geq 1 - p_i^{'} , \quad i = 1,\ldots,n && (ii)\\
&P\left[\sum_{i=1}^{n} t_{ij}x_{ij} \leq b_j\right] \geq 1 - p_j^{''} , \quad j = 1,\ldots,m && (iii)\\
&\sum_{i=1}^{n} x_{ij} \geq x_j^{''} , \quad \sum_{j=1}^{m} x_{ij} \geq x_i^{'} && (iv)\\
&x_{ij} \geq 0 ,\ i = 1,\ldots,n ,\ j = 1,\ldots,m \text{ and integers} && (v)
\end{aligned}\right\} \tag{3.3}$$

Case I: Assume that t_{ij} are independent and identically distributed Gamma Variates , *i.e.*,

t_{ij} ~ G (α, β) then,

$$Mean(t_{ij}) = \frac{\beta}{\alpha} = m(t_{ij}) \text{ and } Variance(t_{ij}) = \frac{\beta}{\alpha^2} = v(t_{ij}).$$

Let us define $S_j = \sum_{i=1}^{n} t_{ij}x_{ij}$, $j = 1, \ldots, m.$

We have

$$E(S_j) = \sum_{i=1}^{n} m(t_{ij})x_{ij} = \frac{\beta}{\alpha}\sum_{i=1}^{n} x_{ij} ,\ j = 1, \ldots, m.$$

Further, as t_{ij} are independently distributed, we have

$$V(S_j) = \sum_{i=1}^{n} v(t_{ij})\, x^2{}_{ij} = \frac{\beta}{\alpha^2}\sum_{i=1}^{n} x_{ij}^2 \quad , \quad j = 1, \ldots, m.$$

Now the constraints 3.3(iii) can be written as

$$P[S_j \leq b_j] \geq 1 - p_j^{''} ,\quad j = 1, \ldots, m. \tag{3.4}$$

Since cluster sizes are assumed to be large we have from Liapounoff's central limit theorem $S_j \sim N(E(S_j), V(S_j))$.

Thus we have (3.4) is equivalent to

$$P\left[Z_j \leq \frac{b_j - E(S_j)}{\sqrt{V(S_j)}}\right] \geq 1 - p_j'' . \tag{3.5}$$

where $Z_j = \dfrac{S_j - E(S_i)}{\sqrt{V(S_j)}}$ is a standard normal variate (SNV).

So we have

$\Phi(Z_j) \geq \Phi(Z_{(1-p''_j)})$, where Φ is the distribution function of SNV and $Z_{(1-p''_j)}$ is *s.t.* $P[Z_j \leq Z_{(1-p''_j)} = 1 - p_j]$.
Since Φ is a non-decreasing function , we have

$$\frac{b_j - E(S_j)}{\sqrt{V(S_j)}} \geq Z_{(1-p''_j)}$$

$$\Rightarrow \frac{b_j - \frac{\beta}{\alpha}\sum_{i=1}^{n} x_{ij}}{\sqrt{\frac{\beta}{\alpha^2}\sum_{i=1}^{n} x^2{}_{ij}}} \geq Z_{(1-p''_j)}$$. Hence

$$\beta\sum_{i=1}^{n} x_{ij} + Z_{(1-p''_j)}\sqrt{\beta\sum_{i=1}^{n} x^2{}_{ij}} \leq \alpha b_j \quad , \; j = 1,\ldots,m \tag{3.6}$$

The inequalities (3.6) are the deterministic equivalents of the constraints 3.3(iii).
In a similar manner the deterministic equivalents of the constraints 3.3(ii) are derived as

$$\beta\sum_{j=1}^{m} x_{ij} + Z_{(1-p''_i)}\sqrt{\beta\sum_{j=1}^{m} x^2{}_{ij}} \leq \alpha a_i \;, \; i = 1,\ldots,n \tag{3.7}$$

Therefore the determinants equivalent of *SLPP* is obtained as

$$\left.\begin{array}{ll}
Minimize\ E(Z) = \frac{\beta}{\alpha}\sum_{i==1}^{n}\sum_{j=1}^{m} x_{ij} & (i) \\
\text{Subject to} & \\
\beta\sum_{j=1}^{m} x_{ij} + Z_{(1-p'_i)}\sqrt{\beta\sum_{j=1}^{m} x_{ij}^2} \leq \alpha a_i \,, i = 1,\ldots,n & (ii) \\
\beta\sum_{i=1}^{n} x_{ij} + Z_{(1-p''_j)}\sqrt{\beta\sum_{i=1}^{n} x_{ij}^2} \leq \alpha b_j \,, j = 1,\ldots,m & (iii) \\
\sum_{i=1}^{n} x_{ij} \geq x''_j \,, \sum_{j=1}^{m} x_{ij} \geq x'_i & (iv) \\
x_{ij} \geq 0 \,, i = 1,\ldots,n \;,\; j = 1,\ldots,m \text{ and integers} & (v)
\end{array}\right\} \tag{3.8}$$

It may be easily seen that the non-linear functions on the left hand side of constraints 3.8(ii) and (iii) are convex. So the problem (3.8) is a convex programming problem.

Case 2: Let us now assume that the parameters t_{ij} , a_i and b_j in *SLPP* (3.3) are all independent random variables following Gamma distributions given as

t_{ij} ~ G(α , β), $i = 1,\ldots,n$, $j = 1,\ldots,m$

a_i ~ G(α_1, β_1), $i = 1,\ldots,n$

b_j ~ G(α_2, β_2), $j = 1,\ldots,m$

The corresponding stochastic linear programming problem is the same *SLPP* as defined in (3.3).Now let us write

$S_j = \sum_{i=1}^{n} t_{ij}x_{ij} - b_j$, $j = 1,\ldots,m$, as

$$S_j = \sum_{i=1}^{n+1} t_{ij}x_{ij} \quad , where \; x_{(n+1)j} = -1, \; t_{(n+1)j} = b_j \,, j = 1,\ldots,m \tag{3.9}$$

Similarly write $S_i = \sum_{j=1}^{m} t_{ij} x_{ij} - a_i$, $i = 1,\ldots,n$, as

$$S_i = \sum_{j=1}^{m+1} t_{ij} x_{ij} \quad , where \;\; x_{i(m+1)} = -1, \;\; t_{i(m+1)} = a_i \,, i = 1,\ldots,n \tag{3.10}$$

The constraints 3.3(ii) and (iii) can thus be written respectively as

$$P[S_i \le 0] \ge 1 - p_i' \;,\;\; i = 1,\ldots,n \tag{3.11}$$

and $$P[S_j \le 0] \ge 1 - p_j'' \;,\;\; j = 1,\ldots,m \tag{3.12}$$

Again using Liapunoffs Central Limit Theorem in (3.11) and (3.12), we get the inequalities

$$P\left[Z_i \le \frac{-E(S_i)}{\sqrt{V(S_i)}} \right] \ge 1 - p_i' \,,\;\; i = 1,\ldots,n \tag{3.13}$$

and $$P\left[Z_j \le \frac{-E(S_j)}{\sqrt{V(S_j)}} \right] \ge 1 - p_j'' , \, j = 1,\ldots,m \tag{3.14}$$

The inequality (3.13) is equivalent to

$$\frac{-E(S_i)}{\sqrt{V(S_i)}} \ge Z_{(1-p_i')} \,, i = 1,\ldots,n \tag{3.15}$$

Now from (3.10) we have

$$E(S_i) = E(\sum_{j=1}^{m} t_{ij} x_{ij} - a_i) \; = \frac{\beta}{\alpha} \sum_{j=1}^{m} x_{ij} - \frac{\beta_1}{\alpha_1}$$

and $V(S_i) = V(\sum_{j=1}^{m} t_{ij} x_{ij} - a_i) = \frac{\beta}{\alpha^2} \sum_{j=1}^{m} x_{ij}^2 + \frac{\beta_1}{\alpha_1^2}$

(3.15) then gives

$$\frac{-\frac{\beta}{\alpha} \sum_{j=1}^{m} x_{ij} + \frac{\beta_1}{\alpha_1}}{\sqrt{\frac{\beta}{\alpha^2} \sum_{j=1}^{m} x_{ij}^2 + \frac{\beta_1}{\alpha_1^2}}} \ge Z_{(1-p_i)} \,,\; i = 1,\ldots,n$$

$$\Rightarrow \alpha_1 \beta \sum_{j=1}^{m} x_{ij} + Z_{(1-p_i)} \sqrt{\beta \alpha_1^2 \sum_{j=1}^{m} x_{ij}^2 + \beta_1 \alpha^2} \le \alpha \beta_1 \,, i = 1,\ldots,n$$

Similarly the inequalities (3.14) are equivalent to

$$\alpha_2 \beta \sum_{i=1}^{n} x_{ij} + Z_{(1-p_i'')} \sqrt{\beta {\alpha_2}^2 \sum_{i=1}^{n} {x_{ij}}^2 + \beta_2 \alpha^2} \le \alpha \beta_2 \,,\;\; j = 1,\ldots,m$$

Therefore the deterministic equivalent of *SLPP* in this case is obtained as the following non-linear programming problem:

$$\left.\begin{aligned}
&Min\, E(Z) = \frac{\beta}{\alpha}\sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij} && (i)\\
&\text{Subject to}\\
&\alpha_1\beta\sum_{j=1}^{m} x_{ij} + Z_{(1-p_i^{'})}\sqrt{\beta\alpha^2\sum_{j=1}^{m} x_{ij}^2 + \beta_1\alpha^2} \le \alpha\beta_1,\ i=1,\ldots,n && (ii)\\
&\alpha_2\beta\sum_{i=1}^{n} x_{ij} + Z_{(1-p_j^{''})}\sqrt{\beta\alpha_2^2\sum_{i=1}^{n} x_{ij}^2 + \beta_2\alpha^2} \le \alpha\beta_2,\ j=1,\ldots,m && (iii)\\
&\sum_{i=1}^{n} x_{ij} \ge x_j^{''},\ j=1,\ldots,m\ ,\sum_{j=1}^{m} x_{ij} \ge x_i^{'}, i=1,\ldots,n && (iv)\\
&x_{ij} \ge 0 \text{ and integers}, i=1,\ldots,n,\ j=1,\ldots,m. && (v)
\end{aligned}\right\} \tag{3.16}$$

4. THE CASE OF NON-IDENTICAL DISTRIBUTIONS

Now we consider the case when t_{ij} are independent but not identically distributed:

Let $t_{ij} \sim G(\alpha_{ij}, \beta_{ij})$

ie $f(t_{ij}) = \dfrac{(\alpha_{ij})^{\beta_{ij}}}{\Gamma(\beta_{ij})} e^{-\alpha_{ij}} t^{\beta_{ij}-1}$, $\alpha_{ij}, \beta_{ij} > 0$

= 0 , elsewhere.

The problem of determining the optimal number of individuals from various clusters to complete the assigned jobs under the prescribed constraints is already defined in (3.3)

The constraint $P[S_j \le a_j] \ge 1 - p_j''$ given in 3.3(iii), is equivalent to

$$P\left[Z_j \le \frac{b_j - E(S_j)}{\sqrt{V(S_j)}}\right] \ge 1 - p_j''. \tag{4.1}$$

Now

$$E(S_j) = \sum_{i=1}^{n} E(t_{ij}).x_{ij} = \sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_{ij}} x_{ij} \text{ and } V(S_j) = \sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_{ij}^{2}} x_{ij}.$$

Therefore, the deterministic equivalent of 3.3(iii) in this case is

$$\sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_{ij}} x_{ij} + Z_{(1-p_j'')}\sqrt{\sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_{ij}^{2}} x_{ij}^{2}} \le b_j\ ,\ j = 1,\ldots,m \tag{4.2}$$

Similarly corresponding to the constraint 3.3(ii) we get

$$\sum_{j=1}^{m}\frac{\beta_{ij}}{\alpha_{ij}} x_{ij} + Z_{(1-p_i')}\sqrt{\sum_{j=1}^{m}\frac{\beta_{ij}}{\alpha_{ij}^{2}} x_{ij}^{2}} \le a_i\ ,\ i = 1,\ldots,n. \tag{4.3}$$

Hence the non-linear deterministic equivalent for SLPP when t_{ij} are independent and non-identically distributed random variables following Gamma distributions is

$$\left.\begin{aligned}
& Min\, E(Z)=\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\beta_{ij}}{\alpha_{ij}}x_{ij} && (i)\\
& \text{Subject to,} \\
& \sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_{ij}}x_{ij}+Z_{(1-p''_j)}\sqrt{\sum_{i=1}^{n}\frac{\beta_{ij}}{\alpha_{ij}^2}x_{ij}^2}\le b_j\ ,\quad j=1,\ldots,m && (ii)\\
& \sum_{j=1}^{m}\frac{\beta_{ij}}{\alpha_{ij}}x_{ij}+Z_{(1-p'_i)}\sqrt{\sum_{j=1}^{m}\frac{\beta_{ij}}{\alpha_{ij}^2}x_{ij}^2}\le a_i\ ,\quad i=1,\ldots,n && (iii)\\
& \sum_{i=1}^{n}x_{ij}\ge x''_j,\ j=1,\ldots,m;\ \sum_{j=1}^{m}x_{ij}\ge x'_i\ ,\ i=1,\ldots,n && (iv)\\
& x_{ij}\ge 0 \text{ and integers } i=1,\ldots,n,\ j=1,\ldots,m. && (v)
\end{aligned}\right\} \quad (4.4)$$

5. EXAMPLES

Suppose that an organization has three types of jobs. They advertised the posts with the qualifications so that every one must be able to do all the three jobs. They received a large number of applications and grouped them into 3 clusters $C_1, C_2, and\ C_3$ using k-means method according to the details of abilities given by the applicants in their applications. Then the applicants from the clusters were examined through the interview and each cluster is divided into binary clusters of selected and non-selected ones. The candidates from the selected clusters were ranked according to their efficiency judged by their performance in the interview

Let the finally selected clusters be $K_1, K_2, and\ K_3$ and the jobs be $J_1, J_2, and\ J_3$.Let the time t_{ij} to complete i^{th} job ($J_1, J_2, and\ J_3$) by an individual from j^{th}cluster ($K_1, K_2, and\ K_3$) follow Gamma distribution with parameters $\alpha = 0.5, \beta = 10$.

. It is also given that the minimum number of candidates to be selected from each of the three clusters should be 2 and the minimum number of persons assigned to a job should be 4.

$$x'_1 = x'_2 = x'_3 = 2.$$
$$x''_1 = x''_2 = x''_3 = 4.$$

It is given that the total available time for completing each job is 160 hrs. Further the upper limit for the expected man hours from each cluster should not exceed 140 hrs.

Let us assume that the tolerable probabilities of violations of the constraints are all 0.05 i.e. $p'_1 = p''_2 = p'_3 = p''_1 = p''_2 = p''_3 = 0.05.$

The corresponding problem to be solved (obtained from 3.7) is

$$Min\, E(Z) = \frac{\beta}{\alpha}\sum_{i=1}^{3}\sum_{j=1}^{3}x_{ij}$$

Subject to

$$10\sum_{i=1}^{3}x_{ij}+Z_{0.05}\sqrt{10\sum_{i=1}^{3}x_{ij}^{\ 2}}\ \le 70\quad j=1,2,3$$

$$10\sum_{j=1}^{3}x_{ij}+Z_{0.05}\sqrt{10\sum_{j=1}^{3}x_{ij}^{\ 2}}\ \le 80\quad i=1,2,3$$

$$\sum_{i=1}^{3} x_{ij} \geq 2\ ,\ j = 1,2,3\ ;\quad \sum_{j=1}^{3} x_{ij} \geq 4\ ,\quad i = 1,2,3\ ,\quad x_{ij} \geq 0\ \&\ \text{integers}$$

The above NLP is solved using the package LINGO. The optimal solution is obtained as follows, with the minimum objective value as 240:

$x_{11} = 1 \qquad x_{12} = 1 \qquad x_{13} = 2$

$x_{21} = 1 \qquad x_{22} = 2 \qquad x_{23} = 1$

$x_{31} = 0 \qquad x_{32} = 2 \qquad x_{33} = 2$

THE CASE OF NON-IDENTICAL DISTRIBUTIONS

Now let us assume that the job completion times t_{ij} follow non-identical gamma distributions with parameters as given in table 5.1. Also the available times for the various jobs and the upper limits on the man hours from each cluster are given in table (5.1) below.

Note that

a_i = Maximum time units to complete i^{th} job.

b_i = Maximum man hours available from j^{th} cluster.

$x_i^{'}$ = Minimum number of individuals to complete the i^{th} job.

$x_i^{''}$ = Minimum number of individuals to be selected from the j^{th} cluster.

Table 5.1: The value of α_{ij}, β_{ij} and other constants

j		1		2		3			
Cluster $\rightarrow$		K_1		K_2		K_3			
i	Jobs $\downarrow$	α_{i1}	β_{i1}	α_{i2}	β_{i2}	α_{i3}	β_{i3}	(a_i)	$(x_i^{'})$
1	J_1	0.5	3	0.5	3	0.6	5	450	15
2	J_2	0.3	3	0.4	2	0.4	2	490	20
3	J_3	0.4	3	0.5	2	0.5	2	900	12
(b_j)		900		800		500			
$(x_j^{''})$		12		10		15			

The Problem to be solved, obtained from inserting the above parameters in (4.4) is given as

$$Min\ E(Z)=\frac{3}{0.5}\times x_{11}+\frac{3}{0.5}\times x_{12}+\frac{5}{0.6}\times x_{31}+\frac{3}{0.3}\times x_{21}+\frac{2}{0.4}\times x_{22}+\frac{2}{0.4}\times x_{23}+\frac{3}{0.4}\times x_{31}+\frac{2}{0.5}\times x_{32}+\frac{2}{0.5}\times x_{33}$$

Subject to

$$\frac{3}{0.5}\times x_{11}+\frac{3}{0.3}\times x_{21}+\frac{3}{0.4}\times x_{31}+1.96\sqrt{\frac{3}{0.5^2}\times x_{11}^2+\frac{3}{0.3^2}\times x_{21}^2+\frac{3}{0.4^2}\times x_{31}^2}\le 900$$

$$\frac{3}{0.5}\times x_{12}+\frac{2}{0.4}\times x_{22}+\frac{2}{0.5}\times x_{32}+1.96\sqrt{\frac{3}{0.5^2}\times x_{12}^2+\frac{2}{0.4^2}\times x_{22}^2+\frac{2}{0.5^2}\times x_{32}^2}\le 800$$

$$\frac{5}{0.6}\times x_{13}+\frac{5}{0.4}\times x_{23}+\frac{2}{0.5}\times x_{33}+1.96\sqrt{\frac{5}{0.6^2}\times x_{13}^2+\frac{5}{0.4^2}\times x_{23}^2+\frac{2}{0.5^2}\times x_{33}^2}\le 500$$

$$\frac{3}{0.5}\times x_{11}+\frac{3}{0.5}\times x_{12}+\frac{5}{0.6}\times x_{13}+1.96\sqrt{\frac{3}{0.5^2}\times x_{11}^2+\frac{3}{0.5^2}\times x_{12}^2+\frac{5}{0.6^2}\times x_{13}^2}\le 450$$

$$\frac{3}{0.3}\times x_{21}+\frac{2}{0.4}\times x_{22}+\frac{2}{0.4}\times x_{23}+1.96\sqrt{\frac{3}{0.3^2}\times x_{21}^2+\frac{2}{0.4^2}\times x_{22}^2+\frac{2}{0.4^2}\times x_{23}^2}\le 490$$

$$\frac{3}{0.4}\times x_{31}+\frac{2}{0.5}\times x_{32}+\frac{2}{0.5}\times x_{33}+1.96\sqrt{\frac{3}{0.4^2}\times x_{31}^2+\frac{2}{0.5^2}\times x_{32}^2+\frac{2}{0.5^2}\times x_{33}^2}\le 900$$

$x_{11}+x_{21}+x_{31}\ge 25,\quad x_{12}+x_{22}+x_{32}\ge 25,\quad x_{13}+x_{23}+x_{33}\ge 20,\quad x_{11}+x_{12}+x_{13}\ge 25$

$x_{21}+x_{22}+x_{23}\ge 20,\quad x_{31}+x_{32}+x_{33}\ge 25$

The above NLP is solved using the package LINGO. The optimal solution is obtained as follows, with the minimum objective value as 238:

$$\begin{array}{lll} x_{11}=12 & x_{12}=3 & x_{13}=0 \\ x_{21}=0 & x_{22}=17 & x_{23}=3 \\ x_{31}=0 & x_{32}=0 & x_{33}=12 \end{array}$$

6. CONCLUSION

In this paper we have studied the recruitment of personnel to various jobs when the job completion times follow identical Gamma distributions. The best group of persons based on their efficiency in completion of jobs are determined by using cluster analysis technique. The Stochastic formulation of the problem has been brought down to a deterministic *NLPP* through Chance constrained programming technique. The model developed has also been extended to the case of non-identically distributed time variables. The illustrative examples are given for both, identical and non-identical distributions. It is seen that the solutions with minimum completion times are easily obtained.

REFERENCES

[1]. A.Charnes and W.W.Cooper (1959): Chance Constrained Programming, *Management Science*, **6,** 227-243.
[2]. Biswal,M.P.,Biswal,N.P and Li,Duan (1998): Probabilistic linear programming problems with exponential random variables: A technical note , *European Journal of Operational research*, **111**,589-597.
[3]. Gorden,A.D.(1981):*Classification*; Chapman and Hall: London, New York.
[4]. Jeeva, M., Rajalaxmi, R., Charles, V. and Yadavalli, V.S.S. (2004): An application of stochastic programming with Weibull distribution- cluster based optimum allocation of recruitment in manpower planning, *Stochastic Analysis and Application*, **22(3)**, 801-812.

[5]. Leonhard, C.A. and Davis, J.S (1995, March); Job-Shop development model: a case study, *IEEE Software*, **12 (2)**, 86-92.

[6]. Yadavalli, V.SS. , Malada,A.,Charles, V.(2007): Reliability stochastic optimization for an *n*-stage series system with m chance constraints, *South African Journal of Science*, **103**,502-504.

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